Mathematical Model of Laser Transmitter in Feed-Forward Mitigating Technique for Radio over Fiber System

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***Abstract*—This paper discusses a compensation system for nonlinear distortion of laser transmitter in Radio over Fiber application at frequency 5.2GHz, where feed-forward linearization technique is the applied technique in the compensation system. The paper focuses on the mathematical model of the laser nonlinear distortion which is an important tool in developing the feed-forward laser transmitter system model. The nonlinearity of the laser diode is modeled using Volterra series analysis, and the magnitudes of the laser 3rd order intermodulation distortion products (IMD3) are computed in MATLAB. The analysis result from the mathematical model has shown considerably good agreement with the results from some of the previous works. Hence, the mathematical model is deemed as useful in the modeling of feed-forward transmitter system.**

***Index Terms*—Radio over fiber, laser transmitter, feed-forward linearization, nonlinear distortion.**

I. Introduction

Wireless communication is entering a new phase where multimedia services are getting increasing demand and wireless user is signing up at increasing rate. Hence, higher bandwidth is needed for higher data transmission capacity. The wideband demands can be met with micro/pico cellular architecture. Radio over Fiber technology (RoF) provides a promising solution for this new cellular architecture by simplifying the structure of the base station (BS). All the processing functions such as modulation and routing are centralized at the central station (CS). The simplification of BSs can be translated into major system installation and operational savings, especially in wide-coverage broadband wireless communication, where a high density of BSs is necessary. However, the RoF links encounter performance degradation due to the laser transmitter nonlinearity which generates harmonic and intermodulation distortions in the modulating RF signal. The distortion and noise generated by the laser limit the achievable dynamic range and performance in a system [1]. Several compensation techniques such as feed-forward, quasi-feedforward, feedback, and predistortion have been considered. Feed-forward linearization is seen as the most effective technique, because it can achieve reduction in all orders of distortion for large bandwidth and high frequency without knowing the nonlinear characteristics of the laser [2]. Even though feed-forward linearization is a relatively complicated and sensitive scheme, it is a promising linearization solution in view of the demand for high channel capacity lightwave systems [3].

This paper will continue as follows, Section II introduces the architecture of the designed system. Section III introduces the mathematical model of the laser nonlinear distortion, while section IV discusses the simulation result. Finally, section V is the conclusion.

II. Architecture of the Designed System

The system architecture of the designed system is shown in Fig. 1. The system consists of two loops, the first loop is signal-cancellation loop and the second loop is error-injection loop. Firstly, at system input, the two tone (or more) RF input signal is split into two paths, where one modulates the primary Laser Diode Circuit LD1, whereas the other is a reference signal. The optical output of LD 1 contains the data signal and the distortion products due to nonlinearity of the laser. After that, the optical output of Laser 1 is further split into two paths, and the other one is transmitted through optical fiber while the other one is detected by Photo Diode Circuit PD1 to convert back to RF signal containing distortion from LD 1. In the first loop, the RF signal with distortion is subtracted with the reference signal to yield only the error RF signal. The variable amplifier in the first loop functions as amplitude matching between the two paths to obtain the optimum signal cancellation.

Whereas in the second loop, the error RF signal with the opposite phase modulates the second Laser Diode Circuit LD2. The optical output of LD 2 is combined to the optical signal from the first path (output of LD 1) to cancel the error signal. The variable amplifier in the second loop functions as amplitude matching between two paths to obtain the optimum error cancellation. Therefore, output of the transmitter contains the data signal and optimum error reduction. After this, the signal is transmitted through the transmission line (fiber line) and is detected by Photo Diode Circuit PD2.

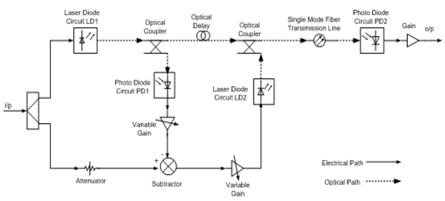

Fig. 1. Feed-forward linearization system architecture.

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III. Mathematical Modelling

A. Volterra Series Model

The Volterra Series is a power series which expresses the output of a nonlinear system with memory in 'powers' of input. Volterra series have found a great deal of use in calculating small but troublesome distortion terms in transistor amplifiers and other systems [4]. Volterra series relates the input and output of a nonlinear system as follow:

$$y(t) = \sum_{n=1}^{\infty} \frac{1}{n!} \int_{-\infty}^{\infty} du_1 \ldots \int_{-\infty}^{\infty} du_n g_n(u_1, \ldots, u_n) \times \prod_{r=1}^{n} x(t-u_r) \tag{1}$$

where $y(t)$ is the output, $x(t)$ the input, and the $g_n(u_1,\ldots,u_n)$ are called the Volterra kernels or impulse responses of the system.

The Volterra Series is chosen for laser modeling because laser transmitters have both system nonlinearity and memory properties. The laser transmitters have only weak nonlinearity, hence only a few Volterra kernels need to be considered.

B. Volterra Series Analysis

The laser diode modeling begins from the simplified laser rate equation, where the injected current I_a can be expressed as a function of the photon density Q in a semiconductor laser [5]:

$$I_a - I_{th} = \frac{V'}{\Gamma} \cdot \frac{dQ}{dt} + \frac{V'}{\Gamma \tau_p} . Q + \frac{V'}{\Gamma g} \cdot \frac{d}{dt} \left\{ \frac{\frac{dQ}{dt} + \frac{Q}{\tau_p}}{(1-\varepsilon Q)Q} \right\} \tag{2}$$

where,

I_a = applied modulation current,
I_{th} = laser threshold current,
V' = volume of the active region times the electron charge,
Γ = optical confinement factor,
Q = photon density in the active region,
τ_p = photon life time,
g = optical gain factor,
ε = gain compression parameter

Both the applied current and photon density can be divided into ac and dc parts, where the time varying part (ac part) is of interest in the laser diode modeling. Considering

$$Q = Q_0 + q(t) \tag{3}$$

$$I_a = I_0 + i(t) \tag{4}$$

where I_0 is the laser's biasing current, $i(t)$ is the modulating current, and Q_0 and $q(t)$ are steady state and time varying parts of the photon density respectively.

By using Taylor expansion on the denominator of the last term in Equation (2), and substituting Equation (3) and (4) into Equation (2), a time varying equation between modulating current and photon density is obtained [6]:

$$\begin{aligned} i(t) = & D.q(t) + E.q'(t) + F.q''(t) \\ & + M.\{q(t).q'(t)\} + N.\{q'(t)\}^2 \\ & + N.\{q''(t)\} \\ & + S.\{q^2(t).q'(t)\} + 2G.\{q(t).(q'(t))^2\} \\ & + G.\{q^2(t).q''(t)\} \end{aligned} \tag{5}$$

where D, E, F, M, N, S, and G are constants express by the laser parameters Γ, τ_p, g, ε, V', and Q_0.

By using the harmonic input method, the first three Volterra kernels of an inverse laser system, G_1, G_2, G_3 can be obtained in frequency domain [7]. G_1, G_2, and G_3 are also known as the output-to-input transfer functions. The transfer functions are determined as follow:

$$G_1(\omega_1) = D + jE.\omega_1 - F.\omega_1^2 \tag{6}$$

$$G_2(\omega_1, \omega_2) = jM.(\omega_1 + \omega_2) - N.(\omega_1 + \omega_2)^2 \tag{7}$$

$$G_3(\omega_1, \omega_2, \omega_3) = j2S.(\omega_1 + \omega_2 + \omega_3) - 2G.(\omega_1 + \omega_2 + \omega_3)^2 \tag{8}$$

In real laser system model, the input current has to be used to find the photon density of the laser. Hence, the input-to-output transfer functions, H_n are needed. $H_n(\omega_1,\omega_2,\ldots\omega_n)$ can be calculated from G_n using the n_{th} order inverse method [8]. H_1, H_2, and H_3 are calculated as follow:

$$H_1(\omega_1) = \frac{1}{G_1(\omega_1)} \tag{9}$$

$$H_2(\omega_1, \omega_2) = \frac{1}{2} . \frac{G_2(\omega_1, \omega_2)}{G_1(\omega_1).G_1(\omega_2).G_1(\omega_1 + \omega_2)} \tag{10}$$

$$H_3(\omega_1, \omega_2, \omega_3) = \frac{1}{6} \left\{ \frac{G_3(\omega_1, \omega_2, \omega_3) H_1(\omega_1) H_1(\omega_2) H_1(\omega_3) - G_2(\omega_1 + \omega_2, \omega_3) Z}{G_1(\omega_1 + \omega_2 + \omega_3)} \right\} \tag{11}$$

where,

$$\begin{aligned} Z \equiv & H_1(\omega_1).H_2(\omega_2,\omega_3) + H_1(\omega_2).H_2(\omega_1,\omega_3) \\ & + H_1(\omega_3).H_2(\omega_1,\omega_2) \end{aligned} \tag{12}$$

By using the forward transfer functions H_1, H_2, and H_3, any selected distortion term can be produced by Volterra series analysis. For example, for a two-tone input signal, the ratio of a third order intermodulation product of the type $2\omega_1$-ω_2 with respect to the carrier is given by

$$\frac{IMD1}{C} = \frac{1}{8} m^2.(i_0 - i_{th})^2 \times \frac{|H_3(\omega_1, \omega_1, -\omega_2)|.|H_1(0)|^2}{|H_1(\omega_1)|^3} \tag{13}$$

where i_0 is the laser's biasing current, m is the optical modulation index per tone.

In a similar way the ratio of a third order intermodulation product of the type $2\omega_2$-ω_1 with respect to the carrier is given by

$$\frac{IMD2}{C} = \frac{1}{8} m^2.(i_0 - i_{th})^2 \times \frac{|H_3(\omega_2, \omega_2, -\omega_1)|.|H_1(0)|^2}{|H_1(\omega_1)|^3} \tag{14}$$

IV. Simulation Result

When the signal input to a laser is a two-tone input

$$i(t) = I\cos\omega_1 t + I\cos\omega_2 t \tag{15}$$

Because of the nonlinear property of the laser, the output signal will be

$$q(t) = \sum_{m,n} B_{mn} \cos(m\omega_1 t + n\omega_2 t) \tag{16}$$

The results of this project will be focused on the third order intermodulation distortions (IMD3), hence the output signal will only include the IMD3 products

$$\begin{aligned} q(t) &= C\cos\omega_1 t + C\cos\omega_2 t \\ &+ IMD1\cos(2\omega_1 - \omega_2)t \\ &+ IMD2\cos(2\omega_2 - \omega_1)t \end{aligned} \tag{17}$$

where C is the magnitude of carrier signal, IMD1 and IMD2 are the magnitudes of the intermodulation products.

The laser transfer functions H_1, H_2, H_3 are computed with the aid of MATLAB and the laser transmitter output is evaluated as the sum of the carrier signal and the 3rd order intermodulation products. The input carrier signal is a two-tone input at frequency 5.20 GHz with 10 MHz frequency spacing. Let the laser diode bias current be 36.75 mA and the optical modulation index be 0.28. By setting ω_1=5.20 GHz and ω_2=5.21 GHZ, the magnitude of C, IMD1, and IMD2 are calculated. The frequency spectrum of the output signal in Equation (17) is obtained by using Fast Fourier Transform (FFT) function in MATLAB. The output spectrum is shown in Fig. 2.

It can be seen from the spectrum that the IMD3 products are 48.7dB lower than the carrier signals. This result is similar to the experimental result in [2], [9], where the IMD3 product is 45dB and 51dB lower than the carrier signal respectively.

V. Conclusion

As a conclusion, the mathematical model developed from the laser rate equation using Volterra Series analysis has shown that the 3rd order intermodulation distortion product is 48.7dB lower than the carrier signal. The result has shown good agreement with some of the previous works. Therefore, the mathematical model can provide a considerably accurate prediction for the laser distortion product. This mathematical model can become a useful tool in the modeling for the laser transmitter feed-forward linearization system design shown in Section II.

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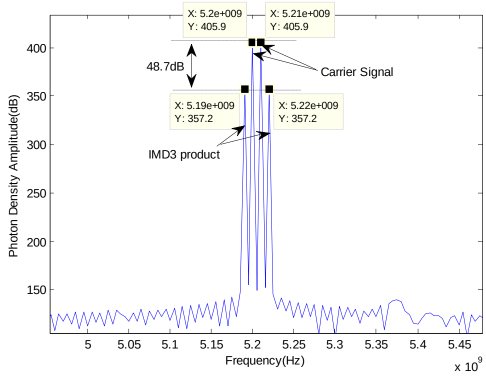

Fig. 2. Output spectrum for the laser transmitter model.

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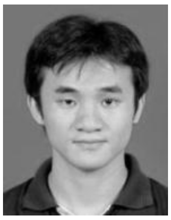

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