Extended Balanced Space-Time Block Coding: Full Rate Full Diversity Wireless Multicasting

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***Abstract*—Multicasting is a spectrally efficient method of supporting group communication by allowing transmission of packets to multiple destinations using fewer resources. To incorporate Multi-Input Multi-Output diversity, Extended Balanced Space-Time Block Codes (EBSTBCs) have been proposed providing full diversity when one or more feedback bits are sent back via feedback channel. However, the EBSTBCs are designed for unicast communication in the literature. This paper presents a novel wireless multicasting scheme which selects the optimum EBSTBC for all mobile users to support wireless multicast. Extensive, detailed simulations are performed to show the feasibility of full rate and full diversity multicast service provisioning in Rayleigh fading channels.**

***Index Terms*—Diversity, extended balanced space-time block coding, MIMO, wireless multicasting.**

I. Introduction

One of the space-time coding scheme is orthogonal space-time block coding (OSTBC) which provides full diversity advantage with low decoding complexity. The transmitted symbols are decoded separately using linear processing [1]. However, full diversity and full rate for more than two antennas cannot be achieved with OSTBC. Several quasi-orthogonal space-time block codes (QOSTBCs) that provide full rate at the expense of some loss in diversity [2],[3], and OSTBCs that provide full diversity with some loss in code rate [1],[4] have been proposed in the literature. In [5], full rate balanced space-time block codes (BSTBCs) have been proposed which achieve full diversity for arbitrary number of transmit antennas when one or more feedback bits are sent back via feedback channel. The main drawback of the BSTBCs is limited coding gain. In [6], [7], the extended balanced space-time block codes (EBSTBCs) scheme has been proposed. In the EBSTBCs, an arbitrary numbers of codes can be generated for improved coding gain.

It is known that multicasting is an efficient method of supporting group communication as it allows for transmission of packets to multiple destinations using fewer network resources [8]. Along with the widespread deployment of wireless networks, the fast-improving capabilities of mobile devices, content and service providers are increasingly interested in supporting multicast communications over wireless networks.

To the best our knowledge, there is no space-time block coding which achieves full rate and full diversity more than one user. In this paper, we propose a novel coding selection scheme for wireless multicasting. Detailed simulations are performed to show the feasibility of the full rate full diversity multicast service provisioning in Rayleigh fading channels. In this regard, in the second section, the system model is described, in the third section, the EBSTBCs are explained, in the fourth section, multicast extended balanced space-time block coding (MEBSTBC) is presented, and in the last Section, the results of the paper and the conclusion are given.

The following notation is used in the paper: The superscript * denotes the conjugate operation; Re {.} and Im{.} are the real and imaginary part of the argument, respectively. The operator *ceil* {.} rounds to the smallest integer greater or equal than its argument. The operator max {.} returns the largest of its operands and the operator min {.} returns the minimum of its operands.

II. System Model

The system model consists of a base station and L mobile multicast users. The base station is equipped with N antennas but the mobile users are equipped with a single antenna. All channels are assumed frequency flat Rayleigh fading channel where channel gains are circularly complex Gaussian random variables and statistically independent from each other. h_{ij} is the channel fading coefficient from the ith antenna of the base station to the jth mobile multicast user where i=1, 2,.., N and j=1, 2,.., L.

The channels are quasi-static, namely, the fading coefficients remain constant over the duration of one frame. The mobile user is assumed to have perfect knowledge of its own channels. The noise is modeled as additive white Gaussian whose components are circular complex random variable with zero-mean and variance σ^2. The base station data bits are mapped by streams of y bits into M-PSK symbols where $M=2^y$.

III. Extended Balanced Space-Time Block Coding

The extended balanced space-time block codes (EBSTBCs) are obtained by multiplying an orthogonal space-time block code with an extension matrix [9]. Since Alamouti`s code [10] is the only orthogonal code with rate one, the EBSTBCs can be obtained as an extension of the Alamouti`s code

$$C=XW \tag{1}$$

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Here $\boldsymbol{X}$ is the Alamouti's code and W is the 2xN extension matrix where $N \geq 2$ and the rank of W must be 2. The following example shows how to generate the EBSTBCs for three transmitters. Consider the EBSTBC pair with transmission matrix

$$\boldsymbol{C}_1 = \begin{bmatrix} s_1 & s_2 & as_2 \\ -s_2^* & s_1^* & as_1^* \end{bmatrix} \tag{2}$$

where $a=e^{j2\pi m/q}$, q is the extension level and m=0, 1,…q-1. The columns and rows of $\boldsymbol{C}_1$ denote symbols transmitted from three transmit antennas in two signaling intervals, respectively. The matrix $\boldsymbol{C}_1$ is obtained from the Alamouti code using Equation (1) where

$$\boldsymbol{X} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & a \end{bmatrix}. \tag{3}$$

In this fashion, an arbitrary number of the EBSTBCs can be generated. It can be shown that the number of possible EBSTBCs is $q^{N-2}(2^{N-1}-1)$ [7]. For that reason, the destination needs $N+d$ feedback bits ($N \geq 3$) to select any possible EBSTBCs where d=*ceil* {(N-2) $\log_2 q$}-1. N-2 feedback bits are needed to achieve full diversity as in BSTBCs. The rest of the d+2 feedback bits provide an additional coding gain [11].

IV. Multicast Extended Balanced Space-Time Block Coding

Multicast extended balanced space-time block coding (MEBSTBC) can be obtained when an optimum EBSTBC is selected for all multicast users. The MEBSTBC contains two phases: Multicast frame initialization phase and multicast transmission phase. In the first phase, the multicast users transmit their channel state information (CSI) to the base station. The base station selects the optimum MEBSTBC for all multicast users and transmits selected MEBSTBC information to the multicast users. In multicast transmission phase, the base station transmits data to the multicast users according to the selected MEBSTBC.

A. *MEBSTBC for Three Transmit Antennas*

When three transmit antennas are present at the base station, then, $\boldsymbol{C}_1$, $\boldsymbol{C}_2$ and $\boldsymbol{C}_3$ are available as MEBSTBC matrices. These matrices are

$$\boldsymbol{C}_1 = \begin{bmatrix} s_1 & s_2 & as_2 \\ -s_2^* & s_1^* & as_1^* \end{bmatrix} \boldsymbol{C}_2 = \begin{bmatrix} s_1 & s_2 & as_1 \\ -s_2^* & s_1^* & -as_2^* \end{bmatrix}$$
$$\boldsymbol{C}_3 = \begin{bmatrix} s_1 & as_1 & s_2 \\ -s_2^* & -as_2^* & s_1^* \end{bmatrix}. \tag{4}$$

The base station picks the MEBSTBC $\boldsymbol{C}_j$, j=1,2,3 that generates the optimum coding gain for all the multicast users. Two bits of feedback is needed to select the MEBSTBC matrices and k bits of feedback is needed to select the feedback a where k=*ceil* {$\log_2 q$}. The optimum MEBSTBC for all multicast users is selected according to

In another words, when the first term of, $\min\left(\mathrm{Re}\left\{ah_{21}^* h_{31}\right\}, \mathrm{Re}\left\{ah_{22}^* h_{32}\right\}, ..., \mathrm{Re}\left\{ah_{2L}^* h_{3L}\right\}\right)$ is maximum, $\boldsymbol{C}_1$ is selected. If the second term of, $\min\left(\mathrm{Re}\left\{ah_{11}^* h_{31}\right\}, \mathrm{Re}\left\{ah_{12}^* h_{32}\right\}, ..., \mathrm{Re}\left\{ah_{1L}^* h_{3L}\right\}\right)$ is maximum, $\boldsymbol{C}_2$ is selected. Otherwise, $\boldsymbol{C}_3$ is selected. After the optimum MEBSTBC is selected, and combining the received signals at the jth mobile multicast user, the estimates are given as

$$\hat{s}_{i,j} = \sqrt{\frac{P}{3}}\left[\left|h_{1j}\right|^2 + \left|h_{2j}\right|^2 + \left|h_{3j}\right|^2 + 2A_1\right]s_i + \eta_{i,j}, \quad i = 1, 2. \tag{6}$$

Here $\hat{s}_{i,j}$ is the estimated ith symbol at the jth mobile multicast user; $\eta_{1,j}$ and $\eta_{2,j}$ are the noise at the jth mobile multicast user.

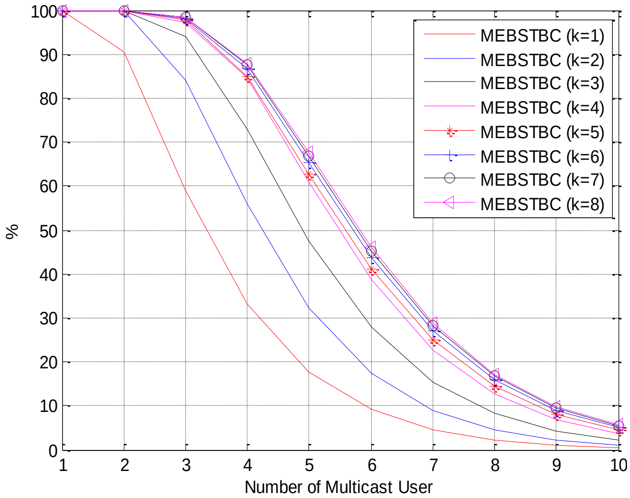

Fig. 1. The percentage of the channels that achieve full diversity for various multicast users when three transmit antennas are present at the base station.

Fig. 1 shows the percentage of the channels that achieve full diversity for various multicast users when three transmit antennas are present in the environment. MEBSTBC with one bit extension of feedback (MEBSTBC (k=1)) achieves full diversity for only one user (unicast communication), since MEBSTBC with one bit extension of feedback yields only 6 different codes. MEBSTBC with two or more bit extension of feedback supports full diversity for two users. When five or more multicast users are present in the wireless environment, full diversity can be achieved in 70% or less of all possible channel conditions.

The followings are the properties of the MEBSTBC for three transmit antennas:

1) One bit extension of feedback (k=1) cannot be achieved full rate and full diversity for two multicast users.
2) Two or more bit extension of feedback ($k \geq 2$) achieves full rate and full diversity for two multicast users.
3) The full diversity can be achieved for an arbitrary number of multicast users, if the below inequality is satisfied for all possible channel conditions.

$$A_1 \geq 0. \tag{7}$$

B. *MEBSTBC for Four Transmit Antennas*

When four transmit antennas are present at the base station, available MEBSTBC matrices are

$$\boldsymbol{C}_1=\begin{bmatrix} s_1 & as_1 & bs_1 & s_2 \\ -s_2^* & -as_2^* & -bs_2^* & s_1^* \end{bmatrix}\ \boldsymbol{C}_2=\begin{bmatrix} s_1 & as_1 & s_2 & bs_1 \\ -s_2^* & -as_2^* & s_1^* & -bs_2^* \end{bmatrix}\qquad \boldsymbol{C}_5=\begin{bmatrix} s_1 & as_1 & s_2 & bs_2 \\ -s_2^* & -as_2^* & s_1^* & bs_1^* \end{bmatrix}\ \boldsymbol{C}_6=\begin{bmatrix} s_1 & s_2 & as_1 & bs_2 \\ -s_2^* & s_1^* & -as_2^* & bs_1^* \end{bmatrix}$$

$$\boldsymbol{C}_3=\begin{bmatrix} s_1 & s_2 & as_1 & bs_1 \\ -s_2^* & s_1^* & -as_2^* & -bs_2^* \end{bmatrix}\ \boldsymbol{C}_4=\begin{bmatrix} s_1 & s_2 & as_2 & bs_2 \\ -s_2^* & s_1^* & as_1^* & bs_1^* \end{bmatrix}\qquad \boldsymbol{C}_7=\begin{bmatrix} s_1 & s_2 & as_2 & bs_1 \\ -s_2^* & s_1^* & as_1^* & -bs_2^* \end{bmatrix}. \tag{8}$$

$$A_2=\max\begin{pmatrix} \min\left(\left[\mathrm{Re}\left\{ah_{11}^*h_{21}\right\}+\mathrm{Re}\left\{bh_{11}^*h_{31}\right\}+\mathrm{Re}\left\{a^*bh_{21}^*h_{31}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{1L}^*h_{2L}\right\}+\mathrm{Re}\left\{bh_{1L}^*h_{3L}\right\}+\mathrm{Re}\left\{a^*bh_{2L}^*h_{3L}\right\}\right]\right), \\ \min\left(\left[\mathrm{Re}\left\{ah_{11}^*h_{21}\right\}+\mathrm{Re}\left\{bh_{11}^*h_{41}\right\}+\mathrm{Re}\left\{a^*bh_{21}^*h_{41}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{1L}^*h_{2L}\right\}+\mathrm{Re}\left\{bh_{1L}^*h_{4L}\right\}+\mathrm{Re}\left\{a^*bh_{2L}^*h_{4L}\right\}\right]\right), \\ \min\left(\left[\mathrm{Re}\left\{ah_{11}^*h_{31}\right\}+\mathrm{Re}\left\{bh_{11}^*h_{41}\right\}+\mathrm{Re}\left\{a^*bh_{31}^*h_{41}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{1L}^*h_{3L}\right\}+\mathrm{Re}\left\{bh_{1L}^*h_{4L}\right\}+\mathrm{Re}\left\{a^*bh_{3L}^*h_{4L}\right\}\right]\right), \\ \min\left(\left[\mathrm{Re}\left\{ah_{21}^*h_{31}\right\}+\mathrm{Re}\left\{bh_{21}^*h_{41}\right\}+\mathrm{Re}\left\{a^*bh_{31}^*h_{41}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{2L}^*h_{3L}\right\}+\mathrm{Re}\left\{bh_{2L}^*h_{4L}\right\}+\mathrm{Re}\left\{a^*bh_{3L}^*h_{4L}\right\}\right]\right), \\ \min\left(\left[\mathrm{Re}\left\{ah_{11}^*h_{21}\right\}+\mathrm{Re}\left\{bh_{31}^*h_{41}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{1L}^*h_{2L}\right\}+\mathrm{Re}\left\{bh_{3L}^*h_{4L}\right\}\right]\right), \\ \min\left(\left[\mathrm{Re}\left\{ah_{11}^*h_{31}\right\}+\mathrm{Re}\left\{bh_{21}^*h_{41}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{1L}^*h_{3L}\right\}+\mathrm{Re}\left\{bh_{2L}^*h_{4L}\right\}\right]\right), \\ \min\left(\left[\mathrm{Re}\left\{ah_{21}^*h_{31}\right\}+\mathrm{Re}\left\{bh_{11}^*h_{41}\right\}\right],\ldots,\left[\mathrm{Re}\left\{ah_{2L}^*h_{3L}\right\}+\mathrm{Re}\left\{bh_{1L}^*h_{4L}\right\}\right]\right) \end{pmatrix}. \tag{9}$$

The optimum MEBSTBC for all multicast users is selected according to Equation (9). After the optimum MEBSTBC is selected, and combining the received signals at the *j*th mobile multicast user, the estimates are given in Equation (10) where $\hat{s}_{i,j}$ is the estimated *i*th symbol at the *j*th mobile multicast user; $\eta_{1,j}$ and $\eta_{2,j}$ are the noise at the *j*th mobile multicast user.

$$\hat{s}_{i,j}=\frac{\sqrt{P}}{2}\left[\left|h_1\right|^2+\left|h_2\right|^2+\left|h_3\right|^2+\left|h_4\right|^2+2A_2\right]s_i+\eta_{i,j},\quad i=1,2 \tag{10}$$

Fig. 2 shows the percentage of channels that achieve full diversity for various multicast users when four transmit antennas are present at the base station. MEBSTBC with one bit extension of feedback (MEBSTBC (*k*=1)) achieves full diversity and full rate for two multicast users. When eight or more multicast users are present in the wireless environment and up to five bit extension of feedback are available, full diversity can be achieved 70% or less of all possible channel conditions.

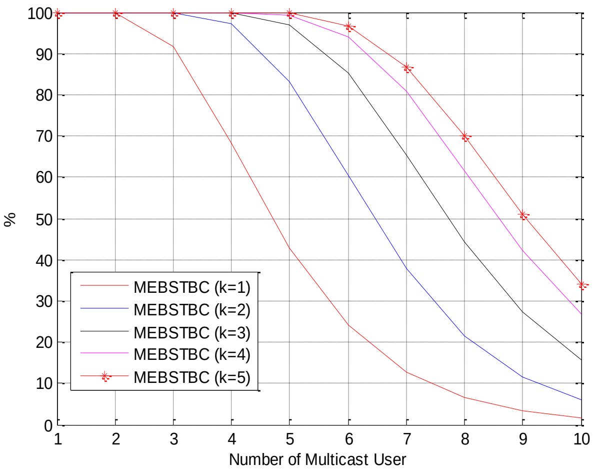

Fig. 2. The percentage of the channels that achieve full diversity for various multicast users when four transmit antennas are present at the base station.

The followings are the properties of the MEBSTBC for four or more transmit antennas:

1) One bit extension of feedback (*k*=1) can achieve full rate and full diversity for two multicast users.
2) When four transmit antennas are present at the base station, full diversity can be achieved for an arbitrary number of multicast users, if the inequality of Equation (11) is satisfied for all possible channel conditions.

$$A_2 \geq 0. \tag{11}$$

V. Performance Evaluations

The bit error probabilities of the MEBSTBC are evaluated for quaternary phase-shift keying (QPSK) modulation by computer simulations. The frame length is 128 symbol duration. For comparison, the bit error rate (BER) curve of the Alamouti code [10] which supports full rate for arbitrary number of multicast users with a diversity order 2, is included in Figure 3-4. Moreover, the BER curves of the unicast EBSTBC [7] are also included in Figure 3-4.

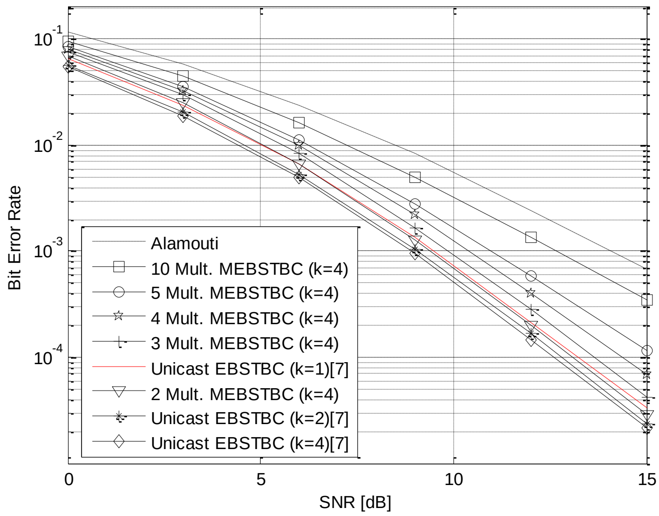

Fig. 3. BER performance of the EBSTBC and the MEBSTBC for three transmit antennas.

Fig. 3 presents the bit error probabilities of the MEBSTBC with four bits extension of feedback for three transmit antennas and various numbers of multicast users. It can be seen from the Figure 3 that the full diversity cannot be achieved more than four multicast users since the slope of curves does not decrease. The MEBSTBC with 10 multicast users (10 Mult. MEBSTBC (*k*=4)) provides better performance than the Alamouti ́s code which supports full rate for arbitrary number of multicast users. Compared to the MEBSTBC with 2 multicast users (2 Mult. MEBSTBC (*k*=4)), the EBSTBC with four bits extension of feedback (Unicast EBSTBC (*k*=4) [7]) has a signal-to-noise ratio (SNR) advantage of only 0.5 dB for a BER value of $2x10^{-4}$. However, the MEBSTBC with 2 multicast users (2 Mult. MEBSTBC (*k*=4)) provides better performance than the EBSTBC with one bit extension of feedback (Unicast EBSTBC (*k*=1) [7]), and the system transmission rate is doubled. Compared to the MEBSTBC with 3 multicast users (3 Mult. MEBSTBC (*k*=4)), the MEBSTBC with 4 multicast users (4 Mult. MEBSTBC (*k*=4)) and the MEBSTBC with 5 multicast users (5 Mult. MEBSTBC (*k*=4)), the EBSTBCs with four bit extension of feedback (Unicast EBSTBC (*k*=4) [7]), has an SNR advantage of just 1.06dB, 1.68dB, and 2.47dB, respectively. The proposed MEBSTBC sacrifices a small amount of coding gain to utilize system resources efficiently.

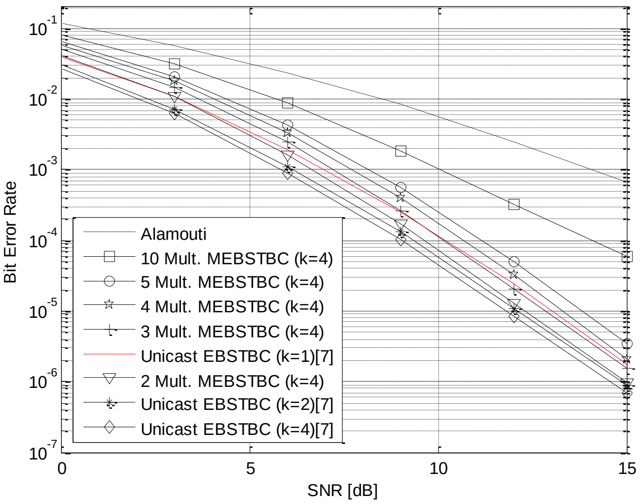

Fig. 4. BER performance of the EBSTBC and the MEBSTBC for four transmit antennas.

Fig. 4 presents the bit error probabilities of the MEBSTBC with four bits extension of feedback for four transmit antennas and various numbers of multicast users. It can be seen from the Figure 4 that the full diversity can be achieved for five multicast users since the slope of the curves does not decrease. Compared to the MEBSTBC with 2 multicast users (2 Mult. MEBSTBC (*k*=4)), the EBSTBC with four bits extension of feedback (Unicast EBSTBC (*k*=4) [7]) has a SNR advantage of only 0.48dB for a BER value of 10^{-5}. However, the MEBSTBC with 2 multicast users (2 Mult. MEBSTBC (*k*=4)) provides better performance than the EBSTBC with one bit extension of feedback (Unicast EBSTBC (*k*=1) [7]) and the system transmission rate is doubled. Compared to the MEBSTBC with 3 multicast users (3 Mult. MEBSTBC (*k*=4)), the MEBSTBC with 4 multicast users (4 Mult. MEBSTBC (*k*=4)), the MEBSTBC with 5 multicast users (5 Mult. MEBSTBC (*k*=4)), the EBSTBCs with four bit extension of feedback (Unicast EBSTBC (*k*=4) [7]), has an SNR advantage of only 1.03dB, 1.5dB, and 2dB, respectively. Compared to the Alamouti ́s code, the MEBSTBC with 10 multicast users (10 Mult. MEBSTBC (*k*=4)) provides a SNR advantage of approximately 4 dB for a BER value of 10^{-3}. Again, the proposed MEBSTBC sacrifices little coding gain to utilize system resources efficiently.

VI. Conclusions

In this paper, full rate and full diversity multicast service provisioning in Rayleigh fading channels is realized. Compared to the unicast EBSTBC, the MEBSTBC employs optimum MEBSTBC for all multicast users. This optimization sacrifices a slight coding gain but utilizes system resources efficiently. Namely, the proposed MEBSTBC sacrifices a slight coding gain but the system transmission rate increases in proportion to the number of multicast users.

References

[1] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. on Information Theory*, vol. 45, pp. 1456-1467, 1999.

[2] H. Jafarkhani, “A quasi-orthogonal space-time block code,” *IEEE Trans. Commun.*, vol. 49, pp. 1-4, 2001.

[3] O. Tirkkonen and A. Hottinen, “Complex space-time block codes for four Tx antennas,” in *Proc. IEEE GLOBECOM 2000*, IEEE, December 2000, pp. 1005-1009.

[4] W. Su and X. G. Xia, “On space-time block codes from complex orthogonal designs,” *Wirel. Pers. Commun*, vol. 25, pp. 1-26, 2003.

[5] M. E. Çelebi, S. Şahin, and Ü. Aygölü, “Increasing diversity with feedback: Balanced space-time block coding,” in *Proc. IEEE ICC*, Istanbul, June 2006, pp. 4836-4841.

[6] A. Ekşim and M. E. Çelebi, “Extended Cooperative Balanced Space-Time Block Coding for Increased Efficiency in Wireless Sensor Networks (Work in Progress),” *Networking 2009*, vol. 5550, pp. 456–467, May 2009.

[7] A. Ekşim and M. E. Çelebi, “Extended Balanced Space-Time Block Coding for Wireless Communications,” *IET Signal Processing*, vol. 3, pp. 476-484, November 2009.

[8] U. Varshney, “Multicast over wireless networks,” *Wirel. Commun. Mob. Comput*, vol. 2, pp. 667–692, 2002.

[9] A. Ekşim, “Extended Balanced Space-Time Block Coding in Wireless Networks,” Ph. D. thesis, Dept. Electronic and Commun. Eng., Istanbul Technical University, Istanbul, 2011.

[10] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE J. Select. Areas Commun*, vol. 16, pp. 1451-1458, 1998.

[11] A. Ekşim and M. E. Çelebi, “Performance Improvement of Binary Sensor Based Statistical STBC Cooperative Diversity Using Limited Feedback,” *IETE Technical Review*, vol. 27, pp. 60-67, Jan-Feb 2010.