# Efficient Maximum Sum-Rate Computation for Multiple Access Channels in CR Networks

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*Abstract*—**This paper considers a cognitive radio (CR) net-work, in which the unlicensed (secondary) users (SUs) are allowed to concurrently access the spectrum allocated to the licensed (primary) users provided that their interference to the primary users (PUs) satisfies certain constraints. It is more general and owns stronger challenge to ensure the quality of service (QoS) of the PUs as well as to maximize the weighted sum-rate of the SUs. By choosing the weighted coefficients, we can find any optimal boundary point of the capacity region. On the other hand, single-antenna mobile users are quite common due to the size and cost limitations of mobile terminals. Thus, we simply term this setting as single input multiple output multiple access channels (SIMO-MAC) in the CR Networks. Subject to the interference constraints of the SUs as well as peak power constraints of the SUs, the weighted sum-rate maximization problem is solved. To effectively and efficiently maximize the achievable weighted sum-rate of the SUs, a tight pair of upper and lower bounds, as an interval, of the optimal Lagrange multiplier is proposed. It can avoid ineffectiveness or inefficiency when the dual decomposition is used. Therefore, not only is convergence of the proposed algorithm guaranteed, but efficient computation is also provided by the proposed algorithm.**

*Index Terms*—**Wireless communications, coganative radio (CR) network, weighted sum-rate, multiple acess channels (MAC), optimization methods.**

## I. Introduction

The radio spectrum is a precious resource that demands effective utilization as the currently licensed spectrum is severely underutilized [1]. Cognitive Radio (CR) [2], which adapts the radios operating characteristics to the real-time conditions, is the key technology that allows flexible, efficient and reliable spectrum utilization in wireless communications. This technology exploits the facts that the licensed spectrum is underutilized by the primary user(s) (PU) and it introduces the secondary user(s) (SU) to operate on the spectrum that is either opportunistically being available or concurrently being shared by the PU and the SU. The proposed paper focuses on the latter case.

Since the multiple-input multiple-output (MIMO) technology uses multiple antennas at either the transmitter or receiver to significantly increase data throughput and link range without additional bandwidth or transmit power, it plays an important role in wireless communications today.

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In our proposed paper, we will combine the CR network with the MIMO technology to fully ensure the quality of service (QoS) of the PUs as well as to maximize the weighted sum-rate of the SUs. This machinery may achieve flexibility and efficiency in spectrum sharing with principle of the CR. Note that by choosing the weighted coefficients, we can find any optimal boundary point. It is easily seen that as the weighted coefficients are all being unity, the above problem is reduced into a sum-rate optimization problem. Since the reduced problem has a simpler structure, it can be solved by the well-known water-filling, but the weighted sum-rate optimization problem cannot use water-filling to compute its solution.

On the other hand, since single-antenna mobile users are quite common due to the size and cost limitations of mobile terminals, we simply term this setting as a single input multiple output multiple access channel (SIMO-MAC) in the CR network, and confine the topic to the SIMO-MAC situation in the CR Networks. By exploiting structure of the weighted sum-rate optimization problem, although water-filling cannot be used, we can still propose an efficient algorithm to compute the optimal input policy and the maximum weighted sum-rate in the proposed paper.

For computing the maximum weighted sum-rate for a class of the Gaussian SIMO systems, [3] has presented some algorithms to provide the max-stability policy. In addition, [4]-[6] have set up the well known duality between the Gaussian broadcast channel and the sum-power constrained Gaussian dual multiple-access channel. Those meaningful works are applied to the non-CR case. For the MIMO MAC under the CR network, in order to compute the optimal input policy, the sum-rate maximization problem for the SIMO-MAC in the CR network is investigated in recent published papers, such as [7].

Motivated by the above work, in this paper, we set up mathematical models for the CR cases, which reflect the systems more practically than others. In addition, we propose a dual decomposition algorithm based on the water-filling principle. The proposed algorithm owns several improvements shown below.

First, to avoid ineffectively using the dual decomposition algorithm and make the proposed algorithm more efficient, a tight pair of upper and lower bounds, asan interval, to the optimal Lagrange multiplier is proposed. Especially when the number of the users is great enough, the benefit to utilizing this interval can be sufficiently exploited.

Second, we reduce the weighted sum-rate problem into solving a decoupled system. Each equation of the decoupled system only contains a scalar variable. Due to the characteristics of both the objective function and the decoupled system with each of the equations containing a scalar variable, although the water-filling cannot be used, any one of the equations is solved by the proposed algorithm at least with exponential convergence rate. To guarantee a fast computation, we construct intervals, each of which contains a corresponding solution to each of the equations.

To the best knowledge of the authors, the above two points have not yet been investigated in the existing literatures.

In addition, convergence of the proposed algorithm can be guaranteed through a rigorous mathematical proof presented in this paper. As a result, the proposed algorithm offers fast convergence. It is important to note that convergence of the proposed algorithm is based on the theoretical advances from the fundamental results of the previously mentioned acclaimed papers.

Key notations that are used in this paper are as follows: $|A|$ and $Tr(A)$ give the determinant and the trace of a square matrix $A$, respectively; $E[X]$ is the expectation of the random variable $X$; the capital symbol $I$ for a matrix denotes the identity matrix with the corresponding size; In addition, for any complex matrix, its superscript + denotes the conjugate transpose of the matrix.

## II. SIMO MAC Under CR Network and its Weighted Sum-Rate

For the SIMO-MAC in the CR network, assume that there are one base-station (BS) with Nr antennas, $K$ SUs and $N$ PUs, each of which is equipped with one single antenna. In this section, assume that the SIMO-MAC under the CR network is described as

$$y=\sum_{i=1}^{K} h_i^+ x^i + \sum_{i=1}^{N} \hat{h}_i^+ \hat{x}^i + Z$$

where $h_i \in C^{1\times N_r}, i=1,\ldots,K,$ and $\hat{h}_i \in C^{1\times N_r}, i=1,\ldots,N,$ are the given fixed channel vectors of the SUs and PUs, respectively. Like regular assumptions, such as those in [7], the i-th entry $x_i$ of $x \in C^{N_r \times 1}$ is a scalar complex input signal from the i-th SU and x is assumed to be a Gaussian random vector having zero mean with independent entries. The j-th entry $\hat{x}_j$ of $\hat{x}$ is a scalar complex input signal from the j-th PU and $\hat{x}$ is assumed to be a Gaussian random vector having zero mean with independent entries. Further, $Z \in C^{N_r \times 1}$ is an additive Gaussian noise random vector, i.e., $Z \sim N(0,\sigma^2 I)$. Thus, $y \in C^{N_r \times 1}$ is the received signal at the BS. Furthermore, $S_i = E[x^i (x^i)^+], i=1,2,\ldots,K.$

The mathematical model of the weighted sum-rate optimization problem for the SIMO-MAC in the CR network can be written as follows, by using the successive interference cancellation (SIC) principle [8]: Given a group of nonnegative real numbers, $\{\alpha_1, \ldots, \alpha_K\}$, at least one of which is nonzero,

$$f_{w\max}(h_1^+,\ldots,h_K^+;P_1,\ldots,P_K;P_t) = \max \sum_{k=1}^{K} \alpha_k \log\left| I + \sum_{j=1}^{k} h_j^+ h_j S_j \right| \quad (1)$$

where the constraint set is $\{\{S_k\}_{k=1}^K : 0 \le S_k \le P_k, \sum g_k S_k \le P_t\}$. The peak power constraints on the SUs are denoted by $P_k$ (> 0), $k = 1, \ldots, K$. In the constraint set, assume $P_t > 0$, and there exists $1 \le j_0 \le K$ such that $h_{j_0} \ne 0$. Further, without loss of generality,

$$\sum_{i=1}^{N} \hat{h}_i^+ \hat{x}^i + Z$$

is assumed to have the identity covariance matrix in model (1). Note that $\alpha k$ is the difference, $\forall k$, between the $k$-th weighted coefficient and the $(k+1)$-th weighted coefficient when the weighted coefficients are assumed to be ones in descending order (users can be arbitrarily renumbered to satisfy this condition). Further, the $(K+1)$-th weighted coefficient mentioned above is assumed to be zero to compute $\alpha_K$.

The constraint

$$\sum_{k=1}^{K} g_k S_k \le P_t$$

of the weighted sum-rate optimization problem (1) of the SIMO-MAC in the CR network is called the sum-power constraint with gains. The gains denoted by $\{g_k\}_{k=1}^K$ are defined as follows. Let

$$H = \left[h_1^+,\ldots,h_K^+\right] \in C^{N_r \times K}$$

and

$$\hat{H} = \left[\hat{h}_1^+,\ldots,\hat{h}_N^+\right] \in C^{N_r \times N}$$

Thus, $y = \hat{H}\hat{x} + (Hx + Z)$, where $Hx + Z$ is the additive interference and noise to the transmitted signal $\hat{H}\hat{x}$, which is transmitted to the BS. To guarantee the QoS for the PUs, the power of the interference and noise is less than the transmitted power by the PUs. That is to say, setting up a threshold Pt limits the power transmitted by the SUs and guarantee the QoS for the PUs. Its mathematical expression can be expressed as

$$Tr(HE(xx^\dagger)H^\dagger + E(ZZ^\dagger)) \le P_t$$

Let $g_k = h_k h_k^+$, ∀k. Then we have:

$$\sum_{k=1}^{K} g_k S_k \le P_t - N_r\sigma^2$$

$P_t \Leftarrow P_t - N_r\sigma^2$, where the symbol "⇐" means the assignment operation. $\{g_k\}_{k=1}^K$ is called gains of the sum-power constraint, and $P_t$ is called the sum-power constraint with the gains.

Further, based on the same principle, a better weighted sum-rate model can be also obtained. Our approach just reflects essence of the issue for the SIMO-MAC in the CR network, compared with others.

In addition, assume that $M$ = rank($H$). Applying the QR decomposition, $H = QR$. Let

$$Q=[q_1, \cdots, q_M]\in C^{N_r\times M}$$

have orthogonal and normalized column vectors. $R \in C^{M\times K}$ is an upper triangle matrix with $r_{m,k}$ denoting the ($m$, $k$)-th entry of the matrix $R$. $Q^\dagger$ is regarded as an equalizer to the received signal by the BS.

Thus, the $i$-th SU should have the rate:

$$\log\left(1+A/B\right),\ \text{where}\ A=\left|r_{i,i}\right|^2 S_i$$

$$B=\sigma^2+\sum_{n=1}^{N}\hat{S}_n q_i^+ \hat{R}_n q_i+\sum_{j=i+1}^{K}\left|r_{i,j}\right|^2 S_j$$

$$\hat{S}_n=E\left[\hat{x}^n(\hat{x}^n)^+\right]\ \text{and}\ \hat{R}_n=\hat{h}_n^+\hat{h}_n, n=1,\ldots,N$$

Compared with [7], since the third term in the denominator above is ignored by the paper [7], thus our model is more complete.

Using the separating hyperplane theorem [9] in convex optimization theory, we may obtain the following proposition.

Proposition II.1. The optimization problem (1) is equivalent to the following optimization problem:

$$\min_{\lambda\geq 0}\left\{\max_{\{S_k\}_{k=1}^K}\sum_{k=1}^{K}\alpha_k\log\left|I+\sum_{j=1}^{k}h_j^+h_jS_j\right| -\lambda(\sum_{k=1}^{K}g_kS_k-P_t)\right\}$$

Subject to $\qquad S_k\geq 0, S_k\leq P_k, \forall k \qquad (2)$

i.e., the optimal objective values of the optimization problems (1) and (2) are equal. Furthermore, with the exception of the part of the dual variable, the restriction of any optimal solution of (2) (to the part of original variable) is the same as the optimal solution of (1).

## III. Algorithm ALW1

To extend the solution to the optimization problem (2) with the weighted sum-rate and more efficiently solve this problem (2) according to Proposition (II.1), a new method is to be proposed as follows.

Given $\lambda \geq 0$ and the optimization problem:

$$\max_{\{S_k\}_{k=1}^K}\sum_{k=1}^{K}\alpha_k\log\left|I+\sum_{j=1}^{k}h_j^+h_jS_j\right| -\lambda(\sum_{k=1}^{K}g_kS_k-P_t)$$

Subject to $\qquad S_k\geq 0, S_k\leq P_k, \forall k \qquad (3)$

an efficient iterative algorithm is proposed here and the optimal objective function value for the problem (3) is denoted by $g(\lambda)$. It is seen that $g(\lambda)$ is a convex function over $\lambda \geq 0$, and $\lambda$ is a scalar. Thus, we may use a line search to obtain the optimal solution $\lambda^*$ to the problem (2). Thus, since the range to search is quite important, a pair of the upper and lower bounds is proposed as follows.

Proposition III.1. For the optimization problem (2), the optimal solution $\lambda^*$ is in the interval (left open and right closed) from 0 to the sum of $\{\alpha_k\}$, summarized from 1 to $K$.

Tightness of the interval or pair of the upper and lower bounds means that there exists a set of channel gains such that its optimal Lagrange multiplier $\lambda^*$ touches either of the ends of the interval. For example, as $K = 1$, $\alpha_1 = 1$, $P_t = 2$, $P_1 = 1$ and $h_1=1$, it is seen that $\lambda^*=\alpha_1$. Let $\lambda$ in the interval mentioned above, and consider the evaluation of $g(\lambda)$. Note that the problem (3) has decoupled constraints. Therefore, the block coordinate ascend algorithm (BCAA)[10], [11] can be used to solve the problem efficiently. The iterative algorithm works as follows. In each step, the objective function is maximized over a single variable $S_k$, while keeping all other $S_k$s fixed, $k = 1, \cdots, K$ and then repeating this process. Since the objective is nondecreasing with each iteration, the algorithm must converge to a fixed point. Using the fixed point theory, the fixed point is an optimal solution to the problem (3). In the detail, let us consider an optimization problem below over $S_k$ , $k = 1$, with respect to all other $S_k$ s being fixed, as follows:

$$\max_{\{S_k\}_{k=1}^K}\sum_{k=1}^{K}\alpha_k\log\left|I+\sum_{j=1}^{k}h_j^+h_jS_j\right| -\lambda(\sum_{k=1}^{K}g_kS_k-P_t)$$

Subject to $\qquad S_k\geq 0, S_k\leq P_k \qquad (4)$

If $g_1 = 0$, then $h_1 = 0$. Let $h_1 \Leftarrow h_2$. Repeating this process, we can obtain a non-zero $h_1$. An optimal solution to the problem (4) satisfies the following relationships:

If $\qquad i\sum_{k=1}^{K}\alpha_k\Lambda_k\leq g_1\lambda,\ \ then\ \ S_1=0$

If $\qquad \sum_{k=1}^{K}\frac{\alpha_k\Lambda_k}{1+\Lambda_k S_1}\geq g_1\lambda,\ \ then\ \ S_1=P_1$

else $\qquad \sum_{k=1}^{K}\frac{\alpha_k\Lambda_k}{1+\Lambda_k S_1}=g_1\lambda$

where $\qquad \Lambda_k=h_1(I+\sum_{j=2}^{k}h_j^+h_jS_j)^{-1}h_j^+,\forall k \qquad (5)$

Since the preceding two cases are trivial and are easy to solve, we will mainly discuss the third case mentioned above without a specific claim. For acquiring fast computation of the solution to the optimality condition (5), Jacobian-Newton method can obtain the exponential convergence at least, resulting from the characteristics of the objective function and the decoupled system with only one equation and a scalar variable. At the same time, efficiency of the fast computation mentioned above stems from choice of an initial point or interval.

Proposition III.2. For the optimality condition (5), the Jacobian-Newton method [9] can obtain the exponential convergence, at least.

Proof: The error between the $n$-th iterative solution $S_1^n$ and the true solution $S_1^*$ is denoted by $e_n$. Then

$$e_{n+1} = e_n^2 \rho, \quad where \quad 0 < \rho < 1$$

If $|e_0| < 1$ is chosen, it implies that $\{S_1^n\}$ exponentially converges at least and much faster the bisection method. On the other hand, we may use the secant method to make $|e_0| < 1$ in a few iterations, which is much faster than the bisection method, especially when the initial interval is provided. #

According to the proposition above, assume $e_0 = 0.8$. Then, $e_1 = 0.8^2 = .64$, $e_2 = 0.8^4 = .41$, $e_3 = .16$, $e_4 = .028$, $e_5 = .0008$, $e_6 = .0000006$, $e_7 = .0000000000004$, $e_8 = .00000000000000000000000016$. Therefore, the convergent rate is quite significant.

Such an interval is just provided by Proposition (III.3) below.

Proposition III.3. For the optimization problem (4) or the optimality condition (5), if

$$\sum_{k=1}^{K} \alpha_k \Lambda_k > g_1 \lambda \quad and \quad \sum_{k=1}^{K} \frac{\alpha_k \Lambda_k}{1+\Lambda_k P_1} < g_1 \lambda$$

The optimal solution $S_1 \in$

$$[\max\{0, \frac{\sum \alpha_k}{g_1 \lambda} - \frac{1}{\min\{\Lambda_k\}_{k=1}^K}\}, \max\{0, \frac{\sum \alpha_k}{g_1 \lambda} - \frac{1}{\max\{\Lambda_k\}_{k=1}^K}\}]$$

For λ, given $\{S_1^{(n)}, ..., S_K^{(n)}\}$, the BCAA is used from the first variable to the $K$-th variable, and we obtain $\{S_1^{(n+1)}, ..., S_K^{(n+1)}\}$

Thus there is a mapping which projects $\{S_1^{(n)}, ..., S_K^{(n)}\}$ to $\{S_1^{(n+1)}, ..., S_K^{(n+1)}\}, \forall n.$ This mapping is denoted by $f$.

With the assumptions and the concepts introduced, a new iterative-water-filling-like algorithm, AlW1, is concisely proposed as follows.

Algorithm AlW1:

1) Given $\varepsilon > 0$, initialize $\{S_1^{(0)} = 0, ..., S_K^{(0)} = 0\}$, $\lambda_{\min}$ and $\lambda_{\max}$.
2) Set $\lambda = (\lambda_{\min} + \lambda_{\max})/2$.
3) Compute $\{S_k^{(n+1)}\}_{k=1}^K = f(\{S_k^{(n)}\}_{k=1}^K)$. Then $n \Leftarrow n + 1$. Repeat the procedure 3) mentioned above until the optimal solution to the problem (3) is reached.
4) If $\sum \alpha_k S_k^* - P_t > 0$, then λmin is assigned by λ;
   If $\sum \alpha_k S_k^* - P_t < 0$, then λmax is assigned by λ;
   If $\sum \alpha_k S_k^* - P_t = 0$, stop.
5) If $|\lambda\min - \lambda\max| \le \varepsilon$, stop. Otherwise, go to step 2). Note that the initial $\lambda_{\min}$ is chosen as 0, and $\lambda_{\max}$ is chosen as $\sum \alpha_k$ respectively. If the initial values $\lambda_{\min} \ge 0$ and $\lambda_{\max} \ge 0$ are chosen as two points at outside of the available range of the $\lambda^*$, there exists an example to account for the fact that algorithms via dual decomposition principle cannot find any optimal solution.

Example 1. If $K = 2$, $P_1 = P_2 = P_t = 2$, $\alpha_1 = \alpha_2 = 1/3$ and $h_1 = h_2 = g_1 = g_2 = 1$, the problem (3) is instanced.

Let the initials $\lambda_{\min} = 6$ and $\lambda_{\max} = 8$. Dual decomposition algorithms cannot be used to find any optimal solution to the weighted sum-rate maximization problem.

If $\sum_{k=1}^{K} \frac{\alpha_k \Lambda_k}{1+\Lambda_k P_1} \ge g_1 \lambda$, then $S_1 = P_1$; if $\sum_{k=1}^{K} \alpha_k \Lambda_k \le g_1 \lambda$, then $S_1 = 0$; else, it is seen that $g_1 \lambda \ne 0$. Thus we utilize the interval

$$[\max\{0, \frac{\sum_{k=1}^{K} \alpha_k}{g_1 \lambda} - \frac{1}{\min\{\Lambda_k\}_{k=1}^K}\}, \max\{0, \frac{\sum_{k=1}^{K} \alpha_k}{g_1 \lambda} - \frac{1}{\max\{\Lambda_k\}_{k=1}^K}\}]$$

with the left end and the right end of the interval mentioned above being denoted by $S_{\min}$ and $S_{\max}$, respectively. The secant method may be applied due to existence of the interval above. It can be proven that this secant method owns a faster convergence.

## IV. Convergence of Algorithm ALW1

First, we will prove the system (5) to be a group of the sufficient and necessary optimality conditions to the optimization problem (4).

Proposition IV.1. The system (5) is a group of the sufficient and necessary optimality conditions to the optimization problem (4).

Proof: For any $\lambda$, $k$ and the optimization problem (4), it has been known that the objective function is maximized over a single variable $S_k$, while keeping all other $S_k$ s fixed. Without loss of generality, assume $k = 1$. It is seen that the optimization problem (4) is equivalent to the following problem:

$$\max_{S_1 : 0 \le S_1 \le P_1} \sum_{k=1}^{K} \alpha_k \log(1 + \sum_{k=1}^{K} \Lambda_k S_1) - \lambda g_1 S_1 \qquad (6)$$

Since the Hessian matrix of the objective function of the optimization problem (6) is strictly negative definite, the objective function is strictly concave with a convex feasible set. Derivative of the objective function is $\sum \frac{\alpha_k \Lambda_k}{1+\Lambda_k P_1} - \lambda g_1$. If $\sum \alpha_k \Lambda_k \le g_1 \lambda$ and $\sum_{k=1}^{K} \frac{\alpha_k \Lambda_k}{1+\Lambda_k P_1} \ge g_1 \lambda$, $S_1 = 0$ and $S_1 = P_1$, respectively, if and only if $S_1$ is the optimal solution to the problem (6); else, $\sum_{k=1}^{K} \frac{\alpha_k \Lambda_k}{1+\Lambda_k P_1} - g_1 \lambda = 0$ if and only if $S_1$ is the optimal solution to the problem (6). #

Second, convergence for the algorithm AlW1 is discussed as follows.

Theorem IV.2. For the weighted sum-rate problem (1), AlW1 is convergent.

Proof: Due to construction of the tight interval or bounds for the optimal Lagrange multiplier $\lambda^*$, convexity of both the optimization problem (2) and $g(\lambda)$, and characteristic the sub-gradient method, the Lagrange multiplier obtained by iterations of the outer loop computation can approximate to $\lambda^*$ only when the inner loop can guarantee convergence. It is seen that the cyclic coordinate ascent algorithm [11] is used by the inner loop computation and it is convergent if the solved problem is a convex optimization problem with a compact Cartesian direct product set as the feasible set and a continuously differential objective function. The problem (1) guarantees the conditions mentioned above. Therefore, AlW1 is convergent. #

## V. Performance Results and Comparision

We end our discussion with some numerical examples to illustrate the simplification and effectiveness of the proposed algorithm. For a clear understanding, the iterative-water-filling-like algorithm in [3] is called Algorithm AW in this paper and can be used to solve the problem (1). The performance for AW was better than others (refer to [3]). Thus, the proposed AlW1 only compares with AW under the same assumption.

Example V.1. The performance of Algorithm AlW1, compared with Algorithm AW of [3], is presented in Fig. 1, where $N_r = 8$. Random data are generated for the channel gain vectors and the number of the PUs. The number, denoted by $K$, of the SUs is 158, and the number of the PUs is 258. The sum-power constraint with the gains is $P_t = 8$ and the peak constraints are chosen at random.

For Fig. 1, the solid curves and the cross markers represent the results of our proposed algorithm AlW1 and the algorithm AW, respectively. These results show that our proposed algorithm AlW1 exhibits faster convergence, although the number of the SUs is great. On the other hand, not only AlW1 guarantees effective and efficient convergence, but it also has a lower computation complexity. Each iteration of AlW1 scales linearly with $K$, the computation complexity of the inner loop is at most $cK \times O(\log(1/\epsilon_1))$, where c denotes the number of the inner loop iterations, and $\epsilon_1$ denotes the error tolerance for computing $S_k$. The outer loop undergoes $O(\log(1/\epsilon_2))$ iterations to satisfy the error tolerance $\epsilon_2$. Compared with complexity $O(K^{3.5} \log (1/\epsilon_3))$ of the interior point algorithm, the complexity of AlW1 is significantly reduced.

## VI. Conclusion

For the model of the SIMO-MAC in the CR network, through the efficient iterative with two sets of the determined bounds, the proposed algorithm exhibits improved convergence rate.

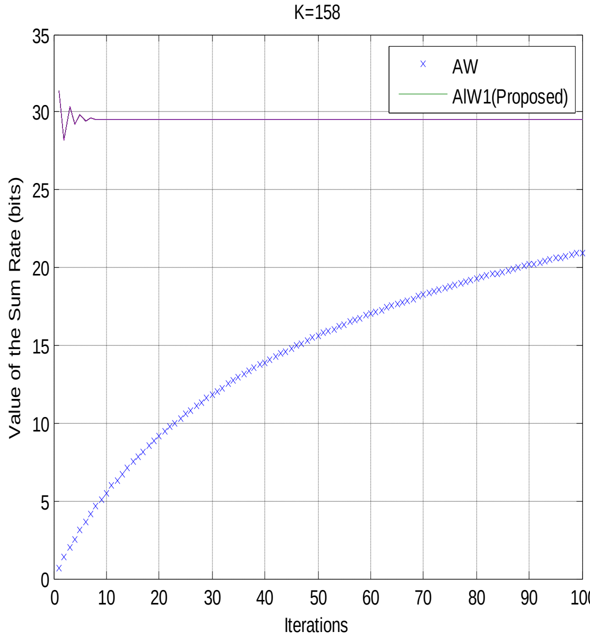


Fig. 1. Algorithm AlW1 compared with algorithm AW, as $K$=158

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