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Gaussianization for Interference Background

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Abstract—**By weakening the bigger and strengthening the smaller, gaussianization can enhance the gaussianity of samples and improve performance of subsequent correlation test. Firstly, an explicit definition on gaussianizing filter and a practical method to evaluate the filtering performance are given. Secondly, two typical gaussianizing filters are proposed and studied. One is so-called U-filter, based on the probability density function and its derivate. The other is so-called G-filter, based on the cumulative distribution function and its inverse. Instances with lake trial data are illustrated. Finally, two applications, one in spectrum estimation and the other in Rao test, are discussed.**

Index Terms—**Gaussianization; Gaussian mixture; Non-Gaussian detection**

I. Introduction

In the classic detection theory, interference back- ground is often assumed as Gaussian. All the conventional active detections, such as match filter, correlation test and likelihood ratio test, just adopt this Gaussian assumption [1]. As widely known, statistical characteristics of the "true" Gaussian process can be described fully just with the first moment (i.e. the mean μ) and the second moment (i.e. the variance σ^2) [2]. So the test problem could be simplified the furthest – the match filter and the general match filter (a match filter succeeding to a prewhitener) are the optimal detectors of deterministic signals in either white or colored Gaussian background respectively [3]. Unfortunately, not all the cases in practice can adopt this simple assumption. The probability distribution of some interference backgrounds, such as reverberation in sonar [4] and clutter in radar [5], has a heavy tail because of its strong correlation with the transmitting pulse. This kind of background is no-Gaussian. If it is still modeled as Gaussian stochastic process in test problem, the detecting performance would be poor. This is to say, the conventional match filter and general match filter are no more optimal detector in non-Gaussian background. In recent years, with developments of high-power transmitting technology, the Gaussian assumption for background is more and more impertinent. It is very pressing to build a new high-performance detector to test signal in non- Gaussian background.

A typical idea with three procedures is as followed. Firstly, a non-Gaussian model, such as Gaussian mixture [6], Class-A [7], KA-distribution [8], K-distribu- tion [9] and so on, is adopted to fit the probability density function (PDF) of interference background. Secondly, a gaussianizing filter is set up based on PDF parameters estimation and used to gaussianize the background. This seems very similar to the prewhitening module which converting the colored background to the white. In fact, these two modules, gaussianization and prewhitening, are often going along mutually. When the background was filtered to be Gaussian and white, the subsequent match filter would be the optimal detector of signal. Therefore, the final step just is, to bring these filtering results into the frame of classic match filter or correlation detection. For written convenience, we called this type of detection "gaussianizing test" which maybe has both gaussianizing and prewhitening modules. The Rao efficient score test (viz. REST) [6] of weak signal in colored non-Gaussian background is such a gaussianizing test.

In brief, on the one hand, the gaussianizing test can get high performance because of its full utilization of non-Gaussian statistical information of background. On the other hand, it can be easily set up because of its frame -consistency with classic test. Obviously, gaussianization is one of the key techniques in gaussianizing test. So it is the most important and urgent task to build a high-performance gaussianizing filter based on accurate non-Gaussian modeling of interference background. In this paper, we would like to set up two typical kinds of gaussianizing filters, viz. *U*-filter and *G*-filter so-called. This paper is organized as followed: Section II gives an explicit definition on gaussianizing filter and a practical criterion on filter performance evaluation. Section III and IV study *U*-filter and *G*-filter in detail, respectively. Section V shows two applications of gaussianization, one in spectrum estimation, the other in REST. Section VI gives a conclusion on gaussianization.

II. Definition and Performance Evaluation

"*Gaussianizing filter*" – this name comes from not structures but functions. Strictly speaking, the structure of a gaussianization module is not always consonant with that of a true filter. However, for convenience, we still call it "*filter*". A processing module which has the function – no matter what distribution the input submits to, the output's distribution must be (or very close to) Gaussian – is generally called gaussianizing filter. Obviously, gaussianizing filter is not Gaussian filter which is just a digital frequency filter with a Gaussian pre-window [10]. Gaussianizing filtering is also different to the Gaussian sampling of Monte-Carlo methods [11], even though the former may utilize some knowledge of the latter. Gaussian sampling is just concern about how to generate a Gaussian process from some non- Gaussian sources whereas

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gaussianizing filter is interested in how to turn non-Gaussian process into Gaussian process with useful information lossless in so far as possible.

No matter how to realize in details, gaussianizing filter should depend on the non-Gaussian PDF parameters. Its typical practical block diagram is as shown in Fig.1 where $\hat{g}$ denotes the estimation of PDF parameters.

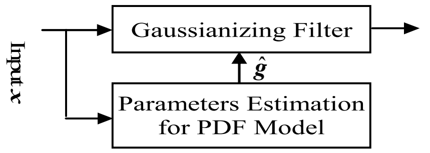


Figure 1. Block diagram of gaussianizing filter

According to actual demands, the non-Gaussian PDF model in Fig.1 can be chosen from Gaussian mixture, Class A, KA-distribution, K-distribution and so on. The exact components of $\hat{g}$ might vary with the PDF model chosen. There may be different estimating approaches for different $\hat{g}$. However, intuitively for engineering applications, the maximum likelihood estimation (MLE) may be the first choose in general. Unfortunately, since being more complicated than the Gaussian, MLE of non-Gaussian PDF is often hard to solve. Some alternatives are widely studied, such as penalized maximum likelihood estimation (PMLE) [12], expectation-maximization iteration (EM) [13], indirect least squares estimation for cumulant generating function (CGF) [14] and so on. In this paper, we'd like to use EM because of its good versatility and high efficiency.

Compared to the Gaussian, non-Gaussian process has wide dynamic range. Therefore, gaussianizing filter must have the function of weakening the big and strengthening the small (For convenience, this function is abbreviated to WBSS) in order to enhance the gaussianity of samples. Typical responding curves of gaussianizing filter are as shown in Fig.2. It can be seen that if only inflection points and curvatures are set accurately, WBSS would be reached.

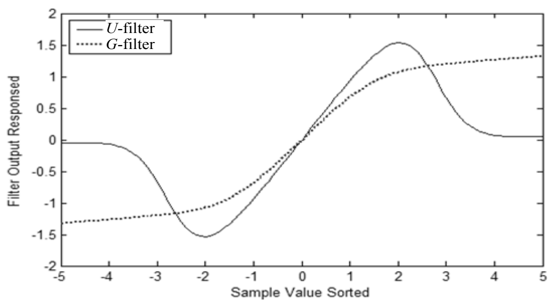


Figure 2. Typical responding curves of gaussianizing filter

How is gaussianization performance evaluated? It is can be imagined intuitively that, no matter what gaussianity the input process has, gaussianity of the output process could not be strong enough. So the performance evaluation of gaussianizing filter becomes the gaussianity test of the output process – high gaussianity indicates high filtering performance. Modern signal processing has told us that the obvious differences between Gaussian and non-Gaussian processes focus on moments and cumulants beyond 3rd order [2]. Based on this knowledge, some well-rounded parametric approaches to test gaussianity have been set up, such as Jarque-Bera test, Lilliefors test, Shapiro-Wilk test, Anderson-Darling test, Kolmogorov-Smirnov test and so on. All these approaches reach a statistic value finally and then compare it with a preset threshold under a preset confidence coefficient to make a decision of being Gaussian against being non- Gaussian. If the confidence coefficient and threshold aren't set correctly, misjudgments would be reached. Moreover, their results are either "yes" or "no", which is not in favor of evaluating in quantity measurement. Therefore, these approaches are neither convenient nor intuitionistic for gaussianizing filter performance evaluation. A graphical approach named quartile- quartile plot (Q-Q plot, viz. normal probability plot) [15] can avoid these disadvantages. In the following text, we'll show how to use Q-Q plot to test gaussianity with an instance.

Fig.3 is a Q-Q plot instance. Originally, the horizontal axis of Q-Q plot is normal order statistic medians while the vertical axis is ordered response values. Now, for observation convenience, they are mapped to sorted samples and probability values respectively. The thick line composed of "+" indicates current process whereas the thin straight line indicates the theoretical normal distribution. If they are coincident, the process is Gaussian. Otherwise, it is non-Gaussian. Departures of thick line from thin line indicate departures from normality. Obviously, process in Fig.3 is non-Gaussian strongly. In conclusion, using Q-Q plot to evaluate gaussianizing filter performance, we wish the two lines in Q-Q plot of output are coincident enough no matter how dispersed they are before gaussianized.

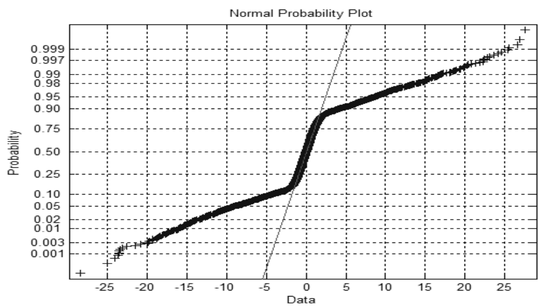


Figure 3. Q-Q plot instance of process

In the next sections, we'll study two typical gaussianizing filters: *U*-filter and *G*-filter. For speaking convenience, a concrete non-Gaussian PDF model must be chosen. We choose the 2nd order zero-mean Gaussian mixture (ZMGM2) in (1) which is a pure mathematical model with concise structure and compact parameters.

$$f(u) = (1-\varepsilon) f_B(u) + \varepsilon \cdot f_I(u) \tag{1}$$

Here, $f_B(u) = (1/\sqrt{2\pi\sigma_B^2})\exp\{-u^2/(2\sigma_B^2)\}$ can describe statistical property of noises in background, where σ_B^2 is much smaller, while $f_I(u) = (1/\sqrt{2\pi\sigma_I^2})\exp\{-u^2/(2\sigma_I^2)\}$ can describe statistical property of impulses in background (such as reverberation, clutter and so on), where σ_I^2 is much bigger. Obviously, $f_B(u)$ and $f_I(u)$ are PDFs of two zero-mean normal distributions, $\mathcal{N}(0, \sigma_B^2)$ and $\mathcal{N}(0, \sigma_I^2)$, respectively. ε is so- called mixture parameter. By a very few parameters, i.e. $\boldsymbol{g} = [\varepsilon, \sigma_B^2, \sigma_I^2]^T$, being adjusted, a very large quantity of background PDFs can be fit perfectly.

III. APPROACH I: *U*-FILTER

U in (2) is firstly introduced in [16] [17] as a weighted function for autoregressive (AR) parameter estimation. However, through deeply theoretical analysis and large quantities of simulational and experimental tests, we confirm that it has relative independence and integrality, and can be used as a gaussianizing filter individually. We call it *U*-filter.

$$U(u \mid \hat{g}) = -f'(u \mid \hat{g}) / f(u \mid \hat{g}) \qquad (2)$$

As we know, if $f(u)$, the PDF of a process *u*, is a campanulate curve as the thin solid line in Fig.4, its derivate, $f'(u) = df / du$, would like the thin dot line. So their minus ratio *U* would like the thick solid line. Obviously, if we use *U* as a filter responding function, it can realize WBSS, i.e. *U* is the system responding function of a gaussianizing filter. We call such gaussianizing filter with responding function *U* in (2) as *U*-filter.

As shown in Fig.4, *U* has two couples of inflexions, the pair near to origin being called the first class inflexions (inflexion-I) and the others being called the second class inflexions (inflexion-II). Let's make an agreement on naming samples between the origin and inflexion-I as "small samples", samples between inflexion-I and inflexion-II as "big samples", and samples beyond inflexion-II as "extra samples" (The term "extra" means they are extra to Gaussian process). This is to say, inflexion-I is separation between big and small samples while inflexion-II is separation between big and extra samples. In general, small samples will be strengthened at a positive slope, big samples will be weakened at a negative slope, and extra samples will be restrained at a low positive slope.

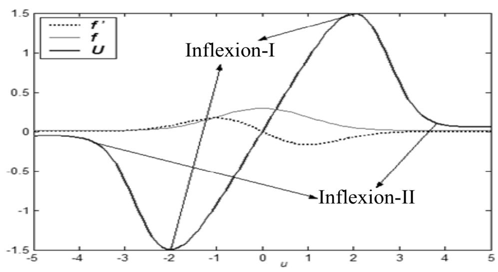


Figure 4. Form of U-filter

From (2) we can see that, just as mentioned in section II, it does be based on PDF parameter estimation $\hat{g} = [\hat{\sigma}_B^2, \hat{\sigma}_I^2, \hat{\varepsilon}]$, to set up *U*-filter. According to different $\hat{g}$, i.e. different non-gaussianity, the shape of *U* will change as shown in Fig.5, where σ_B^2 and ε are fixed to 1 and 0.3 respectively while σ_I^2 varied with different non-gaussianity.

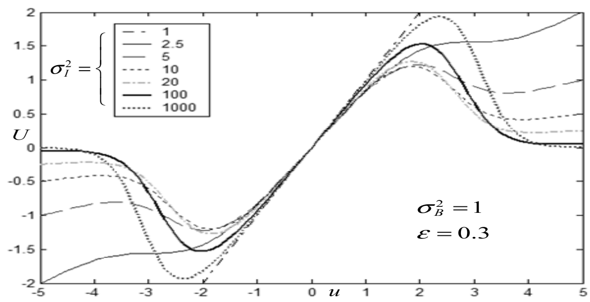
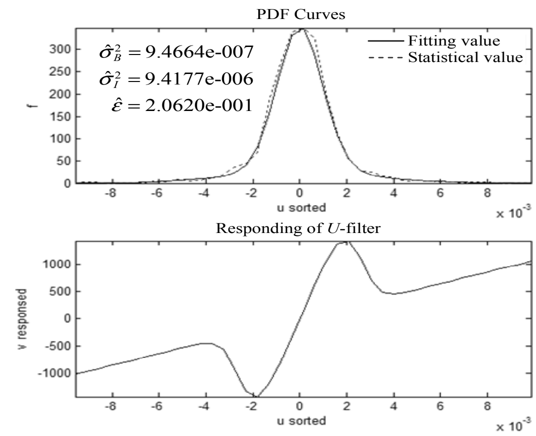


Figure 5. U at different non-gaussianity

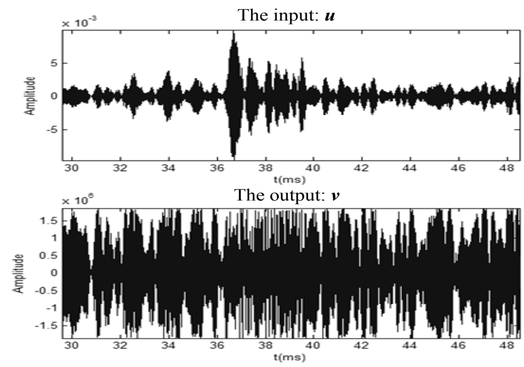
If σ_I^2=1, the process is Gaussian, and *U* is a straight line, so *U*-filter can't change gaussianity of process. If σ_I^2= 2.5, impulse's statistic characteristics are a little different with noise's, and *U*-filter will let small samples pass while weaken big and extra samples little. With increment of σ_I^2, non-gaussianity increases step by step, and *U*-filter will restrain big and extra (especially) samples more and more.

From Fig.5 we can also know that if PDF parameter estimation wasn't accurate enough, *U* would not meet the real requirement of gaussianizing and *U*-filter's performance would decrease. Therefore, perfect gaussianizing performance must be based on accurate fit of non-Gaussian PDF.

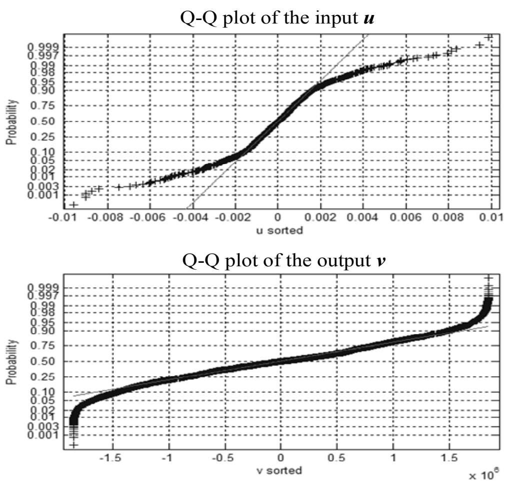
Fig.6 is an example of *U*-filter with a segment of experimental data which were gotten in the Songhua Lake. A sinusoidal pulse was transmit and the echo was received. Through conventional beam forming, 31 beam-outs were gotten from received signals of 17 transducers on a uniform line array. 18.963 ms data of 16th beam (on the abeam direction) are extracted and used to test *U*-filter performance. Suppose ***u*** and ***v*** denoting the samples sequences before and after *U*-filter, respectively.



(a) PDF modeling and responding of *U*-filter



(b) Waveforms of input and output



(c) Q-Q plots of input and output

Figure 6. U-filter for lake trail data processing

The input $\boldsymbol{u}$ is shown in the upper part of (b) which has passed through both band-pass filter and prewhitening filter. Its Q-Q plot is the upper part of (c). ZMGM2 in (1) is used to fit the PDF of $\boldsymbol{u}$. And the parameter estimation $\hat{g}$ is gotten by EM iterative algorithm, which is labeled on the upper part of (a) where the solid curve is the fitting PDF based on $\hat{g}$ while the dot curve is the statistical value by histogram. As can be seen, these two curves are coincident highly. This is to say, parameter estimation is very accurate. From estimation values we can see, σ_I^2 is only about 10 times of σ_B^2 and ε is only about 2, which account for weak non-Gaussianity of $\boldsymbol{u}$. And this is also can be read from the Q-Q plot of $\boldsymbol{u}$. The responding curve of *U*-filter is shown in the lower part of (a). According to the weak non-Gaussianity of $\boldsymbol{u}$, *U*-filter's restraint function on extra samples is not strong. At the same time, *U*-filter enlarges the small samples at a high positive slope. Reviewing the output $\boldsymbol{v}$ in the lower parts of (b), we can see that, the big samples have been restrained down while the small samples have been amplified up. Therefore, gaussianity of the output would be enhanced. This is also can be read from $\boldsymbol{v}$'s Q-Q plot as shown in the lower parts of (c) where most of thick and thin lines are overlap. All of these show that *U*-filter reaches good performance for gaussianization.

It must be pointed out that, *U*-filter's restraint on extra samples is at a low positive slope. This is to say, through *U*-filter's restraint, the bigger of extra samples is still bigger and the smaller is still smaller. The only change is their dynamic range become thinner. Compared with processing of big samples, this kind of restraint is weaker. In some wise, this mapping of extra samples with positive slope seems not appropriate. However, as just shown in Fig.5, it is only happened when the non-gaussianity of input is not quite strong. With increasing of input non-gaussianity, it would disappear. By the way, the main cause for thick line being apart from thin line at ends of $\boldsymbol{v}$'s Q-Q plot in Fig.5(c) is not the unapt mapping for extra samples but the high strengthen for samples around the first class inflexions which is due to the nature of *U*-filter. Fortunately, ratio of the apart to the whole is small enough for regarding process like $\boldsymbol{v}$ as Gaussian.

IV. Approach II: *G*-filter

Scott Chen etc. propose a technique called "gaussianization" for high dimensional density estimation in [18] in the view of image processing. It must be pointed out that their "gaussianization" conception is not mine. We term gaussianization (and gaussianizing filter) as an approach to turning the non-Gaussian background into the Gaussian in active detection whereas they term it as a technique for high dimensional data modeling. However, their means of beginning directly from the normal cumulative distribution's conversion to reduce dimensionality enlightens us to form another gaussianizing filter, i.e. *G*-filter.

G-filter is a gaussianization based directly on the inverse of normal cumulative distribution function. Supposed CDF of the standard normal distribution $\mathcal{N}(0,1)$ is Φ as (3) with the inverse function Φ^{-1},

$$p=\Phi(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u}\exp[-\frac{x^2}{2}]dx \tag{3}$$

the process submitting to $\mathcal{N}(\mu,\sigma^2)$ has CDF as (4)

$$p=\Psi(u)=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{u}\exp[-\frac{(x-\mu)^2}{2\sigma^2}]dx \tag{4}$$

with the inverse function

$$u=\Psi^{-1}(p)=\sigma\Phi^{-1}(p)+\mu$$

Now, if u submits to ZMGM2 as (1), its CDF will be

$$p=F(u)=(1-\varepsilon)\Psi_B(u)+\varepsilon\Psi_I(u) \tag{5}$$

where Ψ_B and Ψ_I are CDFs of $\mathcal{N}(0,\sigma_B^2)$ and $\mathcal{N}(0,\sigma_I^2)$, respectively.

In some sense, gaussianizing filter can be looked upon turning process with CDF (5) into process with CDF (3) or (4). Based on this idea, for ZMGM2 process u with PDF as (1), a gaussianizing filter, so-called *G*-filter, can be set up as (6)

$$G(u\,|\,\hat{g})=\Phi^{-1}\left\{(1-\varepsilon)\Phi(\frac{u}{\hat{\sigma}_B^2})+\varepsilon\Phi(\frac{u}{\hat{\sigma}_I^2})\right\}\sigma \tag{6}$$

where σ is the output's standard variance, which can be fixed to 1 (this is equal to turning non-Gaussian process into standard normal process).

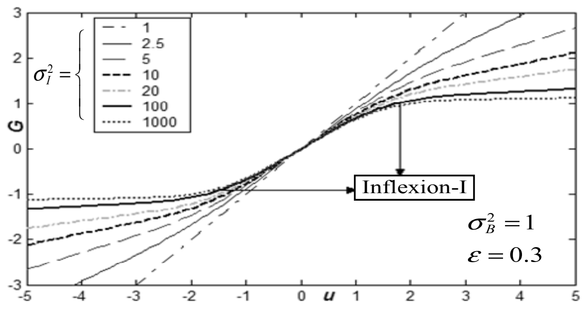
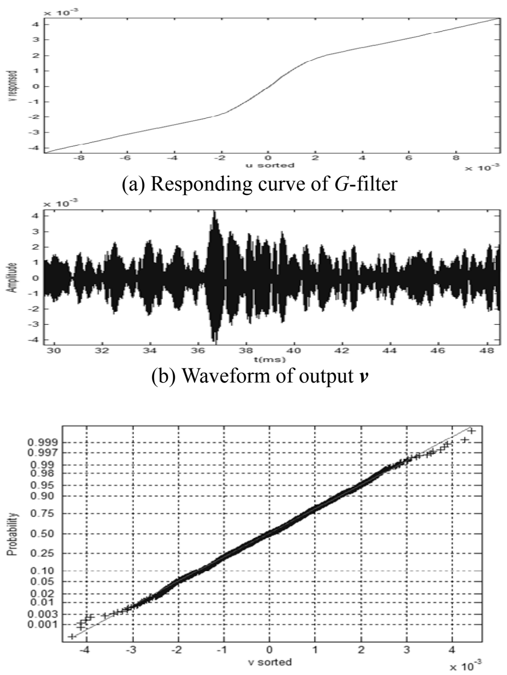


Figure 7. G at different non-gaussianity

G-filter's responding curves at different non- gaussianity are shown in Fig.7, where just like in Fig.5 σ=1, σ_B^2=1, ε=0.3 while σ_I^2 is changed with different non-gaussianity. As we can see, the most distinct difference to *U*-filter is that *G*-filter has only one class inflexions, viz. inflexion-I, while no inflexion-II. This is to say, in *G*-filter, input samples are only divided into two classes – one is small samples, the other is big samples, no such extra samples. And the secondary distinction to *U*-filter is that *G*-filter would not take a negative slope to map big samples. All the restraints are "gentle" so that, we can hope, there would be no obvious departures at ends of output's Q-Q plot, in fact, which can be proved true in Fig.8 (c). Let's continue to study Fig.7. If $\sigma_I^2=1$, as the same as *U*-filter in Fig.5, the process is Gaussian, and *G* is a beeline, so *G*-filter can't change input gaussianity. With increase of σ_I^2 step by step, non-gaussianity is enhanced, and the G curve becomes more and more flexural, the effect of inflexions become more and more prominent, in other words, *G*-filter's restraint function to big samples becomes stronger and stronger.

In the following texts, the lake trial data used in section III will be used to test the performance of *G*-filter. The results are shown in Fig.8. Just as be pointed out in section III, the gaussianity of this data segment is not quite high. Therefore, the responding curve of *G*-filter is not quite flexural, as shown in (a). Passing through this *G*-filter, small samples are amplified in more high degree than big samples being mapped. This leads to an equivalent gaussianizing result-WBSS. Actually, comparing the output $\boldsymbol{v}$ as shown in (b) with the input $\boldsymbol{u}$ as shown in Fig.6(b), we can see that small samples are strengthened more than big samples indeed. Seeing about Q-Q plot of $\boldsymbol{v}$ in (c), we can found, the thick line is highly coincident with the thin line. This is to say that the output has submitted to normal distribution and *G*-filter reaches perfect performance.



(a) Responding curve of *G*-filter

(b) Waveform of output $\boldsymbol{v}$

(c) Q-Q plot of output $\boldsymbol{v}$

Figure 8. G-filter for lake trail data processing

V. APPLICATIONS OF GAUSSIANIZATION

In this section, we'll illustrate two typical applications of gaussianizing filter in active signal detection. One is in AR power spectrum estimation which is the key procedure in prewhitening colored background. The other is in the building of REST after prewhitenning.

A. Gaussianization in AR parameter estimation

As we know, for Gaussian process, the least squares estimation (LSE) of AR parameter is equal to its maximum likelihood estimation (MLE) which is a superior efficient estimation [19]. However, for non-Gaussian process, the efficiency of LSE might at a big discount because of the absence of statistical characteristics beyond the 2nd order. The MLE [20] based on non-Gaussian PDF is still the efficient estimation but it is non-linear seriously with very complicated structure and hard to be solved, so it is impracticable. If gaussianization is introduced into LSE, the efficiency of estimation would be enhanced higher than the conventional LSE while the complexity of solution would be decreased lower than the non-Gaussian MLE. The weighted least squares estimation (WLSE) [16] as (7) is such an excellent AR parameter estimation for colored non-Gaussian process,

$$\begin{bmatrix} \boldsymbol{x}_1^T\boldsymbol{\Gamma}\boldsymbol{x}_1 & \boldsymbol{x}_1^T\boldsymbol{\Gamma}\boldsymbol{x}_2 & \cdots & \boldsymbol{x}_1^T\boldsymbol{\Gamma}\boldsymbol{x}_P \\ \boldsymbol{x}_2^T\boldsymbol{\Gamma}\boldsymbol{x}_1 & \boldsymbol{x}_2^T\boldsymbol{\Gamma}\boldsymbol{x}_2 & \cdots & \boldsymbol{x}_2^T\boldsymbol{\Gamma}\boldsymbol{x}_P \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x}_P^T\boldsymbol{\Gamma}\boldsymbol{x}_1 & \boldsymbol{x}_P^T\boldsymbol{\Gamma}\boldsymbol{x}_2 & \cdots & \boldsymbol{x}_P^T\boldsymbol{\Gamma}\boldsymbol{x}_P \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_P \end{bmatrix} = -\begin{bmatrix} \boldsymbol{x}_1^T\boldsymbol{\Gamma}\boldsymbol{x}_0 \\ \boldsymbol{x}_2^T\boldsymbol{\Gamma}\boldsymbol{x}_0 \\ \vdots \\ \boldsymbol{x}_P^T\boldsymbol{\Gamma}\boldsymbol{x}_0 \end{bmatrix} \quad (7)$$

where, $\boldsymbol{x}_j=[x_{P-j+1},x_{P-j+2},\cdots,x_{N-j}]^T$ is the *j*-th segment of colored GM process $\boldsymbol{x}=[x_1,x_2,\cdots,x_N]^T$ (*N* is the length of $\boldsymbol{x}$, *P* is the order of AR model, every segment $\boldsymbol{x}_j$ is a set of sequential *N-P* samples in $\boldsymbol{x}$); $\hat{\boldsymbol{a}}=[\hat{a}_1,\hat{a}_2,\cdots,\hat{a}_P]^T$ is AR parameter estimation; The weighted matrix inserted in two samples segments is $\boldsymbol{\Gamma}=\mathrm{diag}\{\Gamma(\hat{u}_{P+1}),\Gamma(\hat{u}_{P+2}),\cdots,\Gamma(\hat{u}_N)\}$ where every Γ is a weight coefficient consisted of *U*-filter, just as shown in (8)

$$\Gamma(\hat{u}_n)=U(\hat{u}_n)/\hat{u}_n \quad (8)$$

where $\hat{u}_n$ is rough estimation of the driving gotten by the conventional LSE prewhitenning.

Researches in [16] show that, because of introducing gaussianizing weight coefficients in, WLSE utilizes non-Gaussian information of the observed sequence. So its variance may be smaller than the conventional LSE ten times and very close to Crammer-Rao bound. This is to say, WLSE is an asymptotically efficient estimation for non-Gaussian AR parameter.

B. Gaussianization in the construction of REST

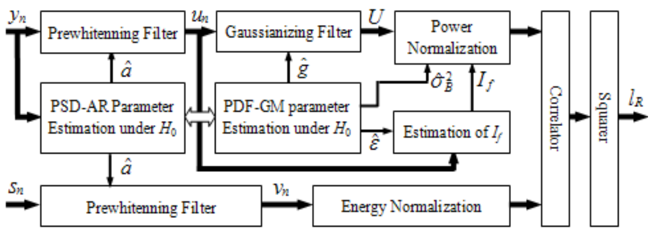


Figure 9. Block diagram of REST

As we know, the general likelihood ratio test (GLRT) is the asymptotically optimal detection of deterministic signal in non-Gaussian background [16] and REST is its efficient approximation at small signal-noise-ratio which can get more high performance than the conventional match filter [6]. Moreover, REST is also representative that applies prewhitenning and gaussianizing techniques successfully. Using these modules, REST in [6] can be re-organized as Fig.9 where y_n is the received sequence, s_n is the copy signal sequence，H_0 against H_1 are the null against alternative hypotheses for existence of echo.

Note that the gaussianizing filter which may be either *U*-filter or *G*-filter is placed after the prewhitening filter. Besides this explicit utilization, gaussianization technology is also implicitly applied into the coupling estimation of PSD-AR and PDF-GM parameters which is shown as double ends of $\Leftrightarrow$ in the figure in order to increase accuracy of parameter estimation. For example, WLSE in section A which utilizes *U*-filter is just a part of such coupling estimation.

From applying researches in underwater acoustic signal processing, we find that, such REST can get more gain of the output signal-reverberation-ratio over 3dB than the conventional match filter for detection of weak signal in reverberation-restricted area.

VI. Conclusion

It is meaningful to compare these two kinds of gaussianizing filters –*U*-filter and *G*-filter. Both of them use WBSS technique to increase the gaussianity of process. However, *U*-filter has two couples of inflexions (inflexion-I and inflexion-II) and classifies samples into three sets (the small, big and extra samples), while *G*-filter has only one couples of inflexions (inflexion-I) and classifies samples into two sets (the small and big samples). To small samples, both of them takes a higher positive slope mapping. To big samples, *U*-filter takes a negative slope mapping while *G*-filter takes a lower positive slope mapping. To extra samples, the particular classification of *U*-filter, *U*-filter takes a lower positive slope mapping. Because of change of slope sign beside inflexion-I, one couple of extreme points will come forth in the output of *U*-filter. Note that these two extreme points are not at minimum or maximum of the input, it is very clear for *U*-filter's output that the effect like "amplitude limiting" would appear in the waveform and departures of the two lines at ends would appear in the Q-Q plot. We call this "extreme point phenomena" (viz. EPP) which is special for *U*-filter whereas not belongs to *G*-filter.

On the face of EPP, *G*-filter looks like better than *U*-filter. In fact, it is not exactly so. Note that the ultimate object of gaussianizing filter that we have mentioned above is to restrain impulses such as reverberation and clutter. Most of them appear as sticks in the waveform. *G*-filter without EPP cannot often weaken these sticks enough whereas *U*-filter with EPP can. Now, what we are concerned about mostly is that whether EPP can make a negative effect on gaussianizing test or not. Considering samples around inflexion-I are very few in the total, values in the output which close to the amplitude bounds would be also very few. Therefore, the succeeding correlation would not be influenced, and even though it does, the negative effect would also be very limited.

Consequently, we can conclude that, both *U*-filter and *G*-filter can be competent for the task of gaussianization although their concrete mechanisms and wises to realize WBSS may have some differences. We can choose one of them to realize gaussianizing filter as circumstances demand in practice. Large quantities of simulational and experimental verifications make it clear that, through these gaussianizing filters integrated some proper prewhitenning filters, interference background would become (or very closed to) white Gaussian. And then, the succeeding correlation detection (or match filter) would do its best.

At last, it is must be pointed out that, despite being studied in the view of active signal detection in this paper, the gaussianization is widely applied in other domains such as communication, image manipulation, speech signal processing and so on.

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