Optimizing Type-I (α) and Type-II (β) Error Probabilities by Game-Theoretic Linear Programming for Sequential Sampling Plans in Quality Control

Mehmet Sahinoglu and Sedat Capar

***Abstract*—A critical step in hypothesis testing at the computer theory and/or engineering decision-making stage is to optimally compute and use type-I (α) and type-II (β) error probabilities. The article's first research objective is to optimize α and β errors, or producer's and consumer's risks, or risks of false positives (*FP*) and false negatives (*FN*) by employing the merits of a game-theoretical framework. To achieve this goal, the cross-products of errors and non-errors model is proposed. The second objective is to apply the proposed model to an industrial manufacturing quality control mechanism, i.e. sequential sampling plans (*SSP*). The article proposes an alternative technique compared to prematurely selecting the conventionally pre-specified type-I and type-II error probabilities. One studies mixed strategy, two-players and zero-sum games' minimax rule derived by von Neumann and executed by Dantzig's linear programming (*LP*) algorithm. Further, one equation for one unknown scenario yielding simple algebraic roots validate the computationally-intensive *LP* optimal solutions. The cost and utility constants are elicited through company-specific input data management. The contrasts between conventional and proposed results are favorably illustrated by tables, figures, individual and comparative plots, and Venn diagrams in order to modify and improve the traditionally executed *SSP*'s final decisions.**

***Index Terms*—Cross-products of errors, minimax rule, accept-reject-continue-terminate, cost and utility.**

I. Introduction

A. Motivations of Research Proposal and Outputs

1) The primary innovative motivation behind this article lies in optimizing *type-I* and *type-II* error probabilities, α and β, using a game-theoretic computationally intensive *LP* algorithm to improve the accuracy and credibility of statistical hypothesis testing outcomes in today's quality control-conscious and information technology-savvy world. This is an alternative to the traditional assumptions of α and β, with no prior data-centric knowledge about hypothesis tests governing any design process.

2) The secondary motivation is to introduce and implement the hereby optimized *alpha* (*producer's*) and *beta* (*consumer's*) risks by employing the business costs and utility input data to item-by-item sequential sampling plans in industrial quality control. The goal is to aim for company-specific and data-centric quality-control inspection rather than heuristic or predetermined. This article, therefore, proposes an alternative to assuming ubiquitous producer's (e.g., α=0.05) and consumer's risk values (e.g., β=0.10). The implementation interface to *SSP* is achieved through case studies and input data elicitation by user-friendly, easy-to-reproduce software algorithms with satisfactory outcomes.

B. Literature Survey for Type I & II Errors, Game Theory

Aside from the usual rule-of-thumb or best-guess or judgment-call-based choices of such as 1-out-of-20 or 1-out-of-50 etc., there have been alternative attempts to compute *alpha* (*type-I* error probability) by deriving the first and second derivatives of the standard *Normal* distribution curve. This is performed by determining the second derivative to reach a maximum at $z = \pm 1.732$ which corresponds to a *p*-value of 0.083. An alternative approach has been to find a point where the concavity in the *Normal* distribution curve is maximal to the first derivative. That is, the maximal curvature $k(z)$ occurs when $z = \pm 1.749$ corresponding to a *p*-value of 0.08. The *p*-value is used to reject H_0 for a given *alpha*. The calculus-based algebraic approaches have been earlier studied by Kelley [1] and Grant [2]. As Kelley quoted [1], "*No one therefore has come up with an objective statistically based reasoning behind choosing the now ubiquitous 5% level, although there are objective reasons for levels above and below it. And no one is forcing us to choose 5% either.*" The history of *type-I* and *type-II* errors goes back to Neyman and Pearson [3] who discovered the problems associated with deciding whether or not a particular sample may be judged as likely to have been drawn from a specific population. They identified two sources of errors, *type-I* (α) and *type-II* (β). They observed that "*... If the probability of obtaining a result as extreme as the one obtained, supposing that the null hypothesis were true, is lower than a pre-specified cut-off probability* (*for example, 5%*)*, then the result is said to be statistically significant and the null hypothesis is rejected.*" Fisher [4] proposed the level *P*=0.05 as a limit of statistical significance where he also originated in his book: "*The value for which P=0.05 or 1 in 20 is 1.96 or nearly 2: it is convenient to take this point as a limit in judging whether a deviation is to be considered significant or not.*" A prominent aspect of Neyman and Pearson's [3] and Fisher's [5] findings was that one never fully justified, or rigorously proved, as to why, e.g. *P*=0.05 or else was selected as a pre-specified cut-off probability. Over a century of *alpha* and *beta* error–related discussions, e.g. by Salkind [6] who wrote that no game-theory was recorded in plain hypothesis testing, and also by Hedberg [7] who recorded that the central theme for *Type-II* error, or the *power,* (1-β), revolves around a symbolic value of β=0.2, as in the SAGE Research: "…*The convention*

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of the social sciences is to design studies with a power of at least 80% chance of detecting an effect…"

Game theory is a branch of mathematical sciences devoted to the logic of decision-making in societal, physical or managerial interactions, and concerns the behavior of decision-makers who influence each other for optimal resource allocations at times subject to budget constraints to maximize utility as studied by Sahinoglu *et al.* [8], Szidarovszky and Luo [9]. The statistical decision theory is a one-person game theory. The *LP* system of equations will optimize *producer's* and *consumer's* risks by two-player, zero-sum and mixed-strategy-based minimax rule by von Neumann [10] and von Neumann *et al.* [11] in 1928 and 1944 respectively. A similar algorithm was adopted in two proceedings by Sahinoglu *et al.* [12], [13] and a monogram [14] and a textbook by Sahinoglu [15]. The effort continued while minimizing *COLLOSS* (Column Loss) in the *Eco-Risk* article, and *Oil-Drilling Spill Risk*-themed article, respectively, in Sahinoglu *et al.* [16], [17]. Next comes what lies behind the *LP* problem by Dantzig [18]. The forward and backward proofs of a general representation theorem (*GRT*) are given by Lewis [19]. Introduction of game theory to risk analysis is by Cox [20].

C. Summary of Sections I to VI

After introduction of goals, outcomes, and an extensive literature survey in section I, the section II studies the game theory-linked *LP* methods to achieve the itemized goals via the *cross-products* of risks and non-risks with related definitions. Section III studies example 1 for statistical sequential sampling plans with an input data management scheme at large. Section IV details example 1 through a thematic Venn diagram in probabilistic terms. Section V verifies and justifies the proposed optimal method via the short-cut algebraic roots to lead to a simple computational procedure. Examples 2 and 3 are added in section V by diversifying the *LOSS* variables to show the game-theoretic *LP* method's input data flexibility. Section VI conclusively summarizes the content with further research suggestions. It is time to compare different analytical approaches as follow.

II. Cross-Products of Errors with Cases

The *cross-products* of errors and non-errors will be proposed and utilized to construct the *LP* algorithm to apply to statistical sequential sampling plans employed in industrial quality control.

A. LP Algorithm with Composite-, Partial- and Non-Risk Errors' Cross-Products

The issue with the classical approaches to hypothesis testing in terms of *alpha* and *beta* errors is that the hand-picked ubiquitous assumptions such as α=0.01 or 0.05, and β=0.10 or 0.20 may be detached from the prevalent data-centric sources. Costs or utility (profit) associated with varying error values: (α and β), or non-error values: (1-α, and 1-β) and their *cross-products*: [$\alpha\ \beta$], [α (1-β)], [(1- α) β] and [(1-α) (1-β)] in the form of partial producer's or consumer's risks, or both, or none, are not hitherto considered. No such errors may have incurred with no whatsoever financial loss for the producers and consumers with a complete market satisfaction due to the error-free pair: (1-β) and (1-α). A common routine as Neyman and Pearson and Fisher practiced, is to select *type-I* error probability (*alpha*) by an existing best-judgement call for H_0, and then, given an alternative set of H_a values, to compute a set of *type-II* error probabilities (*beta*).

Note a critical detail here is such that the utility is a negative cost effect working versus the positive cost effect in the opposite direction, or vice-versa. For cost and utility concepts, which date back to Nicholas Bernoulli, a good argument is laid by Singpurwalla and Wilson [21]. However, game-theoretic *LP* methods have not been studied in hypothesis testing educational curricula. This is mainly because the applications to routine hypothesis tests with pertinent costs associated with *type-I* (α) and *type-II* (β) errors and their *cross products*, including utility or profit with respect to non-errors (i.e. *confidence*=1-α, and *power* =1-β), are not up to date rigorously formulated. In hypothesis testing, this article associates a variety of costs (income lost due to errors) or a utility (revenue profited due to non-error) and observes where the optimal α and β will unfold by employing the basic principles of the game-theoretic minimax rule. This is an alternative computational technique to the usual rule-of-thumb choices, such as $\alpha \approx 0.05$ or $\alpha \approx 0.08$ etc. or those by calculus algebra, as critiqued by Kelley [1] and Grant [2]. The proposed empirical and market-friendlier way is an objective approach compared to the previous subjective rule-of-thumb lacking cost and utility inputs. The hypothesis testing literature as in, e.g. Ostle and Mensing [22], lays two types of errors in Table I and Fig. 1:

1) Type-I error: This is when the analyst rejects a true null hypothesis. The probability of a *type-I* error is α, the significance level; also known as *producer's risk* or *false positive* risk when H_0: Good quality versus H_a: Bad quality.

2) Type-*II* error: This is when the analyst fails to reject a false null hypothesis for an identical hypothesis as above, i.e. H_0 vs. H_a. The probability of committing a *Type-II* error, β, is also known as *consumer's risk or false negative* risk.

The *truth (reality) vs decision (test)* of traditional hypothesis testing is conventionally framed as follows:

TABLE I: *Truth (Reality) vs Decision (Test)* Elements of Test

	Decision	
Truth	Reject Ho	Accept Ho
True Ho	Producer's Risk=α error=*FP*	No Error=*Confidence*=1-α=*TN*
False Ho	No Error=*Power*=1-β=*TP*	Consumer's Risk=β error=*FN*

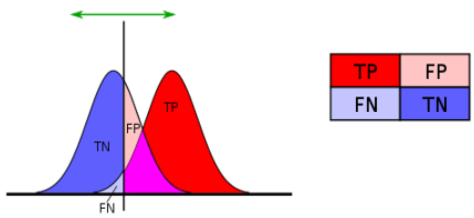


Fig. 1. Hypothesis testing plots of *type-I* (α) and *II* (β) errors by Neyman and Pearson [3] and Fisher [4], [5]; *false positives* (α=*FP*), *false negatives* (β=*FN*), *true positives* (*TP*) and *true negatives* (*TN*) according to Table I.

A. System of Equations to Optimize Type-I & Type-II Error Probabilities

The following subsections will demonstrate the setup of a *LP* system of equations to optimize *type-I* (α) and *type-II* (β) errors, i.e. producer's and consumer's risks, via the game theory as follow:

$$\alpha = P\ \{Type\text{-}I \text{ error} = P\{\text{reject } H_0 | H_0 \text{ is true} \quad (1)$$

$$\beta = P\ \{Type\text{-}II \text{ error}\} = P\{\text{fail to reject } H_0 | H_0 \text{ is false}\} \quad (2)$$

The probability of not committing *Type-I* error and *Type-II* errors are defined as *confidence* or test *specifity*, and *power* or test *sensitivity*, all respectively. The *power* is given in (3).

$$\text{Power} = (1\text{-}\beta) = P\ \{\text{reject } H_0 |\ H_0 \text{ is false}\} \quad (3)$$

Sharma *et al*. extensively studied these two test concepts, known as test *specifity* and test *sensitivity* [23].

The *power* of hypothesis testing is also represented as [1-$\beta(\Theta)$], where Θ denotes the true parameter value, e.g., population mean: μ or population proportion: *P*. The $\beta(\Theta)$, the complement of *power*, is known as the operating characteristics (*OC*) function used in quality control. The *cross-products* of errors and non-errors will be coupled with their associated costs. If the cost is negative, this denotes utility. Let $P_{11} = \alpha\beta$, $P_{12} = \alpha(1\text{-}\beta)$, $P_{21} = (1\text{-}\alpha)\beta$, $P_{22} = (1\text{-}\alpha)(1\text{-}\beta)$ where $\alpha = P_{11} + P_{12}$ and $\beta = P_{11} + P_{21}$. Note, C_{11}, C_{12}, and C_{21} are the corresponding costs due to *cross-products* of errors. C_{22} is the constant due to non-errors while the *cross-products* in (4) sums to unity. Let $\alpha = .05$, $\beta = .10$ such that *Confidence* = 1-α = 1-.05 = .95, and *Power* = 1-β = 1-.10 = .90. Then it follows:

$$\{\alpha\beta\}+\{\alpha(1\text{-}\beta)\}+\{(1\text{-}\alpha)\beta\}+\{(1\text{-}\alpha)(1\text{-}\beta)\}=1;\ 0<\alpha, \beta<1 \quad (4)$$

$$FP(=\alpha) + TN(=1\text{-}\alpha) + FN(=\beta)+TP(=1\text{-}\beta) = 2;\ 0<\alpha, \beta<1 \quad (5)$$

The *cross-products* of cubicles from Table I sums to unity in (4) as follows here: (.05)(10)+(.05)(.9)+(.10)(.95) +(.95)(.9)=.005+ 045+.095+.855 = 1, whereas (5) yields 2. Let the *cross-products* obtained via Table I in (6) to (9) to be:

$$\text{Composite riskiness } (CoR) = P_{11} = \alpha\beta \quad (6)$$

$$\text{Partial riskiness } (PR_I) \text{ due to purely } \alpha \text{ error} = P_{12} = \alpha(1\text{-}\beta) \quad (7)$$

$$\text{Partial riskiness } (PR_{II}) \text{ due to purely } \beta \text{ error} = P_{21} = (1\text{-}\alpha)\beta \quad (8)$$

$$\text{Composite non-riskiness } (CoNR) = P_{22} = (1\text{-}\alpha)(1\text{-}\beta) \quad (9)$$

Nonlinear implies not necessarily linear but includes such functions by Rapcsak [24] for a smooth optimization.

III. SSP and Quantitative Example 1

The entwined goals of this article are set (*i*) to optimize *alpha* and *beta* errors by von Neumann's *LP*-enabled minimax rule to the statistical hypothesis testing of *good quality* of a given lot versus *bad quality*, and (*ii*) to apply the preceeding approach to an attributes-type item-by-item *SSP*.

The test-statistics algorithm in a sequential sampling plan is different than a single-stage sampling by Montgomery [25] and Jamkhaneh and Gildeh [26] such that:

i) When plotted points stay within the limiting boundaries of a single-stage sampling plan, i.e. *AQL* (Acceptable Quality Level) and *RQL* (Rejectable Quality Level), *continue-sampling* decision takes over and another sample must be drawn for continued testing.

ii) When plotted points fall on or above the upper limiting level, *RQL*, the lot is *rejected*.

iii) When plotted points fall on or below the lower limiting level, *AQL*, the lot is *accepted*.

iv) When a threshold sample size (=3n) is reached, and no accept or reject action taken, and continue-sampling prevails, terminate.

Gaus *et al*. [27] rather than making an absolute decision of *accept* or *reject*, refer to lot acceptance/rejection with a confidence interval. However, statistically, a more popular approach is a sequential sampling plan when the analyst keeps testing items from the batch (or lot) and render a decision to either i) *continue sampling* after each item is inspected, ii) *reject*, or iii) *accept*, or iv) finally terminate SSP. The distinction of multiple sampling from *SSP* is that the maximum number of samples for *SSP* is prespecified. With sequential sampling, one could end up conducting 100% inspection on the entire batch. The *SSP* are truncated after the number inspected reaches three times the count with single sampling plan by Beasley [28]. See Theorem 1: Sequential Probability Ratio Test (*SPRT*) under Wald's lemma [29] for the SSP and by Roussas [30]. Graphically, *SSP* can be plotted in Fig. 5 to 9 in subsections III.B and III.C where the cumulative sample size is *n* and the cumulative number of defects is *X* in the Engineering Statistics Handbook by *NIST* [31] and textbook by Montgomery [25].

Acceptance (single-stage) sampling plans were historically first proposed by Dodge and Romig [32]. The *producer's* and *consumer's* risks occur due to the draw of an unrepresentative sample for either wrongly rejecting a lot containing an acceptably small amount of detectives, or accepting a lot containing an unacceptably large amount of detectives, respectively. Here, single-stage acceptance plans are not in focus, but *SSP* are. Briefly, *type-I* error of the *producer's* risk (5% is common) is the probability of rejecting a good lot or batch. *Type-II* error of the *consumer's* risk (10% is typical), whereas, is the probability of not-rejecting a bad lot. The 5% (=α) or 10% (=β) cited are subjective takes per standard assumptions adopted by MIL-STD-1916 [33].

A. Example 1: Input Data Management of Cost and Utility Constants, and LOSS Variable

Take an illustrative example 1 regarding a hypothetical *EAP* (Electric Auto Production) plant as follows with *SSP* on attributes. Critically embedded chips for electric cars are purchased on item-by-item sampling with *n*(sample size)=100 per batch delivery. Let p_1=*AQL*=Acceptable Quality Limit=.01, and p_2=*RQL*=Rejectable Quality Limit=.10 with α=.05 and β=.1 for *producer's* and *consumer's* risks to revisit in section III.B.

How to collect the $C_{ij} = [C_{11}, C_{12}, C_{21}, C_{22}]$ costs, or the input constants by an enterprise, poses several challenging limitations. From the corporate world's sales accounting logs about this hypothetical example 1, this article suggests practical ways to meet the input data demand challenges. Many of the larger merchandisers will break the returns down into four distinct groups, according to a detailed accounting analysis by Hoare [34] as follows in random order.

1) C_{12}*:* This first group reflects solely the consumer's faults or customer-based mistakes. This is attributed to *producer's risk* experienced by the producer due to an accrued financial loss, $\$C_{12}$. As a merchandiser, one needs to monitor the growth rate for this group. If this unwanted trend begins to rise, it might be a sign that the sales staff is unethically forcing the wrong product onto the market, hence ending up with the consumers who are clueless of what they actually purchased and how best to use it.

2) C_{21}: The other form of a return is a type of merchandise that is broken, or has a quality issue. That is, it's not the consumer's fault but that of the producer. This is attributed to the *consumer's risk* experienced by the consumer due to an accrued financial loss, $\$C_{21}$. If the issue is brand-related, the producer or manufacturer may consider discontinuing the brand to substitute it with a higher quality product. Popular examples are recall actions in the automotive industry. Class-actions favoring the consumers have recently become a commonplace event.

3) C_{11}: The two adjustments above in the business world are followed by another elusively described item as allowance, discount or incentive, or occasionally a write-off. These vague adjustments to normal sales reflecting defective items or courtesy calls for failure in delivering the product or service in a timely fashion cause issues. Re-education of the sales representatives may be required if customers' erroneous returns increase since this relates to a wrong kind of purchase. Consumers may not be educated for what they purchased. They claim, the product is defective but it truly is not. This is both *consumer's* (β) and *producer's* (α) risks merged and compounded, bearing an accrued financial loss, $\$C_{11}$.

4) C_{22}*:* This is the uncontested utility, not returned, with 100% customer satisfaction and no serious issues intercepted.

Revisiting the *EAP*-themed example 1 with Hoare [34] taken as a guide, where *Total Sales* subtracted by *Adjustments (Returns + Allowances + Discounts)* denotes the *Net + Other Sales*. Expressed otherwise, let's define elements as follow:

SUM{*Cij*} = Total Sales: $X,XXX,XXX.

C_{12} = Customer-based returns (due to consumer's unjustified faults): $XX,XXX.

C_{21} = Producer-based returns (such as recalls due to company-generated faults): $XX,XXX.

C_{11} = Allowances and Discounts (such as write-offs released by the company after an arbitration process, or else, in case the court case costs more for the ambiguous and non-explicit issues due to consumer bad-debt or vendor's partially unusable bad merchandise, which may not be worth extra re-shelving or re-stocking costs): $X,XXX.

C_{22} = Net Sales (trouble-free) + other revenues, like insurance or warranty agreements: $XXX, where this utility quantity when input into the game-testing software of Table III, is taken as a negative cost (since it is a utility): -$ [XXX, XXX + XXX].

Covering *LOSS* = $5*K* (or $3*K*) in example 1, the following arguments are in place: If the *LOSS* variable constraint is taken as –*LOSS* ≤-$5*K* (or $3*K*) or *LOSS*≥$5*K* (or $3*K*); *LOSS* denotes a tolerance and a company-sponsored minimum indemnity to intercept the damage incurred after deductibles due to each of the risk-related constraints per equations (12) to (15). $P_{ij}*C_{ij}<LOSS$ for *i,j*=1,2 where each of these four constraints are bound not to exceed *LOSS*=$5*K* (or $3*K*), including equation (15) where the utility constant readily obeys. *LOSS*, akin to a company-paid compensation, is a variable different than $C_{ij.}$ The *LOSS* variable is minimized by the *LP*'s objective function of *Min LOSS*.

B. Proposed Method Applied to Attributes-Type Sequential Sampling in Example 1

To recap, let the *Cij* vector to be the most recent averages from the *Electric Automobile Production* (*EAP*) plant of subsection III-A as a set of hypothetical input data:

Total Sales Revenue = $1,000,000 or $1,000*K* where *K*=1,000.

No Adjustments Sale = $800*K*. Uncontested and suspicion-free non-returned income.

Adjustments Returns = $150*K*. Revenue lost from producers' wrong-doings (*consumer's* risk) and consumers' non- compliance (*producer's* risk) are broken down:

Customer-Based = $110*K*. Consumer's noncompliance may cause civil penalties.

Producer-Based = $40*K*. Producer's wrongdoing causes, e.g., recall or class actions.

Allowances or Write-Offs = $50*K*. Consumer's non-adherence can overlap with producer's errors leading to an inseparable blend of *producer's* and *consumer's* risks, yielding arbitration.

Based on this breakdown, one examines what kind of a *SSP* which the *CFO* (Company Financial Officer) in charge of managerial finances, will risk while the *EAP* operates optimally lucrative. **S**elect as preceded, C_{ij} = [C_{11}=$50*K*, C_{12}=$110*K*, C_{21}=$40*K*, C_{22} =-$800*K*] for *EAP*'s input set is used in Tables II to VIII. The *EAP* case study uses *LOSS*≥$3*K* or *LOSS*≥$5*K* after deductibles akin to a company's indemnities to meet any unexpected or emergency rainy-day funds. Table II displays the action-loss based game-theoretic *LP* formulation of the *SSP*:

TABLE II: EXPECTED LOSSES (EL) FOR ACTIONS TAKEN BY PLAYER1 (CONSUMER) INCURRED UPON PLAYER2 (PRODUCER)

Actions Taken by Player1 (Return Policy)	***EL* for action a_i given C_{ij} incurred on Player2**
a_1(Actn 1: Ambigious Fault-based Rtrn)	$EL(a_1) = \$ P_{11}C_{11} \leq LOSS$
a_2 (Actn 2: Consumer's Fault-based Rtrn)	$EL(a_2) = \$ P_{12}C_{12} \leq LOSS$
a_3 (Actn 3: Producer's Fault-based Rtrn)	$EL(a_3) = \$ P_{21}C_{21} \leq LOSS$
a_4 (Actn 4: No Fault No Rtrn Ideal Sale)	$EL(a_4) = \$ P_{22}C_{22} \leq LOSS$

Table II shows how Player2 can find its optimal mixed strategy. The goal here is to calculate probabilities P_{ij} to minimize the expected loss in the *SSP* process incurred upon Player2 regardless of the strategy executed by Player1. In essence, Player2 will protect itself from any strategy selected by Player1 by making sure its expected market loss is as small as possible even if Player1 selects its own optimal strategy to maximize gain. Given the probabilities, P_{ij} for i, j = 1,2 and the expected losses in Table II, the game theory assumes that Player1 will select a strategy that causes the maximum expected loss incurred upon Player2 based on equation (10):

$$Max\ \{EL(a_1), EL(a_2), EL(a_3), EL(a_4)\} \quad (10)$$

However, when Player1 selects its strategy, the value of the game will be the expected maximum gain such that this will maximize Player2's expected loss as well. On the other hand, Player2 will select its optimal mixed strategy using a *minimax strategy* to minimize the maximum expected loss based on (11) to yield the objective function by Anderson *et al.* in [35]:

$$Min\ [Max\ \{EL(a_1), EL(a_2), EL(a_3), EL(a_4)\}] \quad (11)$$

Finally, (11) identifies the Neumann's *MINIMAX* rule. In case the players are reversed, and *GAIN* replaces *LOSS*. *T*hen the *MAXIMIN* rule will replace the *MINIMAX* rule. The *LP* system of equations are governed by an objective function. The following spreadsheets show the input and output with an *LP* algorithm, whereas (19) denoting total cost ($) units accrued is constrained for a maximum net profit. If the *LOSS* variable is as such: $LOSS \geq \$5$ with (12) to (15) and (17); one completes the *LP* system of equations given the binding constraints to minimize the objective function of *Min LOSS (or MAX GAIN)* subject to constraints of (12) to (19) with a solution vector $P_{ij} = [P_{11}, P_{12}, P_{21}, P_{22}]$, *LOSS* variable, $C_{ij} = [C_{11}, C_{12}, C_{21}, C_{22}]$ as inspired by Table II:

$$P_{11}\,C_{11} - LOSS \leq 0 \quad (12)$$

$$P_{12}\,C_{12} - LOSS \leq 0 \quad (13)$$

$$P_{21}\,C_{21} - LOSS \leq 0 \quad (14)$$

$$P_{22}\,C_{22} - LOSS \leq 0 \quad (15)$$

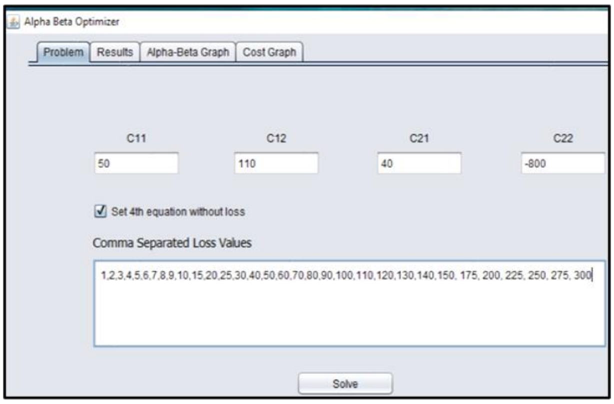
$$0 \leq \text{Pij} \leq 1,\ \text{i,j}=1, 2 \quad (16)$$

$$LOSS \geq LOSS_{\text{min}} \quad (17)$$

$$P_{11} + P_{12} + P_{21} + P_{22} = 1 \quad (18)$$

$$\Sigma\Sigma\ \{P_{ij}\,C_{ij}\} = P_{11}\,C_{11} + P_{21}\,C_{21} + P_{12}\,C_{12} + P_{22}\,C_{22} \leq 0 \quad (19)$$

TABLE III: INPUT COST VECTOR $C_{IJ} = [C_{11}{=}50, C_{12}{=}110, C_{21}{=}40, C_{22}{=}-800]$ USED AS INPUT VECTOR BY *APPENDIX* A



Assume $C_{ij} = [C_{11}{=}\$50,\ C_{12}{=}\$110,\ C_{21}{=}\$40,\ C_{22}{=}-\$800]$, and observe the input and output for Player2's optimal mixed strategy in Table III (input), Table IV (P_{ij} for various *LOSS*), Table V (*LOSS*≥3) and Table VI (*LOSS*≥5). If Player2 uses this optimal mixed strategy, Player2's expected loss for each Player1 strategy follows in Table VII for *LOSS*≥3 and Table VIII for *LOSS*≥5 with constraints regarding (12) to (15). The vector P_{ij} is defined as a minimax mixed-strategy solution.

TABLE IV: SOLUTION VECTOR $P_{ij} = [P_{11}, P_{12}, P_{21}, P_{22}]$ GENERATED BY *APPENDIX* A FOR *LOSS* = 3, 4, 5

The results for loss: 3.0	The results for loss: 4.0	The results for loss: 5.0
P11 = 0.05999981	P11 = 0.07999982	P11 = 0.1
P12 = 0.027272575	P12 = 0.03636355	P12 = 0.045454547
P21 = 0.074999675	P21 = 0.09999968	P21 = 0.125
P22 = 0.83772707	P22 = 0.7836362	P22 = 0.7295455
Expected Total Cost: -661.18	Expected Total Cost: -614.90	Expected Total Cost: -568.63
Alpha: 0.08727238	Alpha: 0.11636337	Alpha: 0.14545456
Beta: 0.13499948	Beta: 0.1799995	Beta: 0.225

TABLE V: FEASIBLE *LP* SOLUTION IN EXAMPLE 1 WITH *LOSS*≥3

Optimal Solution
Objective Function Value = 3.000

Variable	Value
P11	0.060
P12	0.027
P21	0.075
P22	0.838
ALPHA	0.087
BETA	0.135
LOSS	3.000

TABLE VI: FEASIBLE *LP* SOLUTION IN EXAMPLE 1 WITH *LOSS*≥5

Optimal Solution
Objective Function Value = 5.000

Variable	Value
P11	0.100
P12	0.045
P21	0.125
P22	0.730
ALPHA	0.145
BETA	0.225
LOSS	5.000

TABLE VII: EXAMPLE 1 FOR C_{ij}, i,j=1,2; *LOSS*≥3} WITH *EXCEL* SOLVER *LP*

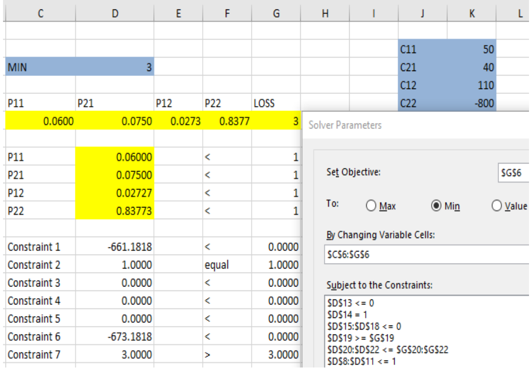


TABLE VIII: EXAMPLE 1 FOR C_{ij}, i,j=1,2; LOSS≥5 WITH *EXCEL SOLVER LP*

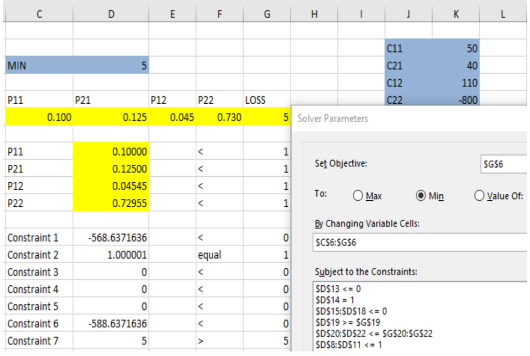


Fig. 2 and Fig. 3 yield the minimax rule-based α and β errors and expected total cost following Table III to VIII.

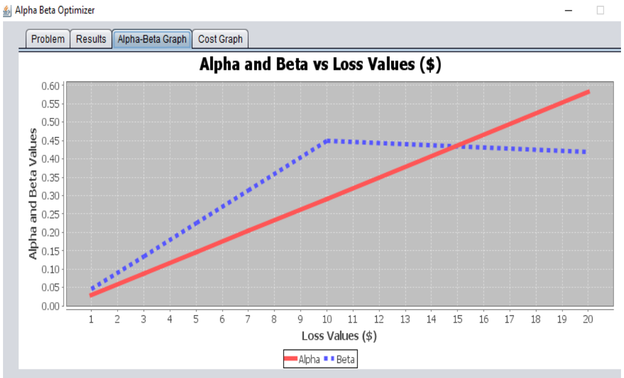


Fig. 2. *LP* solutions: *i) Alpha* ≈ .145 and *Beta* ≈ .225 vs *LOSS*=$5, and ii) *Alpha* ≈ .087 and *Beta* ≈ .135 vs *LOSS*=$3 for Example 1.

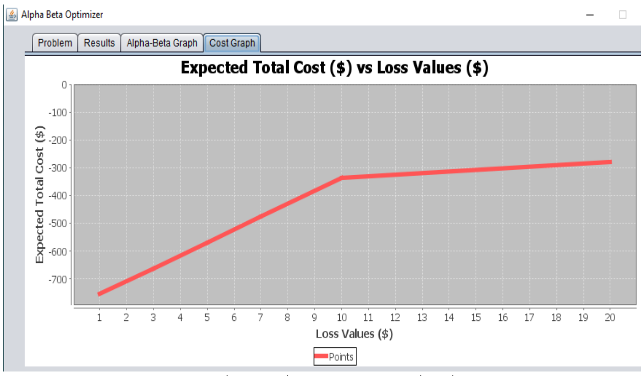


Fig. 3. Total Cost ≈ -$569, -$661 vs *LOSS*=$5, $3 for Example 1.

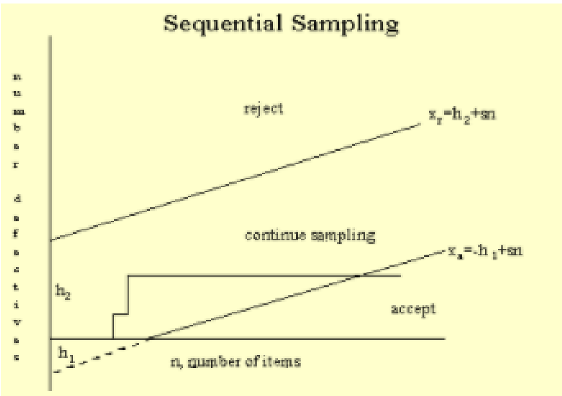


Fig. 4. Representative *SSP* plot of #d effectives (x) vs #items by NIST [31]. No such action exists other than accept-reject-continue-terminate.

Note in the *EXCEL SOLVER* of Table VII and Table VIII, the (non)linear engine serves for constrained minimization problems with differentiable (where partial derivatives of order k are continuous) nonlinear and convex functions, smooth of order k by Rapcsak [24]. This includes the case where all functions are linear, i.e. the *LP* problem. Revisit section III.A's example 1 regarding the previously outlined hypothetical *EAP* (Electric Auto Production) industrial enterprise, which refers to an item-by-item *SSP* on attributes. Let p_1 or *AQL*=Acceptable Quality Limit=.01, and let p_2 or *RQL*=Rejectable Quality Limit=.10, and α=.05 and β=.10 given for the classical *producer's* and *consumer's* risks. Wald [29], Roussas [30] and NIST [31] give the *SSP* equations regarding the *SPRT* (Sequential Probability Ratio Test) for testing H_0: $p=p_1$ vs H_a: $p=p_2$. The equations for the limit lines with parameters p_1, p_2, α, and β for *Exp#1* (Note, *Exp#1* short for *Experiment#1*) follow in (20) to (25). Table IX's *Exp#1 to Exp#5* are plotted individually and pairwise in Fig. 5 to 9. Slope is *s* and intercepts are h_1 and h_2 of Fig. 4. Enter inputs:

$$k = \log[(p_2(1-p_1))/(p_1(1-p_2))] = 1.041 \quad (20)$$

$$h_1 \text{ (accept)} = (1/k)[\log((1-\alpha)/\beta)] = 0.939 \approx 0.94 \quad (21)$$

$$h_2 \text{ (reject)} = (1/k)[\log((1-\beta)/\alpha)] = 1.206 \approx 1.21 \quad (22)$$

$$s = (1/k)\log[(1-p_1)/(1-p_2)] = 0.039747 \approx 0.04 \quad (23)$$

$$X_A \text{ (acceptance line)} = sn - h_1 = 0.04n - 0.939 \quad (24)$$

$$X_R \text{ (rejection line)} = sn + h_2 = 0.04n + 1.206 \quad (25)$$

Apply the solutions to the *SSP* for *Exp#1*, *Exp#2*, *Exp#3*, *Exp#4 and Exp#5* by varying *type-I* and *type-II* errors in Table IX from Fig. 2 and Fig. 3 and Table III to Table VIII.

C. Numerical Results of Attributes-Type Sequential Sampling Plans: Experiments #1 to #5

The solution vector for *LOSS*=$5 based on Tables III, IV, VI, VIII and Fig. 3, as plotted to follow up are, $\alpha = P_{11} + P_{12}$ =.1+.045=.145, $\beta = P_{11} + P_{21}$ = .1 +.125 = .225 for *Exp#2*. Also, ***α'***(disjoint pure alpha) = P_{12} = .045 and ***β'***(disjoint pure beta) = P_{21} = .125 for *Exp#3*. For *LOSS*=$3 by Tables III, IV, V, VII and Fig. 2, the aggregate $\alpha = P_{11} + P_{12}$ = .06 + .027 = .087 and the aggregate $\beta = P_{11} + P_{21}$ = .06 + .075= .135 are for *Exp#4*. Also ***α'***(disjoint pure alpha) = P_{12}= .027 and ***β'***(disjoint pure beta) = P_{21} = .125 for *Exp#5*.

The individually plotted sequential sampling plans in Fig. 5 to Fig. 9, respectively, as revealed by Tables IX to XI such that the number of accepts or rejects when *continue-sampling* ends at n=100 is differing from that of the classical *Exp#1* in Fig. 5. The proposed *Exp#3* and *Exp#5* with varying *LOSS*, such as $5*K* and $3*K* in Tables IX to XI show that as *LOSS* value decreases, the aggregate α and β while reduced to disjoint ***α'*** and ***β'*** lift h_1 and h_2 to mark the difference. Observe Table X with h_1=.848→1.069, h_2=1.238→1.474.

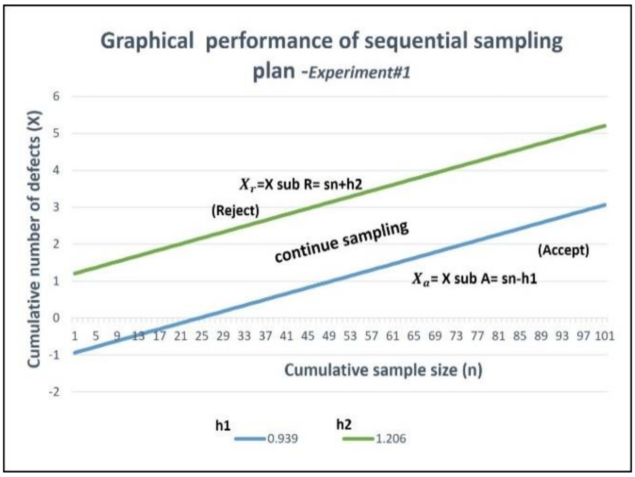


Fig. 5. *Conventional Exp#1* in Tables IX to XI, #defects turns (+) @ *n*=24, *continue sampling* for *n*<24.

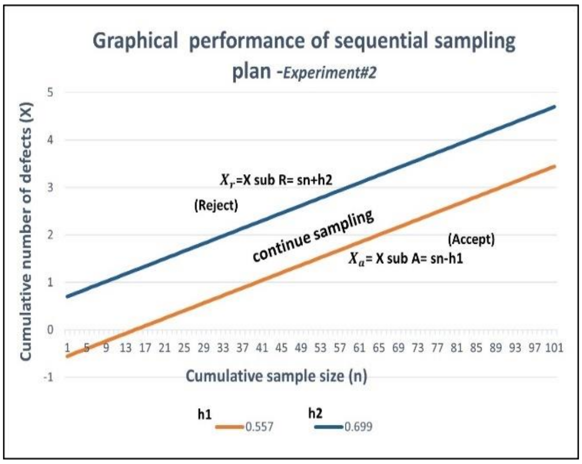


Fig. 6. The aggregate (*LOSS*=5K)*'s Exp#2* in Tables IX to XI, #defects turns (+) @ *n*=14, *continue sampling* for *n*<14.

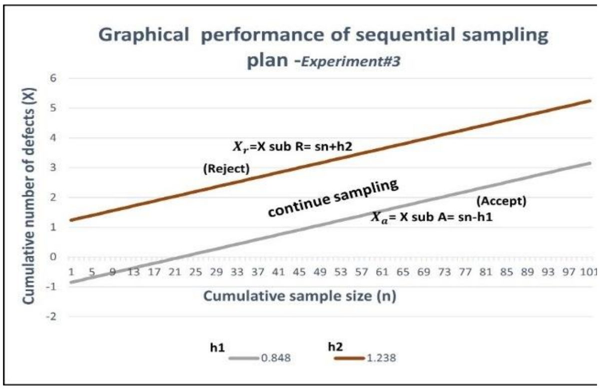


Fig. 7. The disjoint (*LOSS*=5K)*'s Exp#3* in Tables IX to XI, #defects turns (+) @ *n*=22, *continue sampling* for *n*<22.

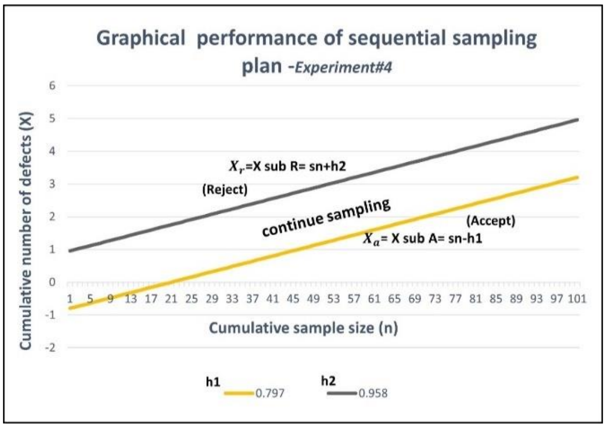


Fig. 8. The aggregate (*LOSS*=3K)*'s Exp#4* in Tables IX to XI, #defects turns (+) @ *n*=20, *continue sampling* for *n*<20.

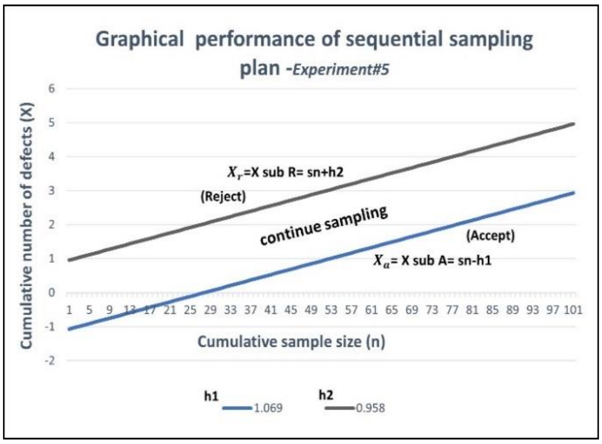


Fig. 9. The disjoint (*LOSS*=3K)'s *Exp#5* in Tables IX to XI, #defects turns (+) @ *n*=27, *continue sampling* for *n*<27.

TABLE IX: THE INPUT AND OUTPUT PARAMETERS FOR THE INDIVIDUAL AND COMPARATIVE PLOTS IN FIG. 5 TO 9 WHERE α and $\alpha' = \alpha - \alpha*\beta$ ARE AGGREGATE AND DISJOINT *TYPE-I* ERRORS, AND β and $\beta' = \beta - \alpha*\beta$ ARE AGGREGATE AND DISJOINT *TYPE-II* ERRORS, RESPECTIVELY

					AQL	RQL				
Experiment	Type I		Type II		p1	p2	k	h1	h2	s
1 (classical)	α=	0.05	β=	0.1	0.01	0.1	1.041	0.939	1.206	0.04
2 (LOSS=$5K)	α=	0.145	β=	0.225	0.01	0.1	1.041	0.557	0.699	0.04
3 (LOSS=$5K)	α'=	0.045	β'=	0.125	0.01	0.1	1.041	0.848	1.238	0.04
4 (LOSS=$3K)	α=	0.087	β=	0.135	0.01	0.1	1.041	0.797	0.958	0.04
5 (LOSS=$3K)	α'=	0.027	β'=	0.075	0.01	0.1	1.041	1.069	1.474	0.04

TABLE X: INPUT ENTRIES AND OUTPUT VALUES FROM TABLE IX ARE PLOTTED IN FIG. 5 TO 9; *EXP#1*'S ACCEPTANCE VALUE IS THE FIRST INTEGER $\leq X_A$= 0.04*n*-0.94 FOR n=1 TO 100 (TABLE X'S 1ST COLUMN FOR $h_1 \approx$.94). ALSO, THE REJECTION VALUE IS THE NEXT INTEGER $\geq X_R$= 0.04*n* +1.21 (THE 6TH COLUMN OF TABLE X FOR $h_2 \approx$1.21). FOR n=1, THE ACCEPTANCE, -1, IS IMPOSSIBLE. THE REJECTION, 2, IS IMPOSSIBLE. AT LAST AT *n*=24, AS IN FIG. 5 AND TABLE X, X_A IS 0 AND X_R IS 3. IN TABLE XI, *X* MEANS *CONTINUE-SAMPLING* WHEN NO ACCEPTANCE OR REJECTION OCCURS. {$n_{inspect}$=100, n_A=3, n_R=6} IS FOR THE CONVENTIONAL *EXP*#1 WHILE {$n_{inspect}$=100, n_A=2, n_R=6} IS FOR THE PROPOSED *EXP*#5 IN TABLES X AND XI. EXP#1 & EXP#3 ARE SAME

n	h1					h2				
	0.94	0.56	0.85	0.80	1.07	1.21	0.70	1.24	0.96	1.47
0	-0.94	-0.56	-0.85	-0.80	-1.07	1.21	0.70	1.24	0.96	1.47
1	-0.90	-0.52	-0.81	-0.76	-1.03	1.25	0.74	1.28	1.00	1.51
2	-0.86	-0.48	-0.77	-0.72	-0.99	1.29	0.78	1.32	1.04	1.55
3	-0.82	-0.44	-0.73	-0.68	-0.95	1.33	0.82	1.36	1.08	1.59
4	-0.78	-0.40	-0.69	-0.64	-0.91	1.37	0.86	1.40	1.12	1.63
5	-0.74	-0.36	-0.65	-0.60	-0.87	1.41	0.90	1.44	1.16	1.67
6	-0.70	-0.32	-0.61	-0.56	-0.83	1.45	0.94	1.48	1.20	1.71
7	-0.66	-0.28	-0.57	-0.52	-0.79	1.49	0.98	1.52	1.24	1.75
8	-0.62	-0.24	-0.53	-0.48	-0.75	1.53	1.02	1.56	1.28	1.79
9	-0.58	-0.20	-0.49	-0.44	-0.71	1.57	1.06	1.60	1.32	1.83
10	-0.54	-0.16	-0.45	-0.40	-0.67	1.61	1.10	1.64	1.36	1.87
11	-0.50	-0.12	-0.41	-0.36	-0.63	1.65	1.14	1.68	1.40	1.91
12	-0.46	-0.08	-0.37	-0.32	-0.59	1.69	1.18	1.72	1.44	1.95
13	-0.42	-0.04	-0.33	-0.28	-0.55	1.73	1.22	1.76	1.48	1.99
14	-0.38	0.003	-0.29	-0.24	-0.51	1.77	1.26	1.80	1.52	2.03
15	-0.34	0.04	-0.25	-0.20	-0.47	1.81	1.30	1.84	1.56	2.07
16	-0.30	0.08	-0.21	-0.16	-0.43	1.85	1.34	1.88	1.60	2.11
17	-0.26	0.12	-0.17	-0.12	-0.39	1.89	1.38	1.92	1.64	2.15
18	-0.22	0.16	-0.13	-0.08	-0.35	1.93	1.42	1.96	1.68	2.19
19	-0.18	0.20	-0.09	-0.04	-0.31	1.97	1.46	2.00	1.72	2.23
20	-0.14	0.24	-0.05	0.003	-0.27	2.01	1.50	2.04	1.76	2.27
21	-0.10	0.28	-0.01	0.04	-0.23	2.05	1.54	2.08	1.80	2.31
22	-0.06	0.32	0.03	0.08	-0.19	2.09	1.58	2.12	1.84	2.35
23	-0.02	0.36	0.07	0.12	-0.15	2.13	1.62	2.16	1.88	2.39
24	0.02	0.40	0.11	0.16	-0.11	2.17	1.66	2.20	1.92	2.43
25	0.06	0.44	0.15	0.20	-0.07	2.21	1.70	2.24	1.96	2.47
26	0.10	0.48	0.19	0.24	-0.03	2.25	1.74	2.28	2.00	2.51
27	0.14	0.52	0.23	0.28	0.01	2.29	1.78	2.32	2.04	2.55
28	0.18	0.56	0.27	0.32	0.05	2.33	1.82	2.36	2.08	2.59
29	0.22	0.60	0.31	0.36	0.09	2.37	1.86	2.40	2.12	2.63
30	0.26	0.64	0.35	0.40	0.13	2.41	1.90	2.44	2.16	2.67
95	2.86	3.24	2.95	3.00	2.73	5.01	4.50	5.04	4.76	5.27
96	2.90	3.28	2.99	3.04	2.77	5.05	4.54	5.08	4.80	5.31
97	2.94	3.32	3.03	3.08	2.81	5.09	4.58	5.12	4.84	5.35
98	2.98	3.36	3.07	3.12	2.85	5.13	4.62	5.16	4.88	5.39
99	3.02	3.40	3.11	3.16	2.89	5.17	4.66	5.20	4.92	5.43
100	3.06	3.44	3.15	3.20	2.93	5.21	4.70	5.24	4.96	5.47

TABLE XI: *EXP#1*(α=.05, β=.1), *EXP#3* (*LOSS=5K*) AND *EXP#5* (*LOSS=3K*) BY TABLES IX AND X WHILE DECISION-MAKING DIFFERENCES ARE MARKED IN ROWS 24, 22, 27 FOR *EXP*#1, #3 AND #5 RESPECTIVELY

n(#1)	n(#1)	n(#1)	n(#3)	n(#3)	n(#3)	n(#5)	n(#5)	n(#5)
(inspect)	(accept)	(reject)	(inspect)	(accept)	(reject)	(inspect)	(accept)	(reject)
1	x	2	1	x	2	1	x	2
2	x	2	2	x	2	2	x	2
3	x	2	3	x	2	3	x	2
4	x	2	4	x	2	4	x	2

5	x	2	5	x	2	5	x	2
6	x	2	6	x	2	6	x	2
7	x	2	7	x	2	7	x	2
8	x	2	8	x	2	8	x	2
9	x	2	9	x	2	9	x	2
10	x	2	10	x	2	10	x	2
11	x	2	11	x	2	11	x	2
12	x	2	12	x	2	12	x	2
13	x	2	13	x	2	13	x	2
14	x	2	14	x	2	14	x	3
15	x	2	15	x	2	15	x	3
16	x	2	16	x	2	16	x	3
17	x	2	17	x	2	17	x	3
18	x	2	18	x	2	18	x	3
19	x	2	19	x	3	19	x	3
20	x	3	20	x	3	20	x	3
21	x	3	21	x	3	21	x	3
22	x	3	22	0	3	22	x	3
23	x	3	23	0	3	23	x	3
24	0	3	24	0	3	24	x	3
25	0	3	25	0	3	25	x	3
26	0	3	26	0	3	26	x	3
27	0	3	27	0	3	27	0	3
28	0	3	28	0	3	28	0	3
29	0	3	29	0	3	29	0	3
30	0	3	30	0	3	30	0	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
95	2	6	95	2	6	95	2	6
96	2	6	96	2	6	96	2	6
97	2	6	97	3	6	97	2	6
98	2	6	98	3	6	98	2	6
99	3	6	99	3	3	99	2	6
100	3	6	100	3	6	100	2	6

A. *Optimal Solutions' Interpretations: Aggregate (Composite) and Disjoint (Pure) Risks*

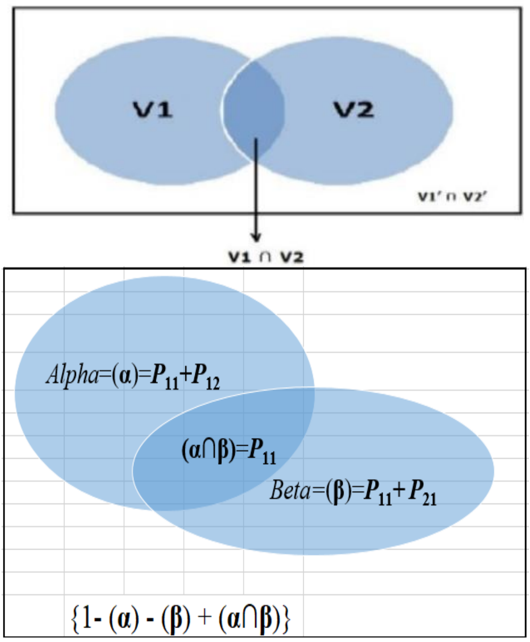
With *LOSS*=$3*K* (in Exp#5) of Table IX if the plotted points stay within the limiting boundaries (*AQL* and *RQL*), the sequential sampling plan continues and hence, another sample to be drawn by {1.0–(***α'***=.027)– (***β'***=.075)}*100% = 89.8% for the percentage of the *continue-sampling* decision. This results from example 1's input in Table III per *SSP* under scrutiny. The expected *total cost* [(=*alpha * relative cost of alpha error + beta * relative cost of beta error + (1-alpha-beta) * relative utility of no errors*)] is -$712*K*. Note, C_{12} (=relative cost of *alpha*) = $110*K* and C_{21} (=relative cost of *beta*) = $40*K*, C_{12} (=relative cost of the *cross-product* or intersection of *alpha* and *beta* errors) = $50*K* and C_{22} (=relative utility of no errors, denoting complete satisfaction with no erroneous returns) = -$800*K*. Disjoint total cost is thus -$712*K* ≈ $\alpha' C_{12} + \beta' C_{21} + (\alpha'\beta') C_{11} + (1-\alpha'-\beta') C_{22}$ = .027 * $110 +.075 * $40 + 0 + .898*-$800 since (***α'β'***) overlap of disjoint ***α'*** and ***β'*** changed to **0** in Fig. 10.c (Venn diagram) from a non-zero in Fig. 10.b (Venn diagram).

For *LOSS*=$5*K* (in Exp#3), disjoint total cost is .045*$110 + .125*$40 + 0 + .83*(-$800) ≈ -$654*K*. See Champerowne [36] on the *SSP* costings for *accept*, *reject* and *continue-sampling*, and Würlander [37] on the *SPRT* performance such as the average sample size (*ASN*).

The company-specific input cost data produces the aggregate *alpha* (α) ≈ .145, and the aggregate *beta* (β) ≈ .225 for *LOSS* = $5 in *Exp#2* per Tables IX to XI. Likely, the aggregate *alpha* (α) ≈.087 and the aggregate *beta*(β) ≈ .135 are for *LOSS* = $3 in *Exp#4*. The proposed ***α'***(disjoint pure alpha) = P_{12} = .045 and ***β'***(disjoint pure beta) = P_{21} = .125 in *Exp#3* are for *LOSS* = $5K. Also, ***α'***(disjoint pure alpha) = P_{12} = .027 and ***β'***(disjoint pure beta) = P_{21} =.125 in *Exp#5* are for *LOSS*=$3K. The deviations between *Exp#1 (classical)* and *Exp#3 (proposed with LOSS = $5)*, and likely *Exp#1 (classical)* and *Exp#5 (proposed with LOSS = $3)* are what the article draws attention to by using C_{ij}, i=1,2, j=1,2 and a *LOSS* variable.

IV. Venn Diagrams to Verify Optimization

In Fig. 10, Venn diagrams constituting all four sample sets of *V* are studied; where *V* stands for vulnerability, which points out to an erroneous decision-making set. Note that [$\alpha\beta + \alpha(1-\beta) + (1-\alpha)\beta + (1-\alpha)(1-\beta)$] =1 via (4) and (18). The composite sample V_1, which aggregates the common-errors intersection $V_1 \cap V_2$, has elements due to the *producer's* risk, such as consumers misusing the vehicle and returning e.g. a hybrid vehicle to the dealership due to customers' user faults per example 1. The composite sample V_2 too contains the common-errors intersection $V_2 \cap V_1$, and denotes the *consumer's* risk such as factory-recalls or class actions due to the producer's faults. The discontent consumer returns e.g. the vehicle, to the vendor as in example 1 of section 3.



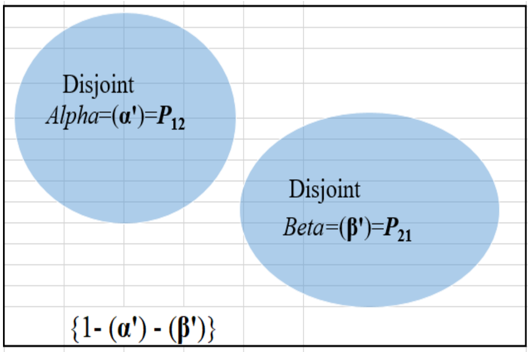


Fig. 10. a, b, c. Venn diagrams: a) Generalized Venn diagram representation of samples. b) Aggregates α *and* β intersected, $P(\alpha\cap\beta) = P(FP\cap FN) = P_{11} \neq 0$. c) Disjoints (mutually exclusive with no ambiguous intersections) $\boldsymbol{\alpha}'$ *and* $\boldsymbol{\beta}'$ with $P(\boldsymbol{\alpha}'\cap\beta') = P(\boldsymbol{FP'}\cap \boldsymbol{FN'}) = \boldsymbol{P}_{11}=0$.

One proceeds to $V_1 \cap V_2$, the intersection of V_1 and V_2 comprising both error regions, α and β, in the realm of an ambiguous or controversial decision described in section III. For the adjustments option, this was classified as allowances or write-offs, which are explained in-depth in subsection III.A. The Venn diagram's blank error-free region is $V_1' \cap V_2'$ for none of α and β errors involved. V_1' and V_2' are complements for V_1' and V_2', respectively. See Sahinoglu [15] where V_1 and V_2 are dependent, i.e. not independent, since $P(V_1 \cap V_2) \neq P(V_1)P(V_2)$. Why? Because $.1 \neq .145 * .225 = .033$, since in the preceding example 1: $P(V_1 \cap V_2) = P_{11} = .1$, $P(V_1) = P_{12} + P_{11} = .145$, $P(V_2) = P_{21} + P_{11} = .225$. Thus, $P(V_1 \cap V_2) \neq P(V_1)P(V_2)$, is equivalent to expressing $\alpha * \beta \neq \alpha$ times β. Conditionally dependent samples V_1 and V_2 are not independent, but $P(V_1 \cap V_2) = P(V_1 | V_2)P(V_2) = P(V_2 | V_1)P(V_1)$. Fig. 10a, 10b, and 10c are the Venn diagram samples.

Let $P(FP \cap FN) = P(\alpha \cap \beta) \neq 0$; let $P(\boldsymbol{FP'} \cap \boldsymbol{FN'}) = P(\boldsymbol{\alpha'} \cap \boldsymbol{\beta'}) = 0$, and d = Disjoint. Let the l.h.s in Fig. 10.b, the light-blue V_1^d =Disjoint *producer's* risk with $P(V_1 \cap V_2') = .045 = P_{12} = \boldsymbol{\alpha'}$. Let the r.h.s. light-blue V_2^d = Disjoint *consumer's* risk with $P(V_2 \cap V_1') = .125 = P_{21} = \boldsymbol{\beta'}$. Let the middle dark-blue $(V_1 \cap V_2)$ = Intersection of *producer's* and *consumer's* risks, and $P(V_1 \cap V_2) = .1 = P_{11}$. Let the blank $V_1' \cap V_2'$ = error-free region with no producer's and no consumer's risks for $P(V_1' \cap V_2') = .73 = P_{22}$. Note, Also, $P(V_1 U V_2) + P(V_1' \cap V_2') = 1$ is identical to $P(V_1^d) + P(V_2^d) + P(V_1 \cap V_2\} + P(V_1' \cap V_2')\} = 1$ or by (18), $P_{12} + P_{21} + P_{11} + P_{22} = 1.0$ or $\{\alpha\beta\} + \{\alpha(1-\beta)\} + \{(1-\alpha)\beta\} + \{(1-\alpha)(1-\beta)\} = 1$. Note $P(\alpha \cap \beta) = P_{11} \neq 0$ in Fig. 10.b is related to *Exp*#2 and *Exp*#4, while $P(\boldsymbol{\alpha'} \cap \boldsymbol{\beta'}) = \boldsymbol{P_{11}} = 0$ in Fig. 10.c is per *Exp*#3 and *Exp*#5. The blank is $P_{22} = (1-\alpha)(1-\beta) = 1 + (\alpha * \beta) - \alpha - \beta$ or $P_{22} = 1 - \alpha - \beta + \alpha \cap \beta$ per Fig. 10.b for $P_{11} = P(\alpha * \beta) \neq 0$; α, β are aggregates. The blank: $P_{22} = 1 - \boldsymbol{\alpha'} - \boldsymbol{\beta'}$ in Fig. 10.c; $\boldsymbol{\alpha'}, \boldsymbol{\beta'}$ are disjoints and $P_{11} = P(\alpha * \beta) = 0$.

V. Algebraic Roots to Verify LP Vector Solution

There exists a favorable shortcut technique to serve as an optimality verification tool without using the software programs so as to validate the *LP*-based feasible solution vector, $P_{i,j}$ given *Cij* and *LOSS* variable(s). What plays a crucial role here is actually the *LOSS* variable constraint. Once the *LOSS* variable is accurately constrained by the financial analyst in (17), it is a simple algebraic task to compute the $\hat{P}_{ij}$ roots. That is, $\hat{P}_{ij} = LOSS/Cij$ given the constant *Cij* for all i and j excluding i=2, j=2. Once $\hat{P}_{11}, \hat{P}_{12}$ and $\hat{P}_{21}$ are calculated, one finds $\hat{P}_{22} = 1 - \hat{P}_{11} - \hat{P}_{12} - \hat{P}_{21}$ by subtraction per equation (18) i.e., $\hat{P}_{11} + \hat{P}_{12} + \hat{P}_{21} + \hat{P}_{22} = 1$ with $\boldsymbol{\alpha}$ (aggregate or composite) = $\hat{P}_{11} + \hat{P}_{12}$ *and* $\boldsymbol{\beta}$ (aggregate or composite) = $\hat{P}_{11} + \hat{P}_{21}$. Fig. 4 clarifies the 3 disjoint actions.

A. Simple Algebraic Root Solutions Applied to Example 1 by Four-Operations Arithmetic

Example 1 delineates that given input vector, $[C_{ij}] = [C_{11}=$ \$50*K*, C_{12}=\$110*K*, C_{21}=\$40*K*, C_{22}= -\$800*K*] with $LOSS \geq \$5K$ and $LOSS \geq \$3K$, respectively; the solution vectors are $[P_{ij}] = [P_{11}=.1,\ P_{12}=.045,\ P_{21}=.125,\ P_{22}=.73]$ for *LOSS*=\$5*K* and $[P_{ij}] = [P_{11}=.06,\ P_{12}=.022,\ P_{21}=.075,\ P_{22}=.84]$ for *LOSS*=\$3*K*. Table VIII for *LOSS*=5*K*, displaying the *EXCEL Solver* input and output shows that the three constraints (#3, #4 and #5) referring to (12) to (14) yield ≈0. $\hat{P}_{ij}$=LOSS/*Cij*. Then, $\hat{P}_{11}$=\$5/\$50=.1, $\hat{P}_{12}$=\$5/\$110=.045, $\hat{P}_{21}$=\$5/\$40=.125, and $\hat{P}_{22} = 1 - \hat{P}_{11} - \hat{P}_{12} - \hat{P}_{21} = .73$ by subtraction per (18). They all concur with the software solution vectors shown in section III. Subsequently, $\hat{\alpha} = \hat{P}_{11} + \hat{P}_{12} = .1 + .045 = .145$, and $\hat{\beta} = \hat{P}_{11} + \hat{P}_{21} = .1 + .125 = .225$ as composite errors in Tables III, IV, VI, VIII and Fig. 3. For *LOSS*=\$3*K in* Table VIII, $\hat{P}_{ij}$=LOSS/C_{ij} → $\hat{P}_{11}$=\$3/\$50=.06, $\hat{P}_{12}$=\$3/\$110=.027, $\hat{P}_{21}$= \$3/\$40=.075, and $\hat{P}_{22} = 1 - \hat{P}_{11} - \hat{P}_{12} - \hat{P}_{21} = .838$ through subtraction per (18). These concur with the software solution vectors in section III's Tables III, IV, V, VII and Fig. 2. Similarly, $\hat{\alpha} = \hat{P}_{11} + \hat{P}_{12} = .087$, $\hat{\beta} = \hat{P}_{11} + \hat{P}_{21} = .135$ are the composite errors.

B. The Analytical Verification with Simple Algebraic Roots, and Examples 2 and 3

The *LP*-based algorithm implemented to *SSP* demonstrates that the feasible solution produced by the three different software algorithms, i.e. i) Microsoft's EXCEL SOLVER, ii) Author's JAVA-coded Game-Testing of Appendix A and iii) LP Software by Anderson *et al.* [35] are validated by the algebraic roots formulated in the preceding subsection V.A. The three simple algebraic roots, $\hat{P}_{11}, \hat{P}_{12}, \hat{P}_{21}$ were calculated and the fourth, $\hat{P}_{22}$, by subtraction of the first three from 1.0 per (18). The algebraic roots verify that $\hat{P}_{ij} = LOSS/C_{ij}$ are identical to the optimal solutions obtained in section II's Fig. 2 and Fig. 3 and Tables III to VIII. Shortcut algebraic roots are, too, optimally best estimates. Generalizations on *LOSS* input variables are given in APPENDIX B and C. That is, for all C_{ij}, $LOSS_{ij}$ can be assigned upon need.

Appendix B (i.e. example 2) uses a new set of $LOSS_{ij}$=\$5, \$6, \$7, \$8 validating *Exp*#2's roots, such as $\hat{P}_{11}$=\$5/\$50=.1, $\hat{P}_{12}$=\$6/\$110=.055, $\hat{P}_{21}$=\$7/\$40=.175, and $\hat{P}_{22}$=.67. Total Cost (disjoint) = .055*\$110 + .175*\$40 + (1-.055-.175) (-\$800) ≈ -\$603 in Table XII of Appendix B.

Appendix C (i.e., example 3) replicates Tables III and IV solution for $LOSS_{ij}$=\$5, i,j=1,2 to replicate subsection V.A's $\hat{\alpha}$ =.145, $\hat{\beta}$=.225, $\widehat{\boldsymbol{\alpha}'}$ =.045, $\widehat{\boldsymbol{\beta}}'$=.125 and Total Cost (disjoint) = .045 *\$110 +.125*\$40 + (1-.045-.125)(-\$800) ≈ -\$654 in Table XIII of Appendix C.

VI. Conclusive Summary, Future Research

This article studies an *LP*-based, and further, simpler linear root-finding solutions, and pertinent industrial applications so to optimize *type-I* (*alpha*) and *type-II* (*beta*) error probabilities in response to employing related cost and utility parameters from input data. These probabilities are otherwise known as *producer's* and *consumer's* risks, or risks of *false positive* and *false negative*. Tables IX to XI and Fig. 5 to Fig. 9 epitomize the tangible differences between the old ubiquitous and newly proposed ways. This is in contrast to the prespecified cut-off values of *alpha* and *beta* (e.g. $\alpha \approx .05$, $\beta \approx .10$) that have been traditionally practiced. Kelly [1] and Grant [2] therefore urged attention to this impasse. The choice for *LOSS* variable(s) and C_{ij}, i,j=1,2, i.e., C_{11}, C_{12} and C_{21} costs and utility constant C_{22}, are dictated by the company-savvy historical data per Sahinoglu [15] to [18] and Hoare [34].

Game theory's extended and value-added approach serves here to contribute to a feasible output vector solution as detailed in example 1 of subsections III.A to III.D. This is why the optimized *alpha* and *beta* errors are objective and data-centric rather than the subjectively popular judgment-call selections, usually prespecified as e.g., α=.05 and β=.10. The apparent limitation of this research topic may involve the data-scientific challenge of estimating *Cij* constants and *LOSS* variables' constraints. These essentially econometric parameters can be estimated either through data mining, and time series modelling, or any viable computationally intensive approach by the analyst to reflect the actual market realities for a profitable sequential sampling plan solely specific to that company.

In the *SSP*, the producer establishes a sequential sampling plan for a continued supply of components with reference to *AQL*, which represents the acceptable upper limit of quality for the supplier's process that the consumer would consider acceptable as a process average at one end. The consumer may also be interested at the other end, i.e. *RQL*, to denote the poorest limit of quality that the consumer is willing to accept with a low probability of acceptance in an individual lot by Montgomery [25]. If both rules do not work, the *continue-sampling* decisive action is adopted to call for a new sample to test. One terminates the *SSP* after 3*n* many samples as a rule of thumb. The author reasons that the proposed technique with attributes-type item-by-item sampling is applicable to the variables-type by Roussas [30]. The proposed method is to upgrade the hypothesis tests from a subjective to an objective stance; while improving industrial control-savvy item-by-item sequential sampling plans. Example 1 of the attributes-type item-by-item *SSP*'s steps are outlined as follow from ***i), i')*** to ***vii), vii')*** in sequence. Note, e.g. for *LOSS*=\$3*K*; ***i), i')*** to ***vii), vii')*** show tasks and outcomes respectively**.** The numerical outcomes are subject to change for a new set of the specific firm's input C_{ij} constants and *LOSS* variable constrained for each case. The step by step algorithm is as follows:

i) Set H_0: p_1(=*AQL*) vs H_a: p_2 (=*RQL*), and *n* = lot sample size.
i') p_1(=*AQL*) = 0.01 vs H_a: p_2 (=*RQL*) =0.10, and *n* =100.

ii) Select C_{ij}= [C_{11}, C_{12}, C_{21}, C_{22}] and provide *LOSS* constraint(s) for numerical examples.
ii') C_{ij}= [\$50*K*, \$110*K*, \$40*K*, -\$800*K*] and *LOSS*=\$3*K* prespecified at the onset.

iii) Optimize the aggregate α and β, and the aggregate total cost, either by game-theory (Section III) or identical algebraic roots (Section V).

iii') α=.087 and β=.135 from Tables III to V. Total Cost (aggregate)=\$661 from Fig. 3 and Table IV for *LOSS*=\$3*K*.

iv) Compute the disjoint pure ***α'*** = *producer's* risk due to α - $\alpha*\beta$, and the disjoint pure ***β'***= consumer's risk due to β - $\alpha*\beta$ and whatsoever no risk due to {1- α'- β'}.
iv') ***α'***=α-α*β=.087-.06=.027, ***β'***=β-α*β=.135-.06=.075 and **{**1-***α'-β'***}=1-.027-.075 = .898 for *LOSS*=\$3*K*.

v) Adopt the optimal ***α'*** and ***β'*** to calculate the parameters: h_1, h_2, *k*, *s* by (20) to (25) to plot the *SSP* (Fig. 5 to 9) to *accept*, *reject* or *continue-sampling* by Table IX to Table XI.
v') X_A (acceptance line) = $sn - h_1$ = 0.04*n* – 1.07; X_R (rejection line) = $sn + h_2$ = 0.04*n* + 1.47 for *LOSS*=\$3*K*.

vi) Mark the *SSP* decision rules, given: C_{ij}, *LOSS*, *AQL*, *RQL* and disjoint ***α'*** and disjoint ***β'***.
vi') See Tables IX to XI to compare *Exp*#3 with *Exp*#5. Accept if plotted points fall below X_A as in {*n*=100, n_{Accept} =3, n_{Reject}=6} per *Exp*#1. Reject if plotted points fall above X_R as in {*n*=100, n_{Accept}=2, n_{Reject}=6} per *Exp*#5. Table XI marks the differences between the usual or conventional *Exp#1* and proposed *Exp#3* for *LOSS*=\$3*K*.

vii) The *SSP* running cost, i.e. ***α'***C_{12} + ***β'***C_{21} +(1- ***α'***- ***β'***)C_{22}, incurred by adopting the proposed algorithm will be authentic based on the nature of inputs, i.e. C_{ij}= [C_{11}, C_{12}, C_{21}, C_{22}] and *LOSS* constraint. Intersection of pure estimates: ***(α')*(β')*** = ***(α')***∩***(β')*** = 0 in Fig. 10.c.
vii') *TC*=Total Cost (disjoint *Exp*#5 in Table IX) =.027*\$110*K* + .075*\$40 + (1-.027-.075) * (-\$800*K*) ≈ -\$712*K* is authentic for Example 1, whereas the same *TC*=-\$613*K* for Example 2 in *APPENDIX* B and finally *TC*=-\$654*K* for Example 3 *APPENDIX* C. The fact that smaller the running total cost becomes, poses no concern. The author's take is that the proposed *Exp#5* uses its proper *SSP's LP*-based *alpha* and *beta*, optimized to ***α'*≈2.7%**, ***β'*≈7.5%**; not the prespecified *alpha* and *beta* errors for *LOSS*=\$3*K*.

VII. Final Remarks

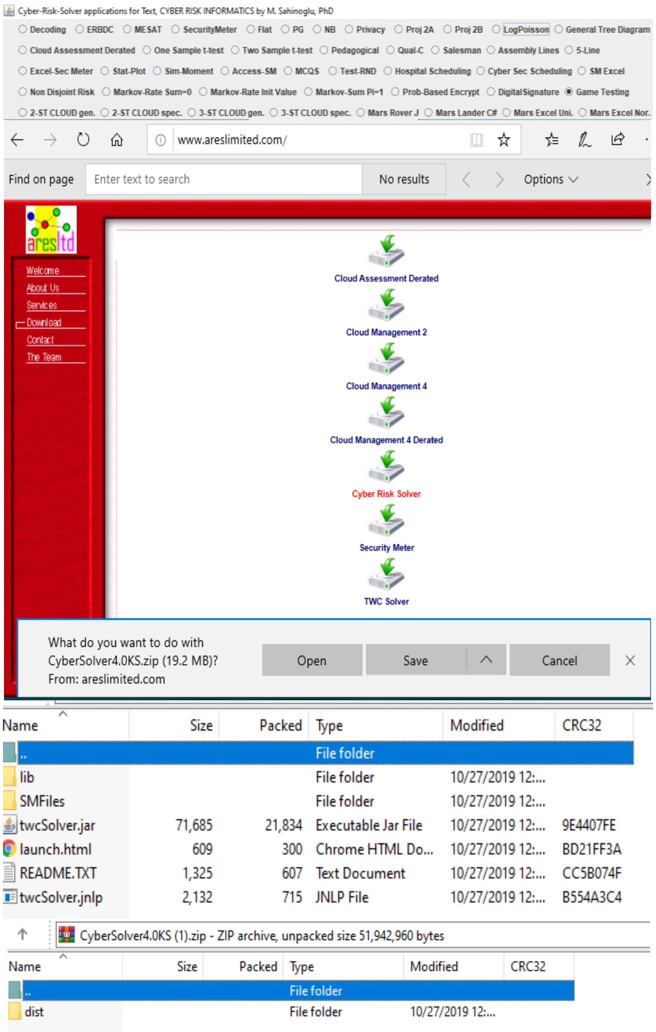
Readers' take per Table XI is such that in the classical approach with the prespecified alpha=.05 and beta=.10 referring to Exp#1 of Table IX at the end of n=100 samples; the testing analyst decides to *accept* 3 and *reject* 6; i.e. {n_{inspect}=100, n_A=3, n_R=6}. Whereas, per Exp#5 of Table IX with an assumed *LOSS*=\$3K, the testing analyst decides to *accept* 2 (instead of 3 in the classical approach) and *reject* 6, saving monetary funds while not accepting one more item out of a lot size of n=100, i.e. {n_{inspect}=100, n_A=2, n_R=6}. Results may amply change if the analyst varies the input parameters.

Appendix A: CyberRiskSolver to Run the Game Testing APPLET

1. Click www.areslimited.com and type in the login user name: *mehmetsuna*, password: *Mehpareanne*, click OK.
2. Go to DOWNLOAD on www.areslimited.com for l.h.s. menu's 4th from the top.
3. Click on the CyberRiskSolver v3.0 in **red** and download the application which a ZIP file. Unzip or extract the

downloaded application into C:\myapp folder. See C:\myapp\dist. Open a Command Prompt and go to C:\myapp\dist folder and run the following command: //For Cyber Risk Solver, java –jar twcSolver.jar. Use license code: **EFE28SEP1986** for twcSolver.jar.

4. Click ***GAME TESTING*** Applet (checked). Click Open.



APPENDIX B

TABLE XII: EXAMPLE 2: $LOSS_{11}$=\$5, $LOSS_{12}$=\$6, $LOSS_{21}$=\$7, $LOSS_{22}$=\$8, C_{IJ} = [\$50, \$110, \$40, -\$800];
DISJOINTS: $\widehat{\alpha}'$ = .055 = P_{12}, $\widehat{\beta}'$ = .175 = P_{21}, TOTAL COST = -\$603

Optimal Solution
Objective Function Value = 26.000

Variable	Value	Reduced Costs
P11	0.100	0.000
P12	0.055	0.000
P21	0.175	0.000
P22	0.670	0.000
LOSS11	5.000	0.000
LOSS12	6.000	0.000
LOSS21	7.000	0.000
LOSS22	8.000	0.000
ALPHA	0.155	0.000
BETA	0.275	0.000

APPENDIX C

TABLE XIII: EXAMPLE 3: $LOSS_{11}$ = $LOSS_{12}$= $LOSS_{21}$ = $LOSS_{22}$ = \$5, C_{IJ} = [\$50, \$110, \$40, -\$800];
DISJOINTS: $\widehat{\alpha}'$ = .045 = P_{12}, $\widehat{\beta}'$=.125 = P_{21}, TOTAL COST = -\$654

Optimal Solution
Objective Function Value = 20.000

Variable	Value	Reduced Costs
P11	0.100	0.000
P12	0.045	0.000
P21	0.125	0.000
P22	0.730	0.000
LOSS11	5.000	0.000
LOSS12	5.000	0.000
LOSS21	5.000	0.000
LOSS22	5.000	0.000
ALPHA	0.145	0.000
BETA	0.225	0.000

CONFLICT OF INTEREST

The author declares no conflict of interest.

AUTHOR CONTRIBUTIONS

The principal author, M.S. began to develop the theory of the original research findings as of 2015 and continued to develop and finalize it up to date. The co-author S.C. contributed as to where he assisted with the working software, web interface, data engineering and computational statistics. Both authors at the final stage approved the current version.

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M. Sahinoglu served as a distinguished professor in the position of founding director of the Informatics Institute and the founder head of the Cybersystems and Information Security Graduate Program (2008-2018) in Auburn University at Montgomery. Formerly, the Eminent Scholar and Chair-Professor at Troy University's Computer Science Department (1999-2008), he holds a BSEE from METU, Turkey (1969-73), and MSEE from University of Manchester, UK as a British Council scholar (1974-75). He completed a joint Ph.D. from Statistics and ECE at Texas A&M University in College Station, TX (1977-81). Full Prof. (1990) by the ABET-accredited METU; he was the founder Dean of the College of Arts & Sciences at Izmir's DEU (1992-97). In 1993-94, Dr. Sahinoglu established the (Applied) Statistics Department to recently have celebrated its 25th anniversary in 2019. Currently conducting Quantitative Risk Assessment and Management-based research, he authored Trustworthy Computing (2007) and Cyber-Risk Informatics: Engineering Evaluation with Data Science (2016) by WILEY Inc. Dr. Sahinoglu, who retired from Auburn University System as of June, 2018, is currently teaching Cybersecurity curriculum at Troy University's CS Dept. in Troy, AL. Dr. Sahinoglu is an ASA Senior (1980-), ISI Elected (1995-), IEEE Senior Life (1978-) and SDPS: Society of Design & Process Science Fellow Member (2003-). He taught at TAMU (1978-81), METU (1977-92), DEU (1992-1997), Purdue (1989-90 Fulbright, 1997-98 NATO) and CWRU (1998-99). One of the world's 14 Microsoft Trustworthy Computing Awardees (2006), and a silver medallist for the DAU's Hirsch Paper Competition on Software Assurance (2015) and Digital Forensics (2016); Mehmet was the 2009 recipient of Software Engineering Society's Excellence in Leadership Award. Dr. Sahinoglu originated, i) S&L (Sahinoglu-Libby) pdf jointly with Dr. David Libby, PhD, of the University of Iowa on repairable hardware (1981), ii) CPSRM: Compound Poisson Software Reliability Prediction Model (1992), iii) MESAT: Cost-Optimal Stopping -Rule in Reliability/Security Testing (2002), iv) SM: Security and Privacy Risk Meter (2005), v) CODING & DECODING of large complex networks using Polish Algorithms (2006), vi) OVERLAP ingress-egress solution for large Complex Block Diagrams jointly with Benjamin Rice (2007), vii) Sahinoglu's 3-State Monte Carlo Simulation Probability Model (2008), and viii) CLOURAM: Cloud Computing Risk Assessor & Manager (2017). Dr. Sahinoglu authored 160+ conference proceedings, 70 peer-reviewed journal articles and managed 20+ international grants. Mehmet delivered 60+ keynotes, invited seminars and radio talks combined since he was invited in 1990's First Kickoff Workshop Conference on Software Reliability Engineering in Washington, DC. He reviewed papers for journals and was a committee member for the RAMS - Reliability & Maintainability Symposium. He presented four ZOOM seminars in Turkey during 11/2021.

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