Controller Design for Tracking Control of an Under-Actuated Surface Ship

Werneld E. Ngongi and Jialu Du

***Abstract*—In this paper an output feedback controller for tracking control of surface ships based on Euler-Lagrange equations has been proposed. It has been assumed that a surface ship is moving in a horizontal plane and under-actuated in sway direction. The change of coordinate's method is applied to overcome the third order component that arises in the Lyapunov function derivatives due to Coriolis and centripetal forces term. The design of the controller is based on the backstepping control technique and Lyapunov stability theory. Firstly, the observer is derived using the change of coordinate method. Next, backstepping control technique is employed to derive the control law. Finally, a global asymptotic convergence is proven using Lyapunov stability theorems. Simulations are provided to demonstrate the performance of the designed controller and prove tracking error of the controller convergence.**

***Index Terms*—Change of coordinates, euler-lagrange equations, output feedback controller, under-actuated surface ship, third order component, backstepping control technique.**

I. Introduction

A conventional ship considers the motion in surge (forward), sway (sideways), and yaw (heading). Normally, we have surge and sway control forces and yaw control moment available for navigating the ship. However, this assumption is not practical for all ships. For example, some ships are either equipped with two autonomous aft-thrusters or with one main aft-thruster and a rudder, but there is no any bow or side thrusters, like, for example, many supply ships. As a result, there is control force in the sway axis. In this paper, we deal with the tracking control for ships with only surge (x-axis) and yaw (z-axis) control moment available. Since we need to control 3DOF (three degrees of freedom) with only two inputs available, thus we are dealing with an under-actuated problem. Since we want to control the ship motion in the horizontal plane, therefore, we neglect the dynamics related to the motion in heave, roll, and pitch [1], [2].

The tracking control of surface ships is compulsory in order to achieve offshore exploitation, activities and various applications such as the drilling, pipe laying, diving support, etc. [3]. Several control strategies have been proposed for surface ships such as global uniform asymptotic stabilization of an under-actuated surface vessel was presented [4]. A Robust adaptive ship autopilot with wave filter and integral action was proposed in [5]. Sliding mode control was presented and experimentally implemented for trajectory tracking of under-actuated autonomous surface vessels [6]. An approximation based control was developed to handle model uncertainties and unknown disturbances for fully actuated ocean surface vessels [7]. Another approach to design a global tracking controller for under-actuated ships with the sway axis unactuated has been introduced [8]. In the proposed work, the assumption that the mass and damping matrices are diagonal is not used as required globally to track the reference trajectory. Also, a pi-type sliding controller for tracking control of the ship is given in [9]. The method applies three components of which one component is to ensure stability in the absence of uncertainties and environmental disturbances. The rest two components are and proportional integral type variable structure controllers respectively. An adaptive feedback controller using neural network feedback-feedforward compensator for a surface ship at high speed has been proposed [10] so as to include the influence of the nonlinear hydrodynamic damping terms on the tracking precision. In [10], the neural network feedback-feedforward compensator is used to for estimation of the uncertainty nonlinear parts of the system where a single layer neural network is used to obtain the adaptive signal online. A trajectory tracking controller design for an underactuated surface ship has been given in [11]. The control system design is based on a linear algebra method and numerical method. Saturated control inputs and an assumption that the reference trajectory is used [11]. The main future of the controller addressed in [11] is that the conditions for tracking errors goes to zero and calculation of control action is done by solving linear equations. Many studies have been presented for the tracking control of surface ships. However, more studies of precise control of trajectory of ships are still in demand. Thus, in this study an observer design for the tracking control of under-actuated surface ships has been presented.

Generally, it is noted that the coriolis and centripetal forces cause third-order components to appear in the Lyapunov function derivative, and these terms can only be dominated on a compact domain about the origin, preventing a global stability conclusion. These high order terms arise due to coriolis and centrifugal forces vector in the Euler-Lagrange equation [12]. Therefore in this paper we utilize the alteration of coordinates to solve the problem.

The key contribution of this paper is based on the design an output feedback controller for tacking control of the under-actuated ship via Euler-Lagrange equation and ensures that the surface ship complies with the desired dynamics and behavior using only available position and heading of the

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The authors are with the School of Information Science and Technology, Dalian Maritime University, Dalian, Liaoning 116026 China (e-mail: werneld@hotmail.com, dujl66@163.com).

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ship. This paper is structured as follows: Section two presents Euler-Lagrange formula in vector form. Mathematical model of under-actuated surface ship is presented in section three. Observer design for under-actuated surface ship is persuasively presented in section four. While backstepping control design and analysis is eloquently presented in section five. Lastly, the conclusion is given in section six.

II. Euler-Lagrange Formula

The Euler-Lagrange equation for dynamical system can be represented in the following form

$$\frac{d}{dt}\left[\frac{\partial L(q,\dot{q})}{\partial \dot{q}}\right]-\frac{\partial L(q,\dot{q})}{\partial L}=\tau_T \tag{1}$$

where Lagrange $L=T(q,\dot{q})-V(q)$, $T(q,\dot{q})$ is the kinetic energy and $V(q)$ is the potential energy function and $\tau_T \in \Re^n$ is the control input. It is assumed that the kinetic energy function is of quadratic form

$$T(q,\dot{q})=\frac{1}{2}\dot{q}^T M(q)q \tag{2}$$

where $M(q)\in\Re^{n\times n}$ is positive definite and uniformly bounded satisfying the following condition

$$0<\lambda_{\min}(M(q))\le\lambda_{\max}(M(q))<\infty \tag{3}$$

Using the Christoffel [9] symbols of the first kind, the equation given in (1) can be written as

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+D(q,\dot{q})\dot{q}+G_q(q)=\tau_T \tag{4}$$

where $C(q,\dot{q})$ is the coriolis and centrifugal matrix, $G_q(q)=\dfrac{\partial V(q)}{\partial q}$ are gravitational and buoyancy forces and $D(q,\dot{q})$ is the hydrodynamic damping matrix.

Remark 1: The coriolis and centrifugal matrix $C(x,y)$ is bounded in x and linear in y having the following properties [9]

$$1)\ \ C(x,y)y=C(q,y)x,\ \text{ for all } x,y,q\in\Re^n \tag{5}$$

$$2)\ \ \|C(x,y)\|\le C_Z\|y\| \tag{6}$$

where $C_Z>0$. In addition, we can parameterize the coriolis and centrifugal matrix in the form of Christoffel symbols [8].

III. Ship Model

A 3 degree of freedom (DOF) dynamic model of a surface ship moving in a horizontal plane (i.e. surge, sway, and yaw modes) is used in this paper. This model is found in [13], and is composed of the kinematics

$$\dot{\eta}=J(\eta_\psi)\upsilon \tag{7}$$

$$M\dot{\upsilon}+C(\upsilon)\upsilon+D(\upsilon)\upsilon=\tau \tag{8}$$

where $\eta=[\eta_x,\eta_y,\eta_\psi]^T\in\Re^2\times S$ represents the earth-fixed position and heading (with $S=[-\pi,\pi]$), $\upsilon=[\upsilon_x,\upsilon_y,\upsilon_\psi]^T\in\Re^3$ represents the ship-fixed velocity, $J(\eta_\psi)\in SO(3)$ is the transformation matrix

$$J(\eta_\psi)=\begin{bmatrix}\cos(\eta_\psi) & -\sin(\eta_\psi) & 0\\ \sin(\eta_\psi) & \cos(\eta_\psi) & 0\\ 0 & 0 & 1\end{bmatrix} \tag{9}$$

Which transforms from the ship-fixed BODY frame (B) to the earth-fixed NED frame (N), M is the inertia matrix, $C(\upsilon)$ is the centrifugal and coriolis matrix, while $D(\upsilon)$ is the hydrodynamic damping matrix. The system matrices satisfy the properties $M=M^T>0$, $C=-C^T$ and $D>0$. The ship-fixed propulsion forces and moment is represented by the vector $\tau=[\tau_x,\tau_y,\tau_\psi]^T\in\Re^3$. The values of M , $C(\upsilon)$, $D(\upsilon)$ are:

$$M=\begin{bmatrix}m_{11} & 0 & 0\\ 0 & m_{22} & 0\\ 0 & 0 & m_{33}\end{bmatrix},\ D=\begin{bmatrix}d_{11} & 0 & 0\\ 0 & d_{22} & 0\\ 0 & 0 & d_{33}\end{bmatrix},\ C(\upsilon)=\begin{bmatrix}0 & 0 & c_{13}\\ 0 & 0 & c_{23}\\ c_{31} & c_{32} & 0\end{bmatrix}$$

where $c_{13}=-m_{22}\upsilon_y$, $c_{23}=m_{11}\upsilon_x$, $c_{31}=m_{22}\upsilon_y$ and $c_{32}=-m_{11}\upsilon_x$. In this paper we assume that there is no thrust force in sway direction, therefore there is no motion in sway direction. Thus, $\tau_y=0$ and $\tau=[\tau_x,0,\tau_\psi]^T$. Furthermore, we assume that the only position (η_x,η_y) and heading η_ψ are available for measurement.

IV. Observer Design

In this part we present an observer based on the general Euler-Lagrange equation. We introduce change of coordinates to overcome or eliminate the problem of third-order terms that remain in the Lyapunov function derivatives. Third-order terms are due to the coriolis and centrifugal forces in (4). These terms have quadratic growth in the velocities which are not measured. Now we introduce the following alteration of coordinates to remove these undesired nonlinearities, thus let

$$z_1=\eta \tag{10}$$

$$z_2=\dot{z}_1=\dot{\eta} \tag{11}$$

Thus, rewriting (4) we obtain the following

$$z_1=z_2 \tag{12}$$

$$M(z_1)\dot{z}_2+C(z_1,\dot{z}_1)\dot{z}_1+D(z_1,\dot{z}_1)\dot{z}_1+G_{z_1}(z_1)=\tau_T \tag{13}$$

Now to remove the nonlinear term $C(z_1,\dot{z}_1)\dot{z}_1$ we can define the following coordinate change

$$w_1=e^{\varphi(t,z_1)}z_2 \tag{14}$$

where $\varphi(t, z_1)$ is invertible matrix to be defined. Taking time derivative of (14) we obtain the following result

$$\dot{w}_1 = e^{\varphi(t, z_1)}\dot{z}_2 + \dot{\varphi}(t, z_1)e^{\varphi(t, z_1)}z_2 \tag{14}$$

Substituting (14) in (12) and (13) yields the following result

$$\dot{w}_1 = e^{\varphi(t, z_1)}M^{-1}(z_1)[\tau_T - C(z_1, \dot{z}_1)z_2 - D(z_1, \dot{z}_1)z_2 - G_{z_1}(z_1)] + \dot{\varphi}(t, z_1)e^{\varphi(t, z_1)}z_2 \tag{15}$$

Further, equation (15) can be written as

$$\dot{w}_1 = e^{\varphi(t, z_1)}M^{-1}(z_1)[\tau_T - D(z_1, \dot{z}_1)z_2 - G_{z_1}(z_1)] + e^{\varphi(t, z_1)} \times[\dot{\varphi}(t, z_1) - M^{-1}(z_1)C(z_1, \dot{z}_1)]z_2 \tag{16}$$

Now we choose a dynamic for $\varphi(t, z_1)$ such that the coriolis matrix will diminish during the observer derivation. Let

$$\dot{\varphi}(t, z_1) = M^{-1}(z_1)C(z_1, \dot{z}_1) \tag{17}$$

The objective is to cancel the effect of the $C(z_1, \dot{z}_1)$ term in the Lyapunov function and to providing negative and radially unbounded term in Lyapunov function derivative.

Remark 2: The solution of (17) is bounded. Proof: The coriolis and centrifugal matrix $C(z_1, .)$ can be selected as

$$C\left(z_1, y\right) = \left\{\sum_{i=1}^{n} c_{ijk}y_i\right\} \tag{18}$$

where $c_{ijk} = \frac{1}{2}\left[\frac{\partial m_{kj}}{\partial z_{1i}} + \frac{\partial m_{ik}}{\partial z_{1j}}\right] - \frac{\partial m_{ij}}{\partial z_{1k}}$ are the Christoffel symbol of the first kind, $M(z_1) = \{m_{ij}(z_1)\}$. Now applying properties (i) and (ii), the Christoffel symbols are bounded and thus the Coriolis and centrifugal matrix satisfy the following conditions

$$\|C(x, y)\| = \le \left\|\left\{\sum_{i=1}^{n} c_{ijk}y_i\right\}\right\|\|y\| \le C_Z\|y\| \tag{19}$$

Hence from equation (15) we have

$$\|\dot{\varphi}(t, z_1)\| \le \left\|M^{-1}(z_1)\right\|\|C(z_1, \dot{z}_1)\| \le \frac{C_Z}{\lambda_{\min}}\|\dot{z}_1\| \tag{20}$$

where $\lambda_{\min}$ is the minimum eigenvalue of the matrix $M(z_1)$. Using Lemma from [14], yields the following results

$$\|\varphi(t, z_1)\| \le \frac{C_Z}{\lambda_{\min}}\int_{t_0}^{t}\|\dot{z}_1\|dt \le \frac{C_Z}{\lambda_{\min}}(z_1(t) - z_1(t_0)) \tag{21}$$

Since the control object is to converge the z_1 state to a smooth trajectory, therefore the solution is bound. Now substituting (17) into (16) result to the following

$$\dot{w}_1 = -e^{\varphi(t, z_1)}M^{-1}(z_1)D(z_1, \dot{z}_1)e^{-\varphi(t, z_1)}w_1 + e^{\varphi(t, z_1)}M^{-1}(z_1)[\tau_T - G_{z_1}(z_1)] \tag{22}$$

Then recalling (12) and (13) and substitute (22) into them we have the following

$$\dot{z}_1 = e^{-\varphi(t, z_1)}w_1 \tag{23}$$

$$\dot{w}_1 = -e^{\varphi(t, z_1)}M^{-1}(z_1)D(z_1, \dot{z}_1)e^{-\varphi(t, z_1)}w_1 + e^{\varphi(t, z_1)}M^{-1}(z_1)[\tau_T - G_{z_1}(z_1)] \tag{24}$$

Based on (23) and (24), the following observer is proposed

$$\dot{\hat{z}}_1 = e^{-\varphi(t, z_1)}\hat{w}_1 + K_1(z_1 - \hat{z}_1) \tag{25}$$

$$\dot{\hat{w}}_1 = -e^{\varphi(t, z_1)}M^{-1}(z_1)D(z_1, \dot{\hat{z}}_1)e^{-\varphi(t, z_1)}\hat{w}_1 + e^{\varphi(t, z_1)}M^{-1}(z_1) \times[\tau_T - G_{z_1}(z_1)] + K_2(t, z_1)(z_1 - \hat{z}_1) \tag{26}$$

where $\hat{z}_1$ and $\hat{w}_1 = e^{\varphi(t, z_1)}\hat{z}_1$ are estimates of z_1 and w_1 respectively, $K_1 = K_1^T$ is a positive diagonal gain matrix. $K_2(t, z_1)$ is defined in remark 3. Now subtracting (23) from (25) and (24) from (26) yield the following error dynamics

$$\dot{\tilde{z}}_1 = e^{-\varphi(t, z_1)}\tilde{w}_1 - K_1\tilde{z}_1 \tag{27}$$

$$\dot{\tilde{w}}_1 = -e^{\varphi(t, z_1)}M^{-1}(z_1)D(z_1, \dot{\hat{z}}_1)e^{-\varphi(t, z_1)}\tilde{w}_1 - K_2(t, z_1)\tilde{z}_1 \tag{28}$$

where $\tilde{z}_1 = z_1 - \hat{z}_1$ and $\tilde{w}_1 = w_1 - \hat{w}_1$ are error dynamics of the observer.

Remark 3: Given a positive definite matrix Γ_1 and Γ_2 such that the matrix gain $K_2(t, z_1)$ is chosen such that

$$K_2(t, z_1) = \Gamma_1^{-1}e^{-\varphi(t, z_1)}\Gamma_2 \tag{29}$$

then the estimation errors $\tilde{z}_1$ and $\tilde{z}_2$ converge globally to the origin. Moreover, (12) and (13) can be written in new coordinates $(z_1, \hat{z}_2)$ as

$$\dot{\chi}_1 = \gamma(\chi_2, \tau_T) + B(\chi_1)\chi_2 \tag{30}$$

$$\dot{\chi}_2 = F(\chi_1, \chi_2)\chi_2 \tag{31}$$

To prove this we define Lyapunov function candidate as

$$V(\tilde{z}_1, \tilde{w}_1) = \frac{1}{2}\tilde{z}_1^T\Gamma_2\tilde{z}_1 + \frac{1}{2}\tilde{w}_1^T\Gamma_1\tilde{w}_1 \tag{32}$$

Taking time derivative of (32) along the trajectory of (27) and (28) together yields the following results

$$\dot{V} = -\tilde{z}_1^T\Gamma_2K_1\tilde{z}_1 - \tilde{w}_1^T\Gamma_1e^{\varphi(t, z_1)}M^{-1}(z_1)D(z_1, \dot{\hat{z}}_1)e^{-\varphi(t, z_1)}\tilde{w}_1 - \tilde{w}_1^T(\Gamma_1K_2(t, z_1) - e^{-\varphi(t, z_1)}K_2)\tilde{z}_1 \tag{33}$$

Choosing $K_2(t, z_1)$ as in (29), then the time derivative of the Lyapunov function of (27) and (28) can be written as

$$\dot{V} = -\tilde{z}_1^T\Gamma_2K_1\tilde{z}_1 - \tilde{w}_1^T\Gamma_1e^{\varphi(t, z_1)}M^{-1}(z_1)D(z_1, \dot{\hat{z}}_1)e^{-\varphi(t, z_1)}\tilde{w}_1 \le 0 \tag{34}$$

Therefore from (34) we conclude that the observer is globally exponentially stable equilibrium, thus

$$\left\|\begin{matrix}\tilde{z}_1(t)\\ \tilde{w}_1(t)\end{matrix}\right\| \le \left\|\begin{matrix}\tilde{z}_1(t_0)\\ \tilde{w}_1(t_0)\end{matrix}\right\|e^{-\delta_0(t-t_0)} \tag{35}$$

Moreover, using (14) and $\hat{w}_1 = e^{\varphi(t, z_1)}\hat{z}_2$, we obtain the following

$$\tilde{w}_1 = e^{\varphi(t, z_1)} \tilde{z}_2 \tag{36}$$

thus using (35) and remark 2, we conclude that the equilibrium of $\tilde{z}_2$ is globally exponential stable, hence

$$\left\| \begin{matrix} \tilde{z}_1(t) \\ \tilde{z}_2(t) \end{matrix} \right\| \leq \zeta_0 \left\| \begin{matrix} \tilde{z}_1(t_0) \\ \tilde{z}_2(t_0) \end{matrix} \right\| e^{-\delta_0(t-t_0)} \tag{37}$$

Therefore based on the above analysis, we shall derive an observer for the under-actuated ship using (7) and (8). First we write (7) and (8) in the new coordinate system, i.e. $(\eta, \hat{\upsilon})$ as follows

$$\begin{bmatrix} \dot{\eta} \\ \dot{\hat{\upsilon}} \end{bmatrix} = \begin{bmatrix} J(\eta_\psi)\hat{\upsilon} \\ -M^{-1}C(\hat{\upsilon})\hat{\upsilon} - M^{-1}D\hat{\upsilon} + M^{-1}\tau \end{bmatrix} + \begin{bmatrix} H_{11} & 0_{3\times3} \\ 0_{3\times3} & J^{-1}(\eta_\psi)H_{22} \end{bmatrix} \begin{bmatrix} \tilde{w}_1 \\ \tilde{\eta} \end{bmatrix} \tag{38}$$

where $H_{11} = e^{-\varphi(t, z_1)}$ and $H_{22} = (K_2(t, z_1))e^{\varphi(t, z_1)}$ with

$$\begin{bmatrix} \dot{\eta}(t) \\ \dot{\hat{\upsilon}}(t) \end{bmatrix} \leq \zeta_0 \begin{bmatrix} \dot{\eta}(t_0) \\ \dot{\hat{\upsilon}}(t_0) \end{bmatrix} e^{-\delta_0(t-t_0)} \tag{39}$$

expanding (38) we have the following results

$$\dot{\eta}_x = (\hat{\upsilon}_x + \tilde{\upsilon}_x)\cos(\eta_\psi) - (\hat{\upsilon}_y + \tilde{\upsilon}_y)\sin(\eta_\psi) \tag{40}$$

$$\dot{\eta}_y = (\hat{\upsilon}_x + \tilde{\upsilon}_x)\sin(\eta_\psi) + (\hat{\upsilon}_y + \tilde{\upsilon}_y)\cos(\eta_\psi) \tag{41}$$

$$\dot{\eta}_\psi = \hat{\upsilon}_\psi + \tilde{\upsilon}_\psi \tag{42}$$

$$\dot{\hat{\upsilon}}_x = \frac{m_{22}}{m_{11}}\hat{\upsilon}_y\hat{\upsilon}_\psi - \frac{d_{11}}{m_{11}}\hat{\upsilon}_x + \frac{1}{m_{11}}\tau_x + \omega_{\upsilon_x} \tag{43}$$

$$\dot{\hat{\upsilon}}_y = -\frac{m_{11}}{m_{22}}\hat{\upsilon}_x\hat{\upsilon}_\psi - \frac{d_{22}}{m_{22}}\hat{\upsilon}_y + \omega_{\upsilon_y} \tag{44}$$

$$\dot{\hat{\upsilon}}_\psi = \frac{m_{11} - m_{22}}{m_{33}}\hat{\upsilon}_x\hat{\upsilon}_\psi - \frac{d_{33}}{m_{33}}\hat{\upsilon}_\psi + \frac{1}{m_{33}}\tau_\psi + \omega_{\upsilon_\psi} \tag{45}$$

where ω_{υ_x} , ω_{υ_y} , ω_{υ_ψ} and first, second and third terms of $J^{-1}(\eta_\psi)H_{22}\tilde{\eta}$. Now we establish the trajectory errors of the ship according to [1], [9], we assume that the ship is on a frame attached to the parameterized trajectory $\alpha(\beta)$ such that

$$[\eta_{xe}, \eta_{ye}, \eta_{\psi e}]^T = J^{-1}(\eta_\psi)[\eta_x - \eta_{xd}, \eta_y - \eta_{yd}, \eta_\psi - \eta_{\psi d}]^T \tag{46}$$

where η_{xe} , η_{ye} and $\eta_{\psi e}$ are the tangential, cross and heading tracking error respectively, η_{xd} , η_{yd} and $\eta_{\psi d}$ are desired tracking in surge, sway and yaw respectively. Furthermore, $\eta_{\psi d}$ is defined by

$$\eta_{\psi d} = \tan^{-1}\left(\frac{\bar{\eta}_{yd}}{\bar{\eta}_{xd}}\right) \tag{47}$$

where $\bar{\eta}_{xd} = \dfrac{\partial \eta_x(\beta)}{\partial \beta}$, $\bar{\eta}_{yd} = \dfrac{\partial \eta_y(\beta)}{\partial \beta}$

Taking time derivative of (46) and (47) along the trajectory of (40) – (42), result in the following tracking error dynamics

$$\dot{\eta}_{xe} = \hat{\upsilon}_x - \upsilon_{xd}\cos(\eta_{\psi e}) + (\hat{\upsilon}_\psi + \tilde{\upsilon}_\psi)\eta_{ye} + \tilde{\upsilon}_x \tag{48}$$

$$\dot{\eta}_{ye} = \hat{\upsilon}_y + \upsilon_{xd}\sin(\eta_{\psi e}) - (\hat{\upsilon}_\psi + \tilde{\upsilon}_\psi)\eta_{xe} + \tilde{\upsilon}_y \tag{49}$$

$$\dot{\eta}_{\psi e} = \hat{\upsilon}_\psi - \upsilon_{\psi d} + \tilde{\upsilon}_\psi \tag{50}$$

$$\dot{\hat{\upsilon}}_x = \frac{m_{22}}{m_{11}}\hat{\upsilon}_y\hat{\upsilon}_\psi - \frac{d_{11}}{m_{11}}\hat{\upsilon}_x + \frac{1}{m_{11}}\tau_x + \omega_{\upsilon_x} \tag{51}$$

$$\dot{\hat{\upsilon}}_y = -\frac{m_{11}}{m_{22}}\hat{\upsilon}_x\hat{\upsilon}_\psi - \frac{d_{22}}{m_{22}}\hat{\upsilon}_y + \omega_{\upsilon_y} \tag{52}$$

$$\dot{\hat{\upsilon}}_\psi = \frac{m_{11} - m_{22}}{m_{33}}\hat{\upsilon}_x\hat{\upsilon}_\psi - \frac{d_{33}}{m_{33}}\hat{\upsilon}_\psi + \frac{1}{m_{33}}\tau_\psi + \omega_{\upsilon_\psi} \tag{53}$$

where

$$\upsilon_{xd} = \sqrt{\eta_{xd}^\beta(\beta)^2 + \eta_{yd}^\beta(\beta)^2}\,\dot{\beta} \tag{54}$$

and

$$\upsilon_{\psi d} = \frac{\eta_{xd}^\beta(\beta)\eta_{yd}^{\beta^2}(\beta) - \eta_{xd}^{\beta^2}(\beta)\eta_{yd}^\beta(\beta)}{\eta_{xd}^\beta(\beta)^2 + \eta_{yd}^\beta(\beta)^2}\,\dot{\beta} \tag{55}$$

where υ_{xd} and $\upsilon_{\psi d}$ are the desired surge and yaw velocities respectively.

V. Control System Design

The knowledge of backstepping design approach is for construction of feedback control law through a recursive construction of a control Lyapunov function. In this section the design of the controller using backstepping approach is presented. We assume that the ship velocities are unmeasurable. Since velocities are unmeasurable, we are going to use $\hat{\upsilon}_x$ as state variable estimate of υ_x . Thus, we apply $\hat{\upsilon}_x$ to stabilize the error dynamics in surge motion. Nevertheless, we cannot apply the estimate of $\hat{\upsilon}_y$ to directly control the error dynamics in sway motion because it is un-actuated. Instead we are going to use $\eta_{\psi e}$ as a virtual control to stabilize the error dynamics in sway motion. Hence we define the following variable as

$$\upsilon_{xe} = \hat{\upsilon}_x - \xi_{\upsilon x} \tag{56}$$

$$\bar{\eta}_{\psi e} = \eta_{\psi e} - \xi_{\upsilon_{\psi e}} \tag{57}$$

$$\upsilon_{\psi e} = \hat{\upsilon}_\psi - \xi_{\upsilon\psi} \tag{58}$$

Now the control is designed through the following steps

1) Stabilizing the υ_{xe}-dynamics, we rewrite equation (48) and (56)-(58) as follows

$$\dot{\eta}_{xe} = \upsilon_{xe} + \xi_{\upsilon x} - \upsilon_{xd}\cos(\eta_{\psi e}) + (\hat{\upsilon}_\psi + \tilde{\upsilon}_\psi)\eta_{ye} + \tilde{\upsilon}_x \tag{59}$$

choosing $\xi_{\upsilon x}$ as the first virtual control to stabilize the υ_{xe} -dynamics as

$$\xi_{\upsilon x} = -\varepsilon_1 + \upsilon_{xd}\cos(\eta_{\psi e}) \tag{60}$$

which gives

$$\dot{\eta}_{xe} = -\varepsilon_1 + (\hat{\upsilon}_{\psi} + \tilde{\upsilon}_{\psi})\eta_{ye} + \upsilon_{xe} + \tilde{\upsilon}_x \tag{61}$$

where $\varepsilon_1 = \dfrac{k_1\eta_{xe}}{\rho_1}$ and $\rho_1 = \sqrt{1+\eta_{xe}^2+\eta_{ye}^2}$

2) Stabilizing the η_{ye} -dynamics, we use (49) as follows

$$\dot{\eta}_{ye} = \upsilon_y + \upsilon_{xd}\sin(\xi_{\upsilon_{\psi e}}) - (\hat{\upsilon}_{\psi} + \tilde{\upsilon}_{\psi})\eta_{xe} + \upsilon_{xd}\Theta + \tilde{\upsilon}_y \tag{62}$$

where

$$\Theta = \sin(\bar{\eta}_{\psi e})\cos(\xi_{\upsilon_{\psi e}}) + (\cos(\bar{\eta}_{\psi e}) - 1)\sin(\xi_{\upsilon_{\psi e}}) \tag{63}$$

By assuming that υ_x cannot be equal to zero, then we choose the virtual control as

$$\xi_{\upsilon_{\psi e}} = \tan^{-1}\left(\frac{\varepsilon_2 + \upsilon_{yd}}{\upsilon_{xd}}\right) \tag{64}$$

where $\varepsilon_2 = \dfrac{k_2\eta_{ye}}{\rho_1}$ and υ_{yd} is the desired sway velocity. Hence we obtain the following

$$\dot{\eta}_{ye} = -\varepsilon_2 - (\hat{\upsilon}_{\psi} + \tilde{\upsilon}_{\psi})\eta_{xe} + \upsilon_{xd}\Theta + \tilde{\upsilon}_y \tag{65}$$

3) We stabilize the $\bar{\eta}_{\psi e}$ -dynamics. Taking time derivative of (57) along the trajectory of (64) and (48)-(50) we have

$$\begin{aligned}\bar{\eta}_{\psi e} &= \left(1 - \frac{k_2\eta_{xe}}{\rho_1\theta}\right)\left(\eta_{\psi e} + \xi_{\upsilon\psi}\right) - \upsilon_{\psi d} - \frac{k_2\eta_{ye}\eta_{xe}}{\rho_1^3\theta}\upsilon_{xe} \\ &+ \frac{k_2(1+\eta_{xe}^2)}{\rho_1^3\theta}\upsilon_{xe} - \frac{\varepsilon_2 + \upsilon_{xd}}{\upsilon_{xd}\theta}\dot{\upsilon}_{xd} + \frac{\dot{\upsilon}_{xd}}{\theta} + \frac{\varepsilon_2}{\theta}\left[\frac{\upsilon_{xd} + \upsilon_{xd}\sin(\eta_{\psi e})}{\rho_1}\right. \\ &\left. - \frac{\eta_{ye}^2(\upsilon_{yd} + \upsilon_{xd})\sin(\eta_{\psi e})) + \eta_{ye}\eta_{xe}(\xi_{\upsilon x} - \upsilon_{xd}\cos(\eta_{\psi e}))}{\rho_1^3}\right] \\ &- \frac{\varepsilon_2\eta_{ye}}{\rho_1^3\theta}(\tilde{\upsilon}_{\psi}\eta_{ye} + \tilde{\upsilon}_x) + \frac{\varepsilon_2(1+\eta_{xe}^2)}{\rho_1^3\theta}(\tilde{\upsilon}_y - \tilde{\upsilon}_{\psi}\eta_{xe}) + \tilde{\upsilon}_{\psi}\end{aligned} \tag{66}$$

where $\theta = \sqrt{\upsilon_{xd}^2 - (\omega_2 + \upsilon_{xd})}$. Now from (66) we can choose the value of ε_2 such that $k_2 = \min(\bar{k}_2, \bar{\bar{k}}_2)$, $\bar{k}_2 := \min(\upsilon_{xd}) - \max(\upsilon_{yd})$ and $\bar{\bar{k}}_2 := \min(\theta)$. Now we design the virtual control $\xi_{\upsilon\psi}$ based on (58) as follows

$$\begin{aligned}\xi_{\upsilon\psi} &= \left(1 - \frac{k_2\eta_{xe}}{\rho_1\theta}\right)^{-1}\left(-\frac{k_3\bar{\eta}_{\psi e}}{\rho_2} + \upsilon_{\psi d} - \frac{\dot{\upsilon}_{yd}}{\theta} - \frac{k_2}{\theta}\left[\frac{\upsilon_{yd} + \upsilon_{xd}\sin(\eta_{\psi e})}{\rho_1}\right.\right. \\ &\left.\left. - \frac{\eta_{ye}\eta_{xe}(\xi_{\upsilon x} - \upsilon_{xd}\cos(\eta_{\psi e}))}{\rho_1^3}\right] - \frac{\upsilon_{xd}\Theta\eta_{ye}}{\bar{\eta}_{\psi e}} + \frac{\varepsilon_2 + \upsilon_{yd}}{\upsilon_{xd}\theta}\dot{\upsilon}_{xd}\right)\end{aligned} \tag{67}$$

Note that, to cancel the cross term in the η_{ye} -dynamics, we added the last term in (67). Note that $\dfrac{\sin(\bar{\eta}_{\psi e})}{\bar{\eta}_{\psi e}} = \int_0^1 \cos(\bar{\eta}_{\psi e}t)dt$ and $\dfrac{\cos(\bar{\eta}_{\psi e}) - 1}{\bar{\eta}_{\psi e}} = \int_0^1 \sin(\bar{\eta}_{\psi e}t)dt$ are smooth functions of $\bar{\eta}_{\psi e}$. Thus, $\bar{\eta}_{\psi e}$ -dynamics becomes

$$\begin{aligned}\dot{\bar{\eta}}_{\psi e} &= \frac{k_3\bar{\eta}_{\psi e}}{\rho_1} + \left(1 + \frac{k_2\eta_{xe}}{\rho_1\theta}\right)\upsilon_{\psi} - \frac{k_2\eta_{xe}\eta_{ye}}{\rho_1^3\theta}\upsilon_{xe} + \frac{k_2(1+\eta_{xe}^2)}{\rho_1^3\theta}\upsilon_{ye} \\ &- \frac{\upsilon_{xd}\Theta\eta_{ye}}{\bar{\eta}_{\psi e}} - \frac{k_2\eta_{ye}}{\rho_1^3\theta}(\tilde{\upsilon}_{\psi}\eta_{ye} + \tilde{\upsilon}_x) + \frac{k_2(1+\eta_{xe}^2)}{\rho_1^3\theta} \\ &\times(\tilde{\upsilon}_y - \tilde{\upsilon}_{\psi}\eta_{ye}) + \tilde{\upsilon}_{\psi}\end{aligned} \tag{68}$$

Now, the desired sway velocity υ_{yd} is obtained by taking time derivative of $\upsilon_{ye} = \hat{\upsilon}_y - \upsilon_{yd}$ along the trajectory of (52)

$$\begin{aligned}\dot{\hat{\upsilon}}_y &= -\frac{d_{22}}{m_{22}}\hat{\upsilon}_y - \frac{m_{11}}{m_{22}}\upsilon_{ye}\xi_{\upsilon\psi} - \frac{m_{11}}{m_{22}}\upsilon_{\psi e}\hat{\upsilon}_x - \frac{m_{11}}{m_{22}}\xi_{\upsilon x}\xi_{\upsilon\psi} \\ &- \dot{\upsilon}_{ye} + \omega_{\upsilon_y}\end{aligned} \tag{69}$$

and from (69) we obtain

$$\dot{\upsilon}_{yd} = -\frac{d_{22}}{m_{22}}\hat{\upsilon}_y - \frac{m_{11}}{m_{22}}\xi_{\upsilon x}\xi_{\upsilon\psi} + \frac{k_2(1+\eta_{xe}^2)}{\rho_1^3\theta}\bar{\eta}_{\psi e} \tag{70}$$

Which gives the following

$$\dot{\upsilon}_{ye} = -\frac{d_{22}}{m_{22}}\upsilon_{ye} - \frac{m_{11}}{m_{22}}\upsilon_{\psi e}\hat{\upsilon}_x - \frac{m_{11}}{m_{22}}\upsilon_{ye}\xi_{\upsilon\psi} - \frac{k_2(1+\eta_{xe}^2)}{\rho_1^3\theta}\bar{\eta}_{\psi e} \tag{71}$$

4) To stabilize υ_{ye} -dynamics, we differentiate with respect to time equation (56) and (60) as

$$\dot{\upsilon}_{ye} = -\frac{d_{22}}{m_{22}}\hat{\upsilon}_y\hat{\upsilon}_{\psi} - \frac{d_{11}}{m_{11}}\hat{\upsilon}_x - \frac{1}{m_{11}}\tau_x + \omega_{\upsilon_x} - \dot{\xi}_{\upsilon x} \tag{72}$$

Since the control law τ_x appears in (72), then we design control law as

$$\begin{aligned}\tau_x = m_{11}\Bigg(&-k_4\upsilon_{ye} + \frac{d_{11}}{m_{11}}\xi_{\upsilon x} - \frac{d_{22}}{m_{22}}\hat{\upsilon}_y\hat{\upsilon}_{\psi} + \frac{k_2\eta_{xe}\eta_{ye}}{\rho_1\theta}\bar{\eta}_{\psi e} \\ &+ \frac{m_{11}}{m_{22}}\upsilon_{ye}\xi_{\upsilon\psi} - \upsilon_{xe} + \frac{\partial\xi_{\upsilon x}}{\partial\eta_{xe}}(\hat{\upsilon}_x - \upsilon_{xd}\cos(\eta_{\psi e}) + \hat{\upsilon}_{\psi}\eta_{ye}) \\ &+ \frac{\partial\xi_{\upsilon x}}{\partial\eta_{ye}}(\hat{\upsilon}_y - \upsilon_{xd}\sin(\eta_{\psi e}) - \hat{\upsilon}_{\psi}) + \frac{\partial\xi_{\upsilon x}}{\partial\eta_{\psi e}}(\hat{\upsilon}_{\psi} - \upsilon_{\psi d}) \\ &+ \frac{\partial\xi_{\upsilon x}}{\partial\upsilon_{xd}}\dot{\upsilon}_{xd}\Bigg)\end{aligned} \tag{73}$$

Differentiating with respect to time (58) along the trajectory of (53) we obtain control law τ_ψ as

$$\begin{aligned}\tau_\psi = m_{33}\Bigg(& -k_5\upsilon_{\psi e} + \frac{d_{33}}{m_{33}}\xi_{\upsilon\psi} - \frac{m_{11}-m_{22}}{m_{33}}\hat{\upsilon}_x\hat{\upsilon}_y - \left(1+\frac{k_2\eta_{xe}}{\rho_1\theta}\right) \\ & \times\bar{\eta}_{\psi e} + \frac{m_{11}}{m_{22}}\upsilon_{ye}\hat{\upsilon}_x + \frac{\partial\xi_{\upsilon\psi}}{\partial\eta_{xe}}(\hat{\upsilon}_x - \upsilon_{xd}\cos(\eta_{\psi e}) + \hat{\upsilon}_\psi\eta_{ye}) \\ & + \frac{\partial\xi_{\upsilon\psi}}{\partial\eta_{ye}}(\hat{\upsilon}_y + \upsilon_{xd}\sin(\eta_{\psi e}) - \hat{\upsilon}_\psi) + \frac{\partial\xi_{\upsilon\psi}}{\partial\eta_{\psi e}}(\hat{\upsilon}_\psi - \upsilon_{\psi d}) \\ & + \frac{\partial\xi_{\upsilon\psi}}{\partial\upsilon_{xd}}\dot{\upsilon}_{xd} + \frac{\partial\xi_{\upsilon\psi}}{\partial\dot{\upsilon}_{yd}}\ddot{\upsilon}_{yd} + \frac{\partial\xi_{\upsilon\psi}}{\partial\dot{\upsilon}_{xd}}\ddot{\upsilon}_{xd}\Bigg) \qquad (74)\end{aligned}$$

Therefore, if the control laws τ_x and τ_ψ derived in (73) and (74) are applied to the under-actuated ship model described by (7)-(9) (together with the observer in (38)), then we can conclude that the trajectory tracking errors (i.e. η_{xe}, η_{ye}, $\bar{\eta}_{\psi e}$, υ_{xe}, υ_{ye} and $\upsilon_{\psi e}$) converges globally asymptotically to zero. The proof of is excluded due to restricted space. Notice that in this paper we have dealt with environmental disturbance free under-actuated ship.

VI. Simulation Study

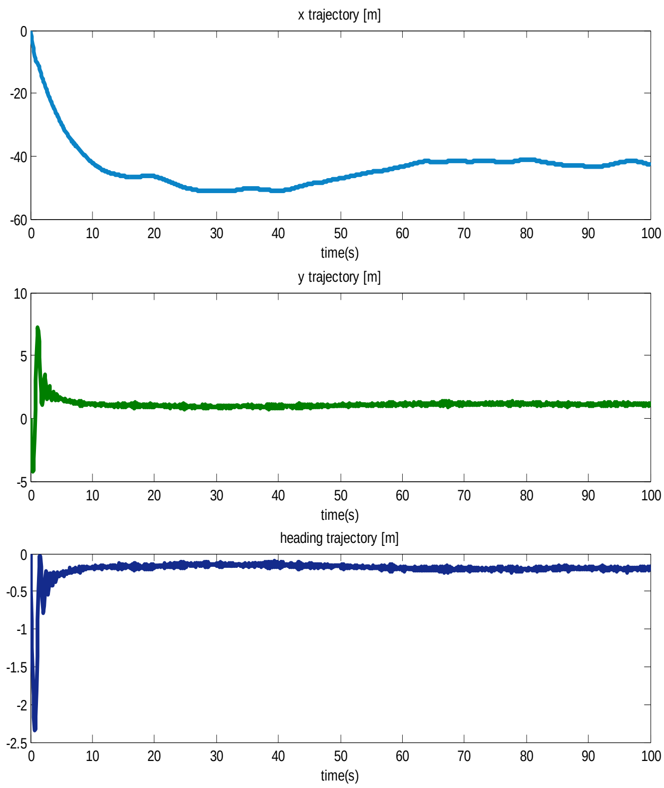

Fig. 1. Trajectory tracking of (x, y, ψ).

In this paper we use the ship proposed in [15] for simulation purposes. The following parameters are used

$$M = \begin{bmatrix} 1.9\times10^1 & 0 & 0 \\ 0 & 3.52\times10^1 & 0 \\ 0 & 0 & 0.42\times10^1 \end{bmatrix}$$

$$C(\upsilon)\upsilon = \begin{bmatrix} 0 & 0 & -(3.52\times10^1)\upsilon_y \\ 0 & 0 & (1.9\times10^1)\upsilon_x \\ (3.52\times10^1)\upsilon_y & -(1.9\times10^1)\upsilon_x & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

The control gains K_1 and K_2 are taken as

$$\Gamma_1 = \Gamma_2 = K_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

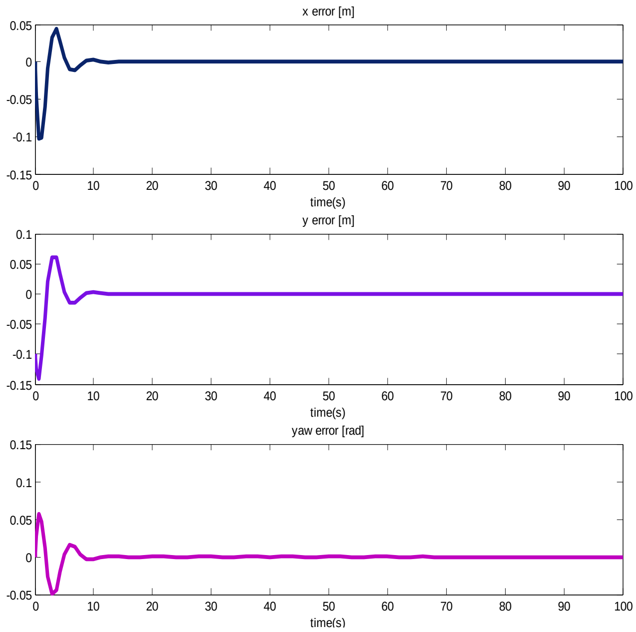

Fig. 2. Trajectory tracking error convergence of (x, y, ψ).

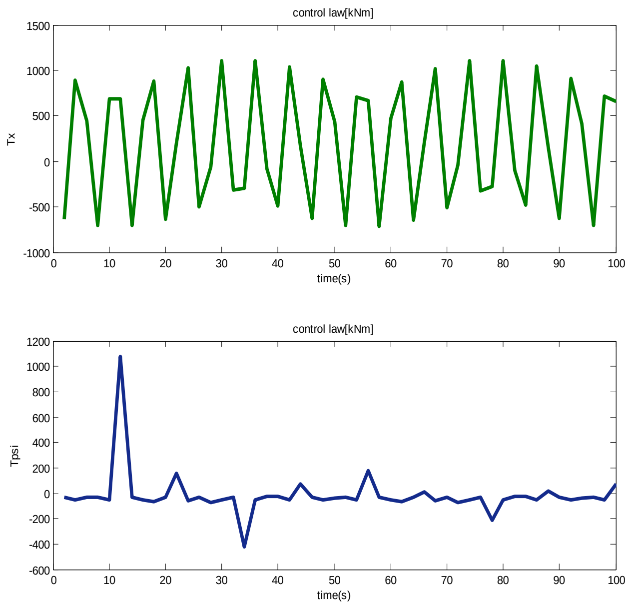

Fig. 3. Control signal (τ_x, τ_ψ).

The initial conditions are set to be $\eta(t) = \eta(0) = [0\text{m}, -2\text{m}, -0.5\text{rad}]^T$ and $\upsilon(0) = \hat{\upsilon}(0) = [0\text{m/s}, 0\text{m/s}, 0\text{rad/s}]^T$. Fig. 1 shows the simulation results of trajectory tracking of the position (x, y) and heading ψ of the ship. Fig. 2 shows the simulation results of tracking error convergence of the observer position and heading. It is clear that from Fig. 2 the error converge as faster as 10 seconds, thus showing that the proposed observer is stable.

Finally, simulation results of Fig. 3 show the control signal (τ_x, τ_ψ) applied.

VII. Conclusion

In this paper an output feedback controller for tracking control of under-actuated surface ships using Euler Lagrange equations has been given. The change of coordinates was applied to overcome the third order that arises in the Lyapunov function derivatives due to Coriolis and centripetal forces term. The controller design was carried out using back stepping control technique and Lyapunov stability theorems. The control law was derived via Lyapunov stability theory and backstepping control technique. The observer was derived using the change of coordinate method; then backstepping control technique was employed to derive the control law. Simulation results presented validates the performance of the controller and prove that the tracking error converges.

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Werneld E. Ngongi was born in April 1976 in Tanzania. He received the MSc. degree in electronics engineering in 2008 and the BSc. degree in electrical engineering in 2002 at the University of Dar es Salaam, Tanzania. He is working as an assistant lecturer at Dar es Salaam Maritime Institute, Tanzania. Presently, he is a PhD candidate in control theory and control engineering at Dalian Maritime University, Dalian, China. His research interests include intelligent control design, nonlinear control design, ship position and motion control.

Jialu Du was born in Liaoning Province, P. R. China, in 1966. She received the PhD degree from Dalian Maritime University, China, in 2005. She was a visiting scholar in Norwegian University of Science and Technology from 2006 to 2007 and in University of California, San Diego from 2012 to 2013, respectively. Currently she is a professor at the School of Information Science and Technology, Dalian Maritime University, China. Her current research interests include nonlinear control theory, intelligent control, and ship motion control.