Deformation Analysis and Simulation of the Cup-Shaped Flexspline for Harmonic Drive Using in Aerocrafts

Li Zu, Aihua Yin, Yanyang Sun, Y. L. Wang, Yi Ou, and Yi Liang

***Abstract*—The harmonic gears used in servo drive systems of aerocrafts have the features of over loading and working for a short time. They are required to be smaller and lighter. Theoretical research and finite element analysis are given to the deformation characteristic of the flexible gear for the harmonic drive. This paper studies the initial deformation of the flexible gear, and compares with the result of finite element analysis. Overload is applied to the flexible gear, and by using ABAQUS FEA software, the distortions on the flexspline can be found. The research results provide theoretical references for improving load capacity of harmonic drive and designing this type of harmonic drive.**

***Index Terms*—Harmonic drive, flexspline, deformation, overload, finite element analysis.**

I. Introduction

With the continuous development of space technology, servo drive systems of aerocrafts are expected to have characteristics such as compactness, light-in-weight property, high bearing capability, excellent dynamic characteristic and reliability. Applied in servo drive systems, the harmonic gears have superiorities in several terms[1]-[3]: high transmission ratio in a limited space which helps to decrease the system in size and in weight, small backlash which contributes to precision and stability, good performance under an oil-free and high temperature condition.

In a large-loaded short-time running aerocraft, the failure modes of harmonic gears differ from those in conventional conditions. The failure of flexspline performs as break caused by low static strength instead of by fatigue fracture, it also performs as engagement failure caused by excessive interference. Both practical and theoretical research demonstrated that flexspline, which is closely related to the stress state caused by its deformation, is the weakest part of harmonic gear drive. Based on a theoretical research, this study will take the example of cup-shaped harmonic drive device and study the deformation of non-loaded and overloaded flexspline.

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II. Theoretical Analysis of Flexspline Deformation

A cup-shaped flexspline with double wave is jointed to the output shaft by bolts at its bottom shown in Fig. 1. The wave generator consists of an elliptical cam and a flexible bearing.

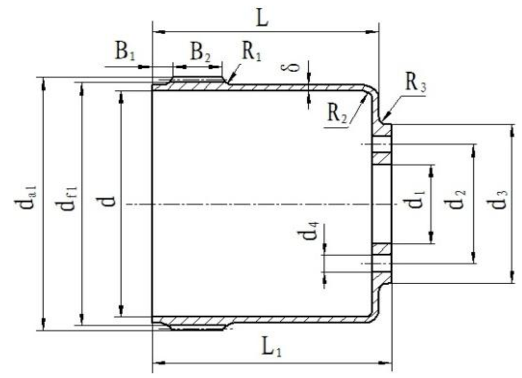

Fig. 1. Structure of flexspline.

A. Initial Deformation of the Flexspline

The initial deformation is the deformation of a non-loaded flexspline when it is assembled with a wave generator. This initial deformation is closely related to the loading capacity of the harmonic gear drive, as well as the form of the wave generator. A wave generator with elliptical cam is suitable for high precision field condition and chosen for this research. It will optimize the engagement between rigid gear and flexspline, which contributes to a good stress distributing state and loading capacity of flexspline.

In the deformation process, the initial curve, which theoretically is the isometric curve of the ellipse, can be presented by radial displacement ω and circumferential displacement υ .To present the relation between β and $\omega_{(\beta)}$, a cosine calculation model as below is used instead of elementary function which could be far more complex [4].

$$\omega_{(\beta)} = \omega_0 \cos 2\beta$$

ω_0 is an important parameter for harmonic gear drive design, which is the maximum radial deformation. β is the rotating angle. With this model, the radial deformations are identical on both macro and minor axis. Fig. 2 compares the radial displacements obtained from this model with those from theoretical model.

According to Fig. 2, these two curves correspond well with each other in the interval of (45°, 135°) and (225°, 315°). When $\beta = 90°$, significant errors appear with a max of 0.006mm (1%), which shows that errors augment around the

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minor axis. It should be stated that if the circular spline is manufactured at a 6 class precision in basic shaft system, its outer diameter tolerance is then 0.019mm which is far greater than the maximum error between the blue and red lines.

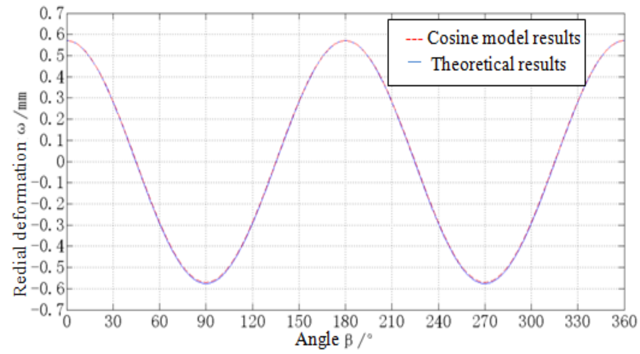


Fig. 2. Radial displacements obtained from two methods.

The cosine calculation model is such simple that the maximum radial deformation appears in minor axis and macro axis direction while the maximum axial deformation, half of the radical deformation, appears in the direction at a 45° angle with the minor or macro axis. The cup part is assumed to have constantly straight generatrix and round cup bottom. Based on the half no-moment theory of cylindrical shell, (1) can be obtained to describe the radial axial and circumferential deformation [5].

$$\begin{cases} \omega_{(\beta,z)} = \dfrac{z}{l}\omega_0 \cos 2\beta \\ \upsilon_{(\beta,z)} = -\dfrac{z}{2l}\omega_0 \sin 2\beta \end{cases} \tag{1}$$

z in (1) is the distance between the studied section and flexspline cup bottom; l is the distance between the loaded plane and cup bottom. It is showed in (1) that ω and υ change in a linear fashion along the axis direction.

B. Deformation of the Flexspline When Bearing Load

Distortion shows in Fig. 3 arises when the flexspline is under load. The flexspline approaches the wave generator in the regions AB and $A'B'$ while it detaches itself from the generator in AB' and $A'B$. Such distortion is caused by flexibility of the wave generator, clearances between different components and extension of the flexspline.

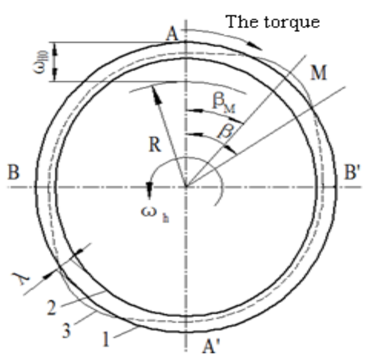


Fig. 3. Flexspline distortion.

The wave generator rotates counterclockwise at a speed of ω_h while the torque is in the clockwise direction. Points A and A' are on the macro axis, B and B' are on the minor axis and M is the point where appears the maximum distortion. R stands for the radius of flexspline midline before deformation and λ stands for clearances between components.

Line 1 represents the initial shape of flexspline in the ideal situation where it has no clearance or deformation. The corresponding nominal radical displacement and circumferential deformation are ω_H and υ_H respectively. Line 2 represents the outline of the wave generator when the clearance λ exists evenly along the contour. Lastly, the line 3 is the initial curve of the flexspline as it is under load.

C. Deformations in the Region AB [5]

In this region, deformations can be presented as below, where υ_p represents the circumferential deformation caused by flexspline extension. λ_{K0} is the radical displacement when $\beta = 0^\circ$.

$$\begin{cases} \omega = \omega_H - \lambda - \lambda_{k0}\cos\beta \\ \upsilon = \upsilon_H + \lambda\beta + \lambda_{k0}\sin\beta + \upsilon_p \end{cases} \tag{2}$$

1) Deformations in the region AB'

AM is approximately a constant radius arc. ω and υ are given, respectively, as

$$\begin{cases} \omega = \omega_{H0} - \lambda - \lambda_{k0} \\ \upsilon = -(\omega_{H0} - \lambda - \lambda_{k0})\beta \end{cases} \tag{3}$$

In the region MB', deformations are more complex:

$$\begin{cases} \omega = (\omega_{H0} - \lambda - \lambda_{k0})\left[B_1(\frac{\pi}{2} - \beta)\cos\beta + B_2\sin\beta + C\right] \\ \upsilon = -(\omega_{H0} - \lambda - \lambda_{k0})\left[B_1(\frac{\pi}{2} - \beta)\sin\beta - (B_1 + B_2)\cos\beta + C\beta + C_1\right] \end{cases} \tag{4}$$

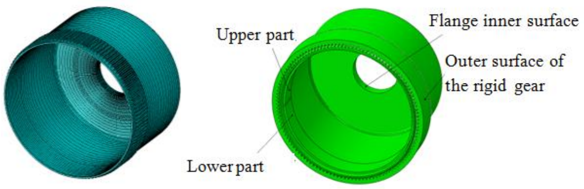
Parameters B_1, B_2, C, C_1, λ_{K0}, λ and υ_p are complex. As it is not practical to get accurate results out from (2) (3) and (4), finite elements software can be used.

III. Finite Element Analysis of Flexspline Deformations

Theoretical solutions have been given previously. However, more factors should be taken in to consideration in practical use, which can be simulated and analyzed by finite element software.

A. Modeling

The wave generator is simplified as a rigid thin-wall ring whose inner and outer surfaces are both equidistant curves of the elliptical cam wave generator. The wave generator is 'cut into' two halves, an upper part and a lower part, which move respectively upwards and downwards for a distance. This helps to simulate the process of wave generator being installed into the flexspline. A torque is applied on the inner surface of the flange to simulate the load on flexspline. The finite element models are shown in Fig. 4.

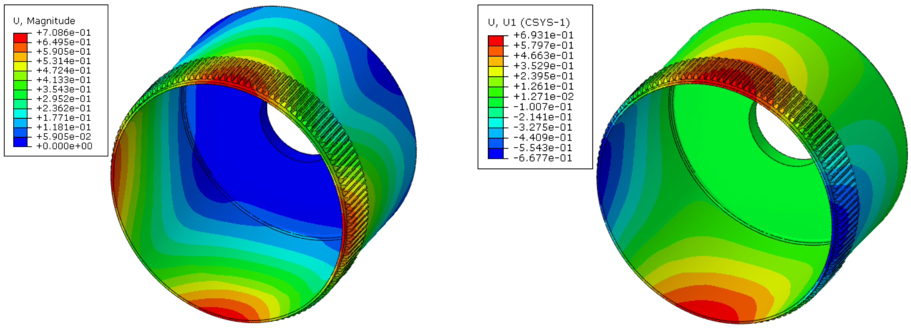


(a) Flexspline (b) Assembly drawing

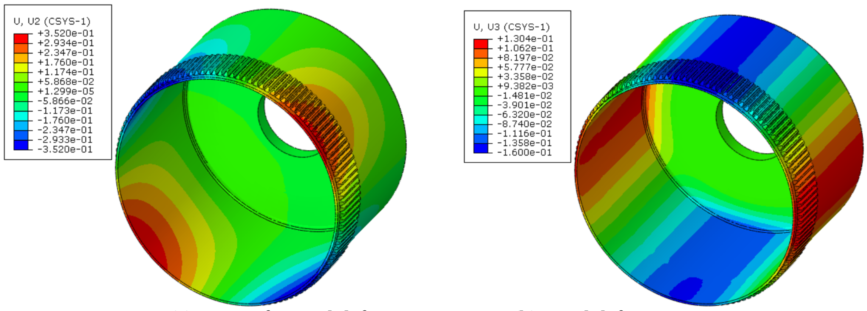
Fig. 4. Finite element models of harmonic drive.

B. Initial Deformation of the Flexspline

Fig. 5 presents the flexspline deformation after the wave generator's installation. When flexspline is non-loaded, the maximum radical deformation appears on the macro and minor axes of the wave generator; the maximum circumferential deformation, about half the large of the maximum radical deformation, is at an angle of 45 degrees with the macro/minor axis; the axial deformation is changeless along the flexspline axis. These results correspond well with (1).

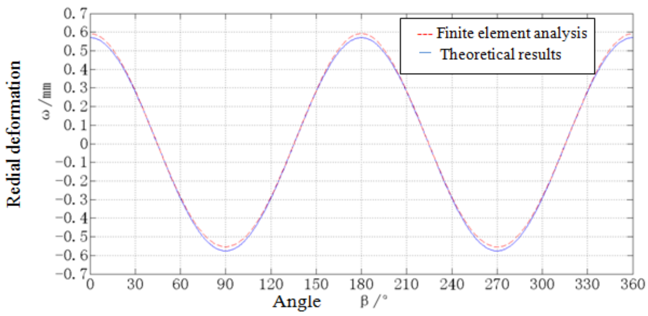


(a) Flexspline deformation (b) Radial deformation

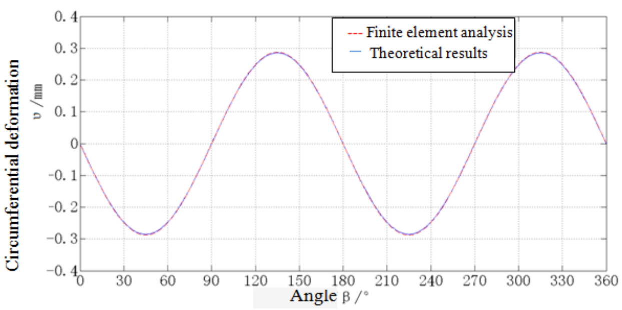


(a) Circumferential deformation (b) Axial deformation

Fig. 5. Pictures of flexspline initial deformation.



(a) Redial deformation



(b) Circumferential deformation.

Fig. 6. Comparison results of theoretical research and finite element analysis.

Taking loaded section of the flexspline as analysis object, where $z = l$, different deformations can be theoretically. Fig. 6 compares the results of theoretical research and finite element analysis.

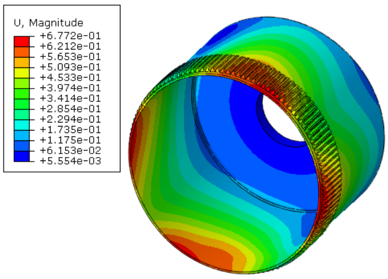
It is shown in Fig. 6(a) that there are certain errors around the macro and minor axes while the rest parts present very well. The theoretical macro-axis deformation is $\omega = \omega_0 = 0.57mm$, while result of finite element analysis is 0.5921mm with an error of 3.9%. The theoretical minor-axis deformation is $\omega_{(90^\circ)} = -0.57mm$ comparing to finite element analysis result -0.5534mm with an error of 2.9%. The cause is that wave generator is assumed to be closely attached to the flexspline inner wall to calculate the theoretical primitive curve. However, the width of the generator cannot be ignored in reality. Because of the limit from cup bottom, deformations along axes are not even so that the generatrix inclines. There is a distance between the inner wall of the flexspline and the bearing outer ring, which verifies with β .

Fig. 6(b) compares the circumferential deformation given by theoretical research and finite element analysis. These two results tally well with each other, except a certain errors at 45 degrees from macro/minor axis with a maximum error less than 0.6%.

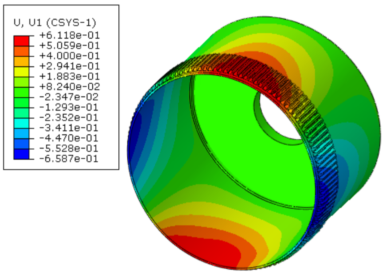
As the errors between two methods are within an acceptable range and the deformations correspond well, (1) can be taken as the deformation regularity of the flexspline loaded section.

C. Deformation of Flexspline When Bearing Load

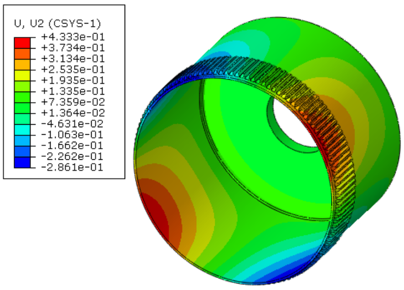
The chosen flexspline has an inner diameter of ϕ72mm and a rated load of 90Nm, which is suitable for long-time running harmonic gear drive. However, for high-loaded short-time running ones, for example aerocrafts, the real load is usually greater than rated load. So a torque T=250Nm, which is 3 times the rated load. Pictures of loaded flexspline deformation are shown in Fig. 7.



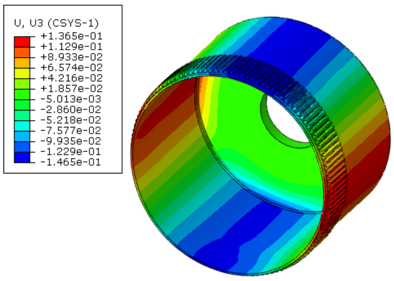
(a) Flexspline deformation.



(b) Radial deformation.



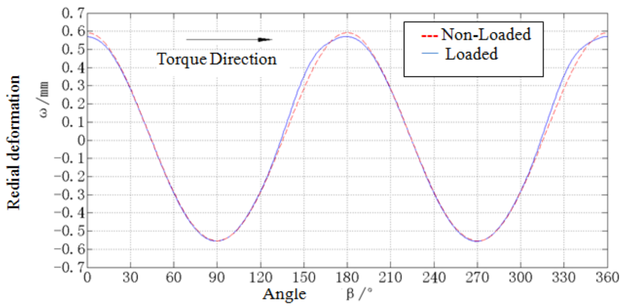
(a) Circumferential deformation



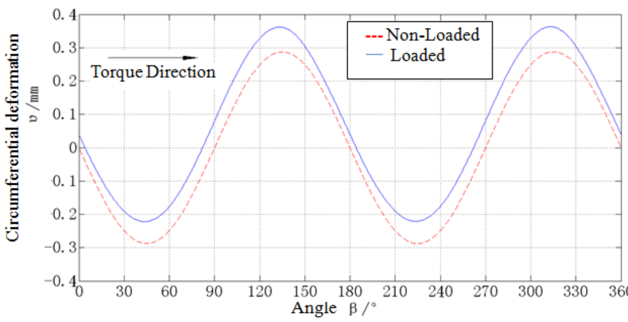
(b) Axial deformation

Fig. 7. Pictures of loaded flexspline deformation.

To study the deformation change of flexspline on its loaded section, Fig. 8 presents the radical and circumferential deformations in both loaded (represented by red lines) and non-loaded (represented by blue lines) situation.



(a) Redial deformation



(b) Circumferential deformation

Fig. 8. Comparison results of non-loaded and loaded deformation of flexspline.

In Fig. 8(a), the blue line moves to the left side of the red line in the region of β =130°-170°. The reason is that the radical deformation here gets bigger under load and the flexspline detaches itself from the wave generator, which is called distortion as mentioned before.

Fig. 8(b) shows that the blue line moves up a little from the red one. Since the positive direction of υ is the same as the direction of the torque, then it can be concluded that the flexspline warps due to the torque, no matter for its rim or bottom, they all distort a certain angle.

In summary, when flexspline is under load, distortion appears so that the inner wall of the flexspline is closely attached to the wave generator in the engaging-in region and the radical deformation decreases. In the contrary, the flexspline is detached form the wave generator in the engaging-out region and the radical deformation increases.

The cup part turns a certain angle leading the circumferential deformation curve to move up a little from the one unloaded.

IV. Conclusion

Harmonic drive used in aerocrafts has features of short running time and large load. The initial deformation is related to stress distribution and load capacity. This paper studies the deformations of a cup-shaped flexspline in the harmonic drive, which lead to the following research results:

1) By analysis of the initial deformation of a harmonic gear drive equipped with an elliptical cam wave generator, a cosine calculation model is used to present the theoretical deformation of a flexspline, errors are within the tolerances of element parts.
2) By using finite element software to simulate the harmonic drive assembly, deformation analysis of the unloaded flexspline can be done. According to the comparison between theoretical study and finite element analysis, results of these two methods are found to be

similar, and the theoretical study is proved to be correct.

3) Finite element analysis of a short-time running flexspline which is loaded by 3 times of the rated load has been done. It shows that distortion arises as the flexspline is under load. The deformation augments in the engaging-out region and forms a hunch-up.

This paper could provide theoretical references for the study of flexspline for harmonic drive using in aerocrafts, also for the optimization design of related parts.

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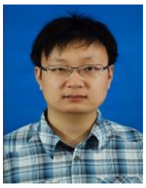

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