A 2-D Second-Order Recursive Inverse Adaptive Filtering Algorithm

Mohammad Shukri Salman, Aykut Hocanin, and Osman Kukrer

***Abstract*—In this paper, a two-dimensional (2-D) version of the recently proposed second order Recursive Inverse (2nd order RI) algorithm is introduced. In the proposed algorithm, instead of estimating the inverse autocorrelation matrix, a second order estimate of the autocorrelation matrix and cross correlation vector, which provide an improved and a more stable performance, are used. Also, the filter coefficients are updated along both the horizontal and vertical directions on a 2-D plane. The performance of the proposed algorithm is compared to that of the 2-D RLS algorithm in Adaptive Line Enhancer (ALE) problem. Simulation results show that the proposed algorithm leads to an improved performance compared to that of the 2-D RLS algorithm.**

***Index Terms*—2-D RLS, adaptive filters, ALE.**

I. Introduction

Due to increased use of digital imaging and video in consumer electronics and multimedia applications, the LMS adaptive algorithm was extended from 1-D to 2-D and has taken many shapes of implementation. However, these ways update the filter coefficients only along the horizontal direction on a 2-D plane [1]. Consequently, these algorithms cannot sufficiently exploit information in 2-D signals.

Different types of 2-D adaptive algorithms, which can update the filter coefficients both along the horizontal and the vertical directions on a 2-D plane, were later developed and applied to reduce noise in image signals. One of the most efficient algorithms that have shown high performance in image noise removal is the 2-D RLS algorithm [2], [3]. This algorithm suffers from its high computational complexity. Also, it suffers from stability problems when the forgetting factor is small and tracking capability when it is high [4].

In this paper, we propose a new 2-D second order recursive inverse adaptive filtering technique, based on the recently proposed second order recursive inverse algorithm [5], that uses the second order recursive estimate of the autocorrelation matrix and cross-correlation vector which in turn provide an improved performance over that of the 2-D RLS algorithm, and a more stable performance since it does not require the inversion of the autocorrelation matrix.

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II. Review of the Second-Order Recursive Inverse Algorithm

In the recently proposed Second-Order Recursive Inverse (RI) adaptive algorithm [5], the weights update equation is given as:

$$w_k = \left[I - \mu_k R_k\right] w_{k-1} + \mu_k \boldsymbol{p}_k, \tag{1}$$

with,

$$\mu_k = \frac{\mu_0}{\gamma_k}, \text{ where } \mu_0 < \mu_{\max} \text{ and } \mu_{\max} = \frac{2}{\lambda_{\max}(R_{xx})} \tag{2}$$

where $R_{xx} = E\left\{x_k x_k^T\right\}$, and γ_k given by

$$\gamma_k = \frac{1}{\beta - 1} + \alpha_1 z_1^k + \alpha_2 z_2^k, \tag{3}$$

where $\alpha_1 = \dfrac{\beta - z_2}{(1-\beta)(z_2 - z_1)}$, $\alpha_2 = \dfrac{\beta - z_1}{(1-\beta)(z_2 - z_1)}$. in (1), $\boldsymbol{R}_k$ and $\boldsymbol{p}_k$ are the instantaneous autocorrelation matrix and cross-correlation vector, respectively. They are estimated recursively, using the second order estimates, as:

$$\boldsymbol{R}_k = \beta_1 \boldsymbol{R}_{k-1} + \beta_2 \boldsymbol{R}_{k-2} + \boldsymbol{x}_k \boldsymbol{x}_k^T, \tag{4}$$

$$\boldsymbol{p}_k = \beta_1 \boldsymbol{p}_{k-1} + \beta_2 \boldsymbol{p}_{k-2} + d_k \boldsymbol{x}_k. \tag{5}$$

where β_1 and β_2 are the forgetting factors, $\mathbf{x}_k$ is the tap-input vector and d_k is the desired response.

III. Proposed Algorithm

Equation (1) describes the filter weights update equation of the Second-Order RI algorithm [5]. It can be generalized into its 2-D form as:

$$\boldsymbol{w}_k(m_1, m_2) = \left[\boldsymbol{I} - \mu_k \boldsymbol{R}_k\right] \boldsymbol{w}_{k-1}(m_1, m_2) + \mu_k \boldsymbol{p}_k, \tag{6}$$

where $\boldsymbol{w}_k(m_1, m_2)$ is the 2-D tap-weight vector with dimensions $N \times N$, $m_1 = 0,1,\ldots,N-1$, $m_2 = 0,1,\ldots,N-1$, $\boldsymbol{R}_k$ and $\boldsymbol{p}_k$ are the instantaneous autocorrelation matrix and cross-correlation vector, respectively. They are estimated

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recursively as:

$$\boldsymbol{R}_k = \beta_1 \boldsymbol{R}_{k-1} + \beta_2 \boldsymbol{R}_{k-2} + \boldsymbol{x}(n_1,n_2)\boldsymbol{x}^T(n_1,n_2), \tag{7}$$

and

$$\boldsymbol{p}_k = \beta_1 \boldsymbol{p}_{k-1} + \beta_2 \boldsymbol{p}_{k-2} + d(n_1,n_2)\boldsymbol{x}(n_1,n_2). \tag{8}$$

where $d(n_1,n_2)$ is the desired output and $\boldsymbol{x}(n_1,n_2)$ is the filter input. The filter input $(\boldsymbol{x}(n_1,n_2))$ and tap-weight vector $(\boldsymbol{w}_k(m_1,m_2))$ can be defined using the following column-ordered vectors [6],

$$\boldsymbol{x}(n_1,n_2) = \begin{pmatrix} x(n_1,n_2) \\ \vdots \\ x(n_1,n_2-N+1) \\ \vdots \\ x(n_1-N+1,n_2) \\ \vdots \\ x(n_1-N+1,n_2-N+1) \end{pmatrix} \tag{9}$$

and,

$$\boldsymbol{w}_k(m_1,m_2) = \begin{pmatrix} w_k(0,0) \\ \vdots \\ w_k(0,N-1) \\ \vdots \\ w_k(N-1,0) \\ \vdots \\ w_k(N-1,N-1) \end{pmatrix}. \tag{10}$$

For 2-D applications, there can be a number of ways that data can be reused. One possible way is shown in Fig. 1 [7]. In this scheme, as shown in Fig. 1b, we consider a mask of 3×3 pixels which move horizontally to the right by one column at a time until the end of each row. Afterward, the same process is repeated with the next row below until the last 9 pixels of the image are reached. At the end of each process of the mask, the data are reshaped as shown in Fig. 1a, starting from the last pixel in the lower right corner.

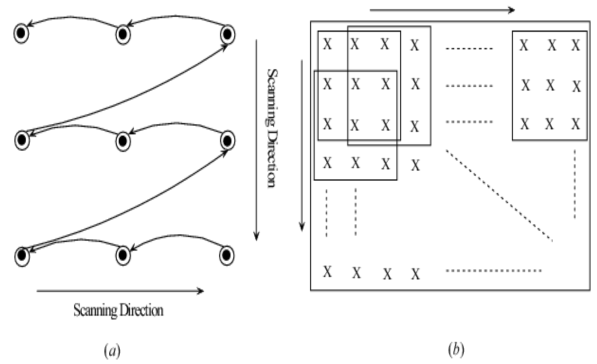

Fig. 1. Rectangular configuration of data-reusing in 2-D, [7].

The filter output is given by the following 2-D convolution:

$$y(n_1,n_2) = \sum_{m_1=0}^{N-1}\sum_{m_2=0}^{N-1} w(m_1,m_2)x(n_1-m_1,n_2-m_2). \tag{11}$$

IV. Simulation Results

In order to see the performance of the proposed algorithm, it is applied to a 2-D ALE in [7]. In this setting, the original image with noise, $\boldsymbol{x}(n_1,n_2)$ is the same as the desired response, $d(n_1,n_2)$. The adaptive filter input, $\boldsymbol{u}(n_1,n_2)$ is a delayed version of the desired response. The noise added to the original image, $\boldsymbol{I}$ was multiplicative noise using (12),

$$\boldsymbol{x} = \boldsymbol{I} + \upsilon \boldsymbol{I}. \tag{12}$$

where υ is uniformly distributed random noise with zero mean and variance $\sigma_\upsilon^2 = 5$.

The proposed algorithm is compared to the well-known 2-D RLS algorithm. Both algorithms were implemented using a 2-D FIR filter of size 3×3 taps. For the proposed algorithm, $\mu_0 = 0.00065$, $\beta = 0.995$ for Fig. 2c and $\beta = 0.999$ for Fig. 2e. For the 2-D RLS algorithm, $\beta = 0.995$ for Fig. 2d and $\beta = 0.999$ for Fig. 2f.

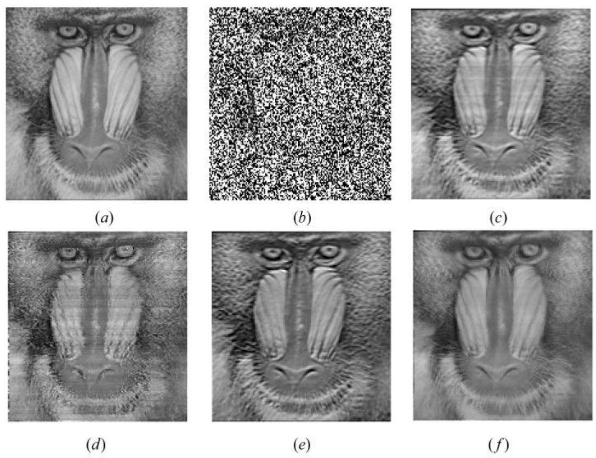

Fig. 2. (a) Original image. (b) Image with noise. (c) Restored image using the proposed algorithm $(\beta = 0.995)$. (d) Restored image using the 2-D RLS algorithm $(\beta = 0.995)$. (e) Restored image using the the proposed algorithm $(\beta = 0.999)$. (f) Restored image using the 2-D RLS algorithm $(\beta = 0.999)$.

Fig. 2a shows the original image, Fig. 2b shows the image with noise ('speckle'), Fig. 2c shows the restored image by the proposed 2-D 2nd order RI algorithm with $\beta = 0.995$, and Fig. 2d shows the restored image by the 2-D RLS algorithm with $\beta = 0.995$. We note that the RLS algorithm provides very low performance compared to the proposed algorithm with relatively low values of β. Fig. 2e shows the restored image by the proposed algorithm with $\beta = 0.999$, and Fig. 2f shows the restored image by the 2-D RLS algorithm with $\beta = 0.999$. Even though the value of β is very high (close to unity), by inspection, it is clear that the image restored by the proposed algorithm has sharper edges than that recovered by the 2-D RLS algorithm.

V. Conclusion

In this paper, a two-dimensional (2-D) version of the recently proposed second order Recursive Inverse (2^{nd} order RI) algorithm is introduced. The proposed approach avoids the use of the inverse autocorrelation matrix in the coefficient update. Instead, a second order estimate of the autocorrelation matrix and cross correlation vector, which provide an improved and a more stable performance, are used. Also, the filter coefficients are updated both along the horizontal and vertical directions on a 2-D plane Simulations show that the proposed algorithm outperforms the 2-D RLS algorithm.

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