Designing Passive Filters for Harmonic Reduction in a Noisy System Based on Discrete Wavelet Transform

Hamid Rahimi Esfahani, Rasoul Amirfattahi, Farshad Kiyoumarsi, and Ebrahim Borzabadi

Abstract—**In this paper, among different methods of signal processing, wavelet transform is chosen due to its advantages over other methods. To show wavelet transform capabilities, first an HVDC system which has noise on its output current is simulated. In the first step noise is removed by applying discrete wavelet transform. In the next steps, harmonic problem is resolved thorough appropriate passive filters. In this paper, we suppose that not only low order harmonics exist in the output current, but also high order ones. The results indicate that we can obtain a good passive filter design for harmonic reduction by decomposing a signal into its harmonic components via applying discrete wavelet transform.**

Index Terms—**Wavelet multi-resolution decomposition, passive filter for harmonic reduction, harmonic detection, wavelet-based threshold de-noising method.**

I. Introduction

The most common tool, used up to now for wave-shape analysis, has been the Fourier transform (FT). It transforms a signal into fundamental and high-order harmonic components. FT, or its discrete version (DFT), which has been developed for computer applications [fast FT (FFT)], has some disadvantages, such as aliasing, spectral leakage, picket fence effect, etc.[1]. FT gives the exact frequency spectrum of stationary and periodical signals. However, in modern variable-speed drives, changes in developed torque and angular velocity are often required. Therefore, the drive passes through numerous transient states and information about harmonics is inaccurate. To deal with this problem, the windowed FT (WFT), or short-time FT, has been developed. It decomposes the signal into smaller parts of exact length first, and then applies the FT. However, as the width of the window is fixed, the signal is assumed periodical and stationary in the window, so harmonics are obtained as rows of discrete values with limited accuracy. The WFT solves the initial problem, but the mentioned disadvantages remain, so accuracy is not satisfactory. In the last ten years, the wavelet transform (WT) has been introduced, as a new approach in signal analysis [2], [3]. The wavelet theory says that a signal can be represented by superposition of some special signals called wavelets. Wavelets are waveforms of limited duration, with zero average value. WT is similar to finite response filters, so it does not transform the signal into discrete harmonics, but into frequency bandwidths, which cover all significant harmonics. WT eliminates the drawbacks of WFT and is able to track fast amplitude variations of certain harmonics. This feature is enabled by its characteristics of having a narrow window for higher frequencies, and wider window for lower frequencies. Appearing noise on output signal is undesirable, although this important problem is resolved at the first step before harmonic reduction by applying Discrete Wavelet Transform (DWT). The method will be described in part II.B Wavelet Transform is of localization in both time and frequency domains, and the frequency distribution of certain time can be calculated, also the mixed signal which is composed of different frequencies can be decomposed into different frequency bands with different frequency ranges, consequently different harmonic currents can be gained through wavelet transform for a good passive filter design. In next steps, this paper introduces a method of signal decomposition through wavelet transform for the detection of 5^{th} , 7^{th}, 11^{th} , 13^{th} and 24^{th} harmonics of current. (low and high order harmonics) . The results show that this method can be useful for obtaining an acceptable passive filter design for harmonic reduction.

Manuscript received June 23, 2012; revised August 8, 2012.

H. R. Esfahani is with the Department of Electrical Engineering, Young Researchers Club, Najafabad Branch, Islamic Azad University, Najafabad, Isfahan, Iran (e-mail: Rahimi_ham@yahoo.com)

R. Amirfattahi is with the Digital Signal Processing Research Lab, Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, 84156-83111, Iran (e-mail: Fattahi@cc.iut.ac.ir)

F. Kiyoumarsi is with the Department of Computer Engineering, Shahrekord Branch, Islamic Azad University, Shahrekord, Iran (e-mail: Kumarci-farshad@iaushk.ac.ir)

E. Borzabadi is with the Department of Electrical Engineering Najafabad Branch, Islamic Azad University, Najafabad, Isfahan, Iran (e-mail: eborzu@yahoo.com)

II. Wavelet Theory and Wavelet-Based Threshold De-Noising

A. Wavelet Theory

The basic idea underlying wavelet analysis consists of expressing a signal as a linear combination of a particular set of functions (wavelet transform, WT), obtained by shifting and dilating one single function called a mother wavelet. The DWT is a mathematical method of decomposing the signal in the time domain into several scales at different levels of resolution (time-scale domain) through dilations and translations. The wavelet transformation coefficients (WTCs) at the several scales reveal the time-localizing information about the variation of the signal from high- to low-frequency bands. The wavelet transform of a time-continuous signal is defined as [4]:

$$\mathrm{CWT}_{\psi} x(a,b) = |a|^{\frac{1}{2}} \int x_t \psi^* \left(\frac{t-b}{a}\right) dt \quad (1)$$

where a is called the scaling factor, b is the translation parameter, and ψ^* is the window function or wavelet.

Discrete wavelet transform can be implemented as a set of filter banks comprising a high-pass and a low-pass filters, each followed by down sampling by two. The low-pass filtered and decimated output is recursively passed through similar filter banks to add the dimension of varying resolution at every stage. In practical applications, the mathematically expressed of DWT is defined as:

$$\mathrm{CWT}_{\psi}x(m,n) = \frac{1}{\sqrt{2^m}}\sum_k x_k \psi^*\left(\frac{k-n}{2^m}\right) \tag{2}$$

where k is an operating index; m is a scaling number; n is a sampling number, n = 1,2, ...,N. N is the total number of sampling points.

B. Basic Wavelet-Based Threshold De-noising Method

We develop the basic ideas of thresholding the wavelet transform using Donoho's formulations. Assume a finite length signal with additive noise of the form as:

$$y_i = x_i + \varepsilon n_i, \quad i = 1,2, \dots N \tag{3}$$

as a finite length signal of observations of the signal x_i that is corrupted by i.i.d. zero mean, white Gaussian noise n_i with standard deviation ε, i.e.. The goal is to recover the signal x from the noisy observations y. Here and in the following, v denotes a vector with the ordered elements v_i if the index i is omitted. Let W be a left invertible wavelet transformation matrix of the discrete wavelet transform (DWT). Then (3) can be written in the transformation domain

$$Y = X + N, \quad or, \quad Y_i = X_i + N_i \tag{4}$$

where capital letters denote variables in the transform domain, i.e., $Y = Wy$. then the inverse transform matrix W^{-1} exists, and we have

$$W^{-1}W = I \tag{5}$$

Let $\hat{X}$ denote an estimate of X, based on the observations Y. We consider diagonal linear projections

$$\Delta = diag(\delta_1, \dots, \delta_N), \quad \delta_i \in \{0,1\}, \quad i = 1, \dots, N \tag{6}$$

Which give rise to the estimate

$$\hat{x} = W^{-1} = W^{-1}\Delta Y = W^{-1}\Delta W_y \tag{7}$$

The estimate $\hat{X}$ is obtained by simply keeping or zeroing the individual wavelet coefficients. Since we are interested in the l_2 error we define the risk measure

$$R(\hat{X}, X) = E\left[\|\hat{x} - x\|_2{}^2\right] = E\left[\left\|W^{-1}(\hat{X} - X)\right\|_2{}^2\right] = E\left[\left\|(\hat{X} - X)\right\|_2{}^2\right] \tag{8}$$

Notice that the last equality in (8) is a consequence of the orthogonality of W. The optimal coefficients in the diagonal projection scheme are $\delta_i = 1_{x_{i>\epsilon}}$, i.e., only those values of Y where the corresponding elements of X are larger than ε are kept, all others are set to zero. This leads to the ideal risk:

$$R_{id}(\hat{X}, X) = \sum_{n=1}^{N} \min(X^2, \varepsilon^2) \tag{9}$$

The ideal risk cannot be attained in practice, since it requires knowledge of X, the wavelet transform of the unknown vector x. However, it does give us a lower limit for the l_2 error [1]. Donoho [5] proposes the following scheme for de-noising:

1) Compute the $DWTY = W_y$.
2) Perform thresholding in the wavelet domain, according to so-called hard-thresholding.

$$\hat{X} = T_h(Y,t) = \begin{cases} Y, & |Y| \geq t \\ 0, & |Y| < t \end{cases} \tag{10}$$

or according to so-called soft-thresholding.

$$\hat{X} = T_s(Y,t) = \begin{cases} sgn(Y)(|Y| - t), & |Y| \geq t \\ 0, & |Y| < t \end{cases} \tag{11}$$

III. De-Noising Output Current By DWT

With this brief introduction about Basic Wavelet-based Threshold De-noising Method, now we are ready to de-noise the output current signal on bus 2 of Fig. 1, which is an HVDC System and will be described more in part VII. Fig. 2 shows the full of noise output current which is successfully de-noised via mentioned method. For better indication of the differences between the full of noise output current and the de-noised output current, a phase of current before and after de-noising is chosen and rescaled and is shown with the 3phase noise in Fig. 3.

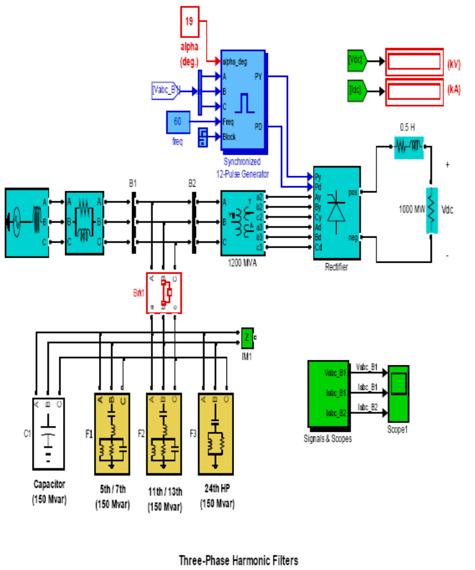

Fig. 1. An hvdc system modelling with appropriate filters

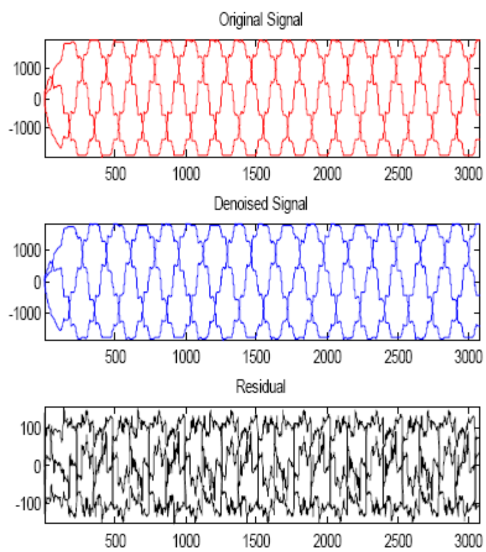

Fig. 2. Full of noise current before and after de-noising with figure of the noise signal

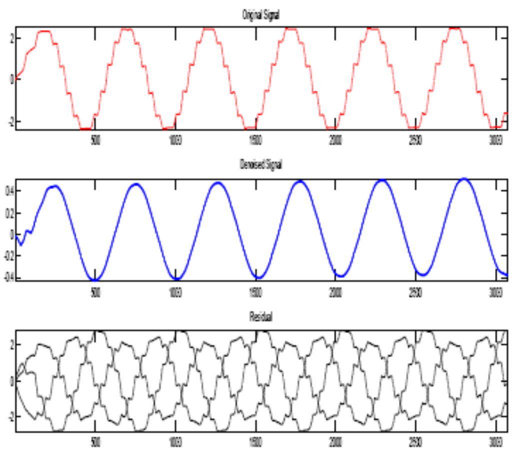

Fig. 3. Full of noise current before and after de-noising with figure of the noise signal after rescaling

Now we are ready to start passive filter design for harmonic reduction, but before that writing about the theory of this method is necessary.

IV. Wavelet Multi-Resolution Analysis

The concept of multi-resolution is described as follows: square integrable function $f(t) \in L^2(R)$ can be regarded as the limit case of certain gradual approach, every approximation is the result of the smoothness of low-pass smooth function $\varphi(t)$ towards $f(t)$, the smooth function $\varphi(t)$ is also expanding and contracting gradually while approaching by degrees, that is, the analyzed function $f(t)$ is approached gradually by using different resolution.

V. Wavelet Multi-Resolution Decomposition

S.Mallat proposed the concept of multi-resolution analysis while constructing orthogonal wavelet base in 1988. The meaning of multi-resolution can also be apprehended from the view of function space. If the sampling frequency of analyzed signal meets the Sampling Proposition, the normalization frequency band must be limited between $-\pi$ and $+\pi$, the total frequency band (($0\sim\pi$) (for positive frequency)) of the analyzed signal can be defined as space V_0 shown in Fig. 4[6]. After the first scale decomposition, V_0 is divided into two subspaces: low-frequency V_1 (frequency band for $0\sim\pi / 2$) and high-frequency W_1 (frequency band for $\pi / 2\sim\pi$), and so on. [2]. The dividing process of frequency subspace can be marked as follows:

$$V_0 = V_1 \oplus W_1, V_1 = V_2 \oplus W_2, \ldots\ldots, \qquad V_{j-1} = V_j \oplus W_j$$

$$V_0 = W_1 \oplus W_2 \oplus W_3 \oplus \ldots \oplus W_j \oplus V_j$$

where W_j is high-frequency subspace reflecting space W_{j-1}'s signal details, V_j is low-frequency subspace reflecting space V_{j-1}'s signal approach, also it can be regarded that W_j is the orthogonal complement space of V_j in V_{j-1}, V_j *and* W_j are respectively called scale space and wavelet space on scale j. In order to further apprehend multi-resolution analysis, here takes a decomposition of 3 scales for example, the wavelet decomposing tree is shown in Fig. 5[6]. The decomposition has a relationship that is:

$$f(n) = a_3(k) + d_3(k) + d_2(k) + d_1(k)$$

If the decomposition needs to be conducted further, the low-frequency component $a_3(k)$ can be sequentially decomposed into low-frequency $a_4(k)$ and high-frequency $d_4(k)$, and so on. It can be seen from the block diagram of multi-resolution decomposing tree above that, for multi-resolution analysis, just the low-frequency component is decomposed, whose frequency resolution is becoming higher and higher, while the high-frequency component is not decomposed at all [3], which can be well demonstrated in Fig. 6[6]. Fig. 6 is a real example of multi-resolution decomposition.

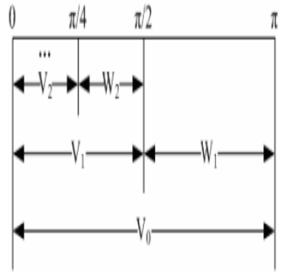

Fig. 4. The gradual division of function space and frequency band (scale *j*=2).

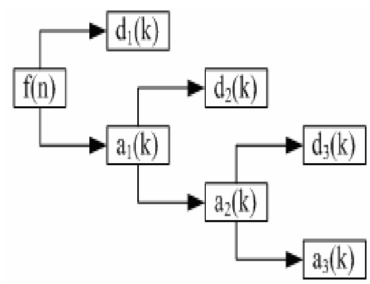

Fig. 5. The block diagram of 3 scales multi-resolution decomposing tree.

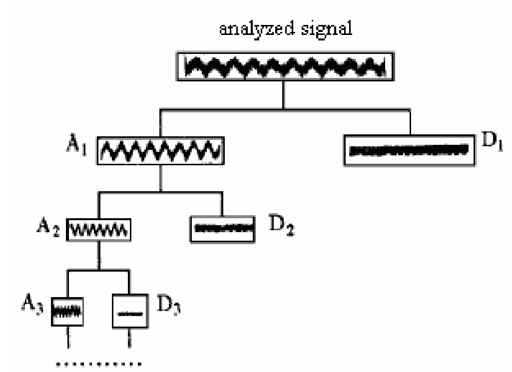

Fig. 6. The demonstration of multi-resolution decomposition.

VI. Selection of Wavelet Function

One of the differences that WT differs from traditional Fourier Transform is that WT doesn't have fixed wavelet function. Therefore, different wavelet functions have quite different errors for first-harmonic component and harmonics detection, which means the selection of wavelet function is rather important while using WT for signal processing. The 'db' family gives better accuracy through minimizing the spectral leakage problem when using mother wavelet with high order N.[5]. In case of low distortion level the 'db' family is the suitable one and the most suitable mother wavelet is either ‘db9’ or ‘db10’, therefore better analysis performance can be achieved with high order.[7]. According to what was studied in [6], [7] Daubechies wavelet is adopted in this paper, which was constructed by world famous analyzing wavelet scholar Inrid Daubeahies. Daubechies wavelet has character of depicting the global and local singular change of signal, especially that of the local singular change. Daubechies wavelet is usually short for dbN, where N means the scale of wavelet. In this paper db10 is chosen for simulation analysis.

VII. MATLAB Simulation for Designing Passive Filters

In this part first of all an HVDC System is modelled by MATLAB Modelling Section shown in Fig. 1. The HVDC rectifier is built up from two 6-pulse thyristor bridges connected in series. The converter is connected to the HVDC rectifier is built up from two 6-pulse thyristor bridges connected in series. The converter is connected to the system with a 1200-MVA Three-Phase transformer (three windings). A 1000-MW resistive load is connected to the DC side through a 0.5 H smoothing reactor. The output current signal in Fig.7 is composed of 5^{th}, 7^{th}, 11^{th}, 13^{th} and 24^{th} harmonics and it is decomposed into $D1$, $D2$, and $D3$. Considering aliasing effect let sampling frequency be 2 KHz. If we continue the main signal decomposition through DWT as mentioned before in Fig. 6, we can see that the frequency bands of sequences $D1$, $D2$, and $D3$ are $1000{\sim}2000$, $500{\sim}1000$ *and* $250{\sim}500$, respectively. If we choose the main frequency 60 Hz, it is clear that the frequency bands related to D1 can be considered for 24^{th} harmonic (1440 Hz), the frequency bands related to D2 can be considered for 11^{th} and 13^{th} harmonics(660 Hz and 780 Hz) and the frequency bands related to $D3$ can be considered for 5^{th} and 7^{th} harmonics(300 Hz and 420 Hz). According to [8], [9] the double–tuned filter is appropriate for lower order harmonics and the high-pass filter is appropriate for high order ones(in this paper 24^{th} harmonic). Now in this part we try to obtain a good passive filter design for harmonic reduction in several steps based on figures related to $D1$, $D2$, $D3$ and S(main signal). It is notable that the reactive power for all passive filters is 150 Mvar. In first step, shown in Fig. 7, no filter is added to the system. In second step, as shown in Fig. 8, a filter is designed with Q=2 for both double-tuned passive filters and the high-pass one. In third step, shown in Fig. 9, Q=14, 14, 5 is chosen for double-tuned and high-pass filters, respectively. (14, 14, 5 is related to passive filters for "5^{th} and 7^{th}", "11^{th} and 13^{th}" and 24^{th} harmonics, respectively). In fourth step, as shown in Fig. 10, Q=20, 30, 7 is chosen for double-tuned and high-pass filters, respectively. If we compare these four figures we can see that we have reached our goal in final step, relying on main signal wave shape and reduction in wavelet coefficients, in which reduction of current harmonics is clear.

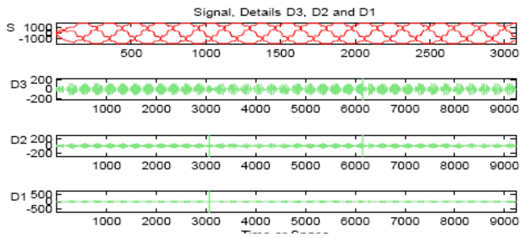

Fig. 7. First step of decomposing main signal into its harmonics before using a passive filter

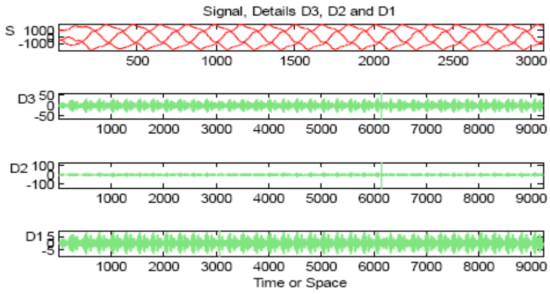

Fig. 8. Second step of decomposing main signal into its harmonics after using a passive filter with related Q=2, 2, 2.

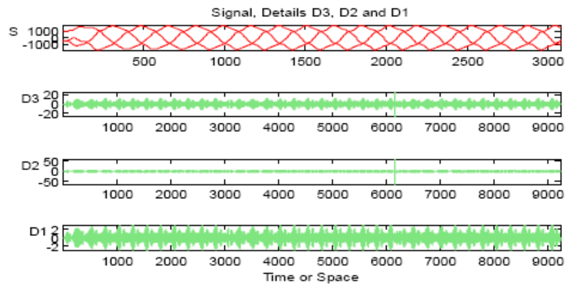

Fig. 9. Third step of decomposing main signal into its harmonics after using a passive filter with related Q=14, 14, 5.

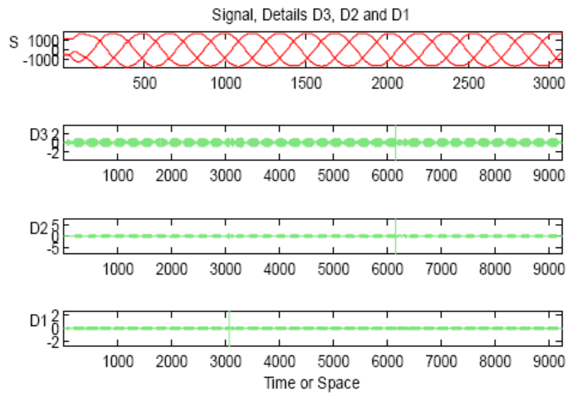

Fig. 10. Fourth step of decomposing main signal into its harmonics after using a passive filter with acceptable related Q=20, 30, 7.

VIII. Conclusion

Firstly, the basic theory of WT is stated at and basic wavelet theory for threshold de-noising was explained in summary and finally the results indicate that we can use the advantage of wavelet transform (for a successful de-noising) over other signal processing transforms, which insists on choosing wavelet transform for this paper.

Secondly, the theory of wavelet multi-resolution analysis is also summarized in detail and the method of harmonics detection through wavelet multi-resolution analysis (another advantage) based on MATLAB is proposed in this paper, and simulation analysis results are shown.

Finally, it was shown that by decomposing a full of harmonic signal into its components and designing an acceptable passive filter in several steps, we can obtain an acceptable filter design. It is notable that the whole done last works was focused just on detecting harmonics but in this paper it was proved that not only we can design passive filter for low order and high order harmonic reduction through a branch of signal processing which is called Discrete Wavelet Transform(DWT) but also, de-noising signals by using DWT is possible. For future works this method can be applied to modern variable-speed drives which has time variable harmonics.

REFERENCES

[1] V. Katic´, "Computer based harmonic measurement systems: Discussion and a realization," in *Proeedings of IEEE International Conference on Harmonics in Power Systems*, Atlanta, GA, ICHPS V. Sept. 1992, pp. 16–22.

[2] A. Galli, G. Heydt, and P. Ribeiro, "Exploring the power of wavelet analysis," *IEEE Comput*. Appl. Power, vol. 9, pp. 37–41, Oct. 1996.

[3] A. Graps, "An introduction to wavelets," *IEEE Comput*. Sci. Eng. Mag.,vol. 2, pp. 50–61, Summer 1995.

[4] H. Yang and C. Liao, "A de-noising scheme for enhancing wavelet based power quality monitoring system," *IEEE Trans*. Power Del., vol. 16, no. 3, pp. 353-360, Jul. 2001.

[5] D. L. Donoho, "De-noising by soft-thresholding," *IEEE Trans*. Inf. Theory, vol. 41, no. 3, pp. 613-627, May, 1995.

[6] B. Y. Zhang, H. A. Chen, G. H. Feng, and H. N. Zhang, "Application of wavelet multi-resolution analysis to harmonics detection based on MATLAB in power system," in *Proeedings of IEEE International Conference on Automation and Logistics*, Jinan, ICAL. Aug. 2007, pp. 137-143.

[7] W. G. Morsi and M. E. El-Hawary, "The Most suitable mother wavelet for steady-state power system distorted wave forms," in *Proeedings of IEEE International Canadian Conference on Electrical and Computer Engineering*, Niagara Falls, CCECE. May 2008, pp. 1-4.

[8] A. B. Nassif, W. Xu, and W. Freitas, "An investigation on the selection of filter topologies for passive filter applications," *IEEE Trans*. Power Del., vol. 24, no. 3, pp. 1710-1718, Jul. 2009.

[9] J. C. Das, "Passive filters—Potentialities and limitations," *IEEE Trans*. Ind. Appl., vol. 40, no. 1, pp. 232-241, Jan/Feb. 2004.

Hamid Rahimi Esfahani was born in Isfahan, Iran. in 1984. He got his BSc in Electrical Engineering (power) from Isfahan University of Technology (IUT), Iran. His major field of study is power quality, Two mass systems, and Attitude and energy control of small satellites. He is a post graduate Student of MSc in Najafabad Branch, Islamic Azad university. His publications are: Rahimi-Esfahani. H, Kiyoumarsi. F, 'Designing Passive Filters for Harmonic Reduction in a Noisy System Based on Wavelet Transform Compared to Traditional Method' *Journal of Computing* Vol. 3, No. 7, pp. 1-7. Jul. 2011. Rahimi-Esfahani.H, Kiyoumarsi.F, 'Designing Passive Filters for Harmonic Reduction Based on Discrete Wavelet Transform Combined with Computational Methods' *IREMOS* Vol.4, No.5, pp. 2317-2323. Oct.2011. He is a member of Young Researchers Club. He is also a member of Institute of Electrical and Electronics Engineers of Iran. He used to work in Sepahan Almas Electronics on power quality issues for 1 year. He has been teaching math in Hakim Sanaei and Imam Sadegh Guidance Schools since 3 years ago. His contact information is: e_mail:Rahimi_ham@yahoo.com tel: +98 3116250984, +989131027928