Throughput Analysis for a Dynamic Spectrum Sharing Model with Finite Primary Users and Infinite Secondary Users

Suman Bhatta charjee and Subhasree Bhatta charjee

***Abstract*—In this paper we present throughput analysis for a dynamic spectrum-sharing model. We assume that number of primary users allocated to a channel is fixed all the time. But the number of primary users is different for different channels. Primary users allocated to a specific channel compete to access the channel and the secondary users or cognitive users sense the channel whether the channel is free or not. The transmission probabilities of primary and secondary users are different. The packet transmission delay of secondary users is considered in this paper. Our objective is to find the optimal number of secondary users to maximize the total throughput of primary and secondary users.**

***Index Terms*—Primary users, secondary users, throughput analysis, transmission probability.**

I. INTRODUCTION

Fixed radio spectrum allocation causes under utilization of some licensed radio spectrum. The limitation of unlicensed spectrum creates a major problem in the development of next generation radio system. Reports from Federal communication commission (FCC) [1] shows that over 70% of the allocated spectrum is unused in a given time. Cognitive radio technology provides us the facility of sharing the licensed radio spectrum between licensed users (primary users) and unlicensed users (secondary users). This is termed as dynamic spectrum sharing in our paper. In the scenario of dynamic spectrum sharing, primary users or licensed users have the privilege of using licensed spectrum band. Cognitive users or secondary users periodically sense the channel whether it is free or not and opportunistically use the spectrum band when it is not used by primary users. Thus spectrum utilization improves.

There are many research works regarding dynamic spectrum sharing [2]-[4]. In [5], channel occupancy by the primary users is modeled by Markov chain model. Then a slotted transmission protocol for secondary users using periodic sensing strategy is proposed. Many researchers propose protocols and algorithms to optimize the performance of dynamic spectrum sharing.

In [6], the authors model the interaction between primary and secondary users using continuous time Markov chain approach. The proposed primary prioritized Markov approach scheme provides efficient utilization of spectrum among unlicensed users. In [7] throughput trade off problem for a multiple channel cognitive radio network is studied. For maximization of the throughput of cognitive radio network they have designed the optical sensing time and power allocation methods. The effect of cooperation overhead on throughput of secondary network is studied in [8]. Total sensing time and throughput of cognitive network is derived. In [9], throughput and delay for random channel selection scheme base CR system and idle channel selection based CR system is derived. Delay analysis based on success probability of transmission has been studied here. Different MAC protocols are used in CR network for efficient utilization of spectrum. An adaptive MAC protocol for throughput enhancement in cognitive radio network is proposed in [10], [11]. In [11], a decentralized adaptive medium access control protocol with no dedicated global common control channel is proposed. In [12], L. Wang et al. propose the concurrent transmission MAC protocol. This MAC protocol is used to identify the possibility of second link in the presence of first link in an unlicensed spectrum environment. In [13] authors designed a full duplex multi channel MAC protocol for multi-hop transmission in cognitive radio network.

In [14] authors discussed the problem of dynamic sharing of channels between primary and cognitive users with one primary user in each channel. In [15] authors allow variable number of primary users compete for channels. They derive throughput model for primary and secondary users and finally calculate the optimal number of secondary users needed to maximize the throughput.

The rest of the paper is organized as follows. In Section II, the network model is discussed. In Section III, throughput analysis for primary and secondary users is done. Section IV deals with results and discussion. Section V concludes the paper.

II. NETWORK MODEL

We consider a network with finite number of primary users and infinite number of secondary users. We also consider that secondary users periodically check whether the channel is occupied by primary users or not. If the channel is accessible by secondary users then the users transmit packets with probability q. Primary users transmit with probability p. Let, N and $\widetilde{N}$ are the number of primary users and secondary users respectively. Here, N is assumed as finite and $\widetilde{N}$ is infinite. M is the number of channels. Number of primary

Manuscript received June 3, 2012; revised July 3, 2012.
Subhasree Bhattacharjee is with the Narula Institute of Technology, Kolkata, India (e-mail: bhattacharjeesubhasree@gmail.com).
Suman Bhattacharjee is with the IBM India Pvt. Ltd (e-mail: suman.only@gmail.com).

users allocated into a channel is fixed but the number of primary users in each channel may be different. As in [15], a special type of fixed allocation is considered here. This is called even allocation. In this paper we assume the packet transmission delay of secondary users. That means each secondary users sense the channel and if the channel is free then it sends the packet with a delay d. Sensing channel also causes delay, but that delay is not considered here. Only, packet transmission delay is considered. In order to compete for idle channels secondary users use carrier sense multiple access with collision avoidance (CSMA/CD) scheme [15]. If the secondary users sense the channel as busy, then transmission is deferred for random time. We assume that for secondary user's detection time of idle channel is negligible compared to the access time of secondary users [14]. Thus, detection time of a channel i.e. the time spent to detect that whether the channel is free or not is negligible compared to the total slot time of transmission. So, the total slot time is contributed towards throughput calculation [15].

III. Throughput Analysis

From [15], total throughput for primary users for M channel is

$$T = \sum_{i=1}^{M} T(i) = \sum_{i=1}^{M} X_i . p.(1-p)^{X_i - 1} \quad (1)$$

Here X_i = number of primary users in the ith channel, p = Transmission probability for Primary user,

Throughput of secondary users in a time slot depends on number of accessible channels and number of active secondary users that have data to transmit.

For each channel, the channel will be accessible for secondary user if primary users have no data to transmit. So for ith channel the success probability is $(1-p)^{X_i}$.

Let, Y is a random variable. Yi = 1, if channel is accessible for Secondary user and 0, otherwise.

From [15], *p.m.f* for Y_i is

$$f_{Y_i}(1) = P_r(Y_i = 1) = (1-p)^{X_i}$$

$$f_{Y_i}(0) = P_r(Y_i = 0) = 1-(1-p)^{X_i}$$

Let, $V = \sum_{i=1}^{M} Y_i$

So, *p.m.f.* for V is

$$f_V = f_{Y_1} * f_{Y_2} * .... * f_{Y_M}$$

where * means convolution.

Now, $f_{Y_1} * f_{Y_2}$ involves 3 cases, λ = 0, 1, 2

When λ =0, no channel is accessible,

λ =1, one channel is accessible,

λ =2, two channels are accessible.

So for λ =0, $f_{Y_1} * f_{Y_2}$

$$= \{1-(1-p)^{X_1}\}\{1-(1-p)^{X_2}\}$$

For λ =1, $f_{Y_1} * f_{Y_2}$

$$= (1-p)^{X_1}\{1-(1-p)^{X_2}\} + (1-p)^{X_2}\{1-(1-p)^{X_1}\}$$

For λ =2, $f_{Y_1} * f_{Y_2}$

$$= (1-p)^{X_1}(1-p)^{X_2}$$

For M channels if λ =M-1

$$f_V = f_{Y_1} * f_{Y_2} * .... * f_{Y_M} = \sum_{j=1}^{M} (1-p)^{N-X_j}\{1-(1-p)^{X_j}\} \quad (2)$$

Now we define a random variable z to denote the number of active secondary users in a time slot. Active secondary users are those whose have data to transmit. We consider z is infinite and follow Poisson distribution. Its *p m f* is

$$f_z(k) = \frac{e^{-m}.m^K}{K!} \quad (3)$$

where $m = \widetilde{N}q$

Here K is the number of active secondary users and each secondary user has transmission probability q.

Let among M channels, h channels are accessible. So the probability of selecting each channel is $1/h$. There should be no collision if it selects an accessible channel that is not selected by any other active secondary user. So the probability of no collision for each channel is $\frac{1}{h}(1-\frac{1}{h})^{K-1}$ [15]. For h channel the probability is $(1-\frac{1}{h})^{K-1}$. For K active Secondary users the throughput can be defined as

$$\tilde{T}(h,k) = K(1-\frac{1}{h})^{K-1} \quad (4)$$

From [9], the packet transmission delay = 1/ successful packet transmission probability.

So total delay for h channel = $\frac{1}{(1-\frac{1}{h})^{K-1}}$

So if we conceder the delay then equation (4) becomes,

$$\tilde{T}(h,k) = K(1-\frac{1}{h})^{K-1}.\frac{1}{Delay} = K(1-\frac{1}{h})^{K-1}.(1-\frac{1}{h})^{K-1} = K(1-\frac{1}{h})^{2(K-1)} \quad (5)$$

Now Throughput of a Secondary Users at time t is given as

$$\widetilde{T}_t = K_t(1-\frac{1}{h_t})^{2(K_t-1)} \quad (6)$$

Now at time t+1, throughput is denoted as $\widetilde{T}_{t+1}$. So

$$\widetilde{T}_{t+1} = K_{t+1}(1-\frac{1}{h_{t+1}})^{2(K_{t+1}-1)}$$

Here we conceder $K_{t+1} = K_t + \nabla K.K_t$ and $h_{t+1} = h_t - \nabla h.h_t$

Solving the equation we find

So $$\tilde{T}_{t+1} = \tilde{T}_t(1+\nabla K).\frac{(h_t.A-1)^{2(K_{t+1}-1)}}{A^{2(K_{t+1}-1)}.(h_t-1)^{2(K_t-1)}.h_t^{2\nabla K.K_t}} \quad (7)$$

Similarly we can find

$$\widetilde{T}_{t+2} = \widetilde{T}_{t+1}(1+\nabla K).\frac{(h_{t+1}.A-1)^{2(K_{t+2}-1)}}{A^{2(K_{t+2}-1)}.(h_{t+1}-1)^{2(K_{t+1}-1)}.h_{t+1}^{2\nabla K.K_{t+1}}} \quad (8)$$

So we can write

$$\tilde{T}_{t+n} = \tilde{T}_{t+n-1}(1+\nabla K).\frac{(h_{t+n-1}.A-1)^{2(K_{t+n}-1)}}{A^{2(K_{t+n}-1)}.(h_{t+n-1}-1)^{2(K_{t+n-1}-1)}.h_{t+n-1}^{2\nabla K.K_{t+n-1}}} \quad (9)$$

So, throughput of a secondary user at time *t+n* depends on throughput at time *t+n*-1.

Now from (3) and (5) we have,

$$\widetilde{T}(h) = \sum_{k=0}^{\alpha} f_z(k).\widetilde{T}(h,k)$$

$$= \sum_{k=0}^{\alpha} \frac{e^{-m}m^k}{k!}.k(1-\frac{1}{h})^{2(k-1)}$$

$$= e^{-m}\sum_{k=1}^{\alpha} \frac{m^k}{(k-1)!}.(1-\frac{1}{h})^{2(k-1)}$$

$$= me^{-m}\sum_{k=1}^{\alpha} \frac{\{m(1-\frac{1}{h})^2\}^{k-1}}{(k-1)!} ...let, k-1=l$$

$$= me^{-m}\sum_{l=0}^{\alpha} \frac{\{m(1-\frac{1}{h})^2\}^{l}}{l!}$$

$$= me^{-m}.e^{m(1-\frac{1}{h})2}$$

$$= m.e^{-m}.e^{m(1-\frac{2}{h}+\frac{1}{h^2})}$$

$$= me^{m(\frac{1}{h^2}-\frac{2}{h})} \quad (10)$$

$$= m.e^{\frac{m}{h}(\frac{1}{h}-2)}$$

$$= \tilde{N}qe^{\frac{\tilde{N}q}{h}(\frac{1}{h}-2)}$$

Now combining (2) and (10) we have,

$$\tilde{T} = \sum_{h=1}^{M} f_v(h)\tilde{T}(h)$$
$$= \sum_{h=1}^{M}[\sum_{j=1}^{M}(1-p)^{N-X_j}\{1-(1-p)^{X_j}\}].\tilde{N}qe^{\frac{\tilde{N}q}{h}(\frac{1}{h}-2)} \quad (11)$$

From (10) if throughput is maximum, then we have, $\frac{\partial \widetilde{T}}{\partial \widetilde{N}} = 0$

So $$qe^{\frac{\tilde{N}q}{h}(1/h-2)} + \widetilde{N}q(\frac{q}{h})(1/h-2).e^{\frac{\tilde{N}q}{h}(1/h-2)} = 0$$

So $$\widetilde{N}(h) = -\frac{h}{q(\frac{1}{h}-1)}$$

$$\tilde{N}(h) = -\frac{h^2}{q(1-2h)}$$
$$= \frac{h^2}{q(2h-1)} \quad (12)$$

So $$\widetilde{N} = \sum_{h=1}^{M} f_v(h)\widetilde{N}(h)$$

$$\tilde{N} = \sum_{h=1}^{M}[\sum_{j=1}^{M}(1-p)^{N-X_j}\{1-(1-p)^{X_j}\}].\frac{h^2}{q(2h-1)} \quad (13)$$

Now Total throughput *T* = Throughput for Secondary user + Throughput for Primary user

$$T = \sum_{h=1}^{M}\sum_{j=1}^{M}(1-p)^{N-X_j}\{1-(1-p)^{X_j}\}\tilde{N}qe^{\frac{\tilde{N}q}{h}(\frac{1}{h}-2)} + \sum_{i=1}^{M}X_i.p.(1-p)^{X_i-1}$$

IV. Results and Discussion

In this section we simulate the model to analyze the performance of the model. We examine the total throughput of both primary and secondary users vs. fraction of primary users, i.e. (no of primary users / total number of users). Our objective is to find the optimal number of secondary users for which total throughput is maximized. In Fig. 1, for even allocation total throughput is plotted with fraction of primary users. Here, number of channel is 10 and number of primary users is 5. By varying the number of secondary users we get the plot. We consider three cases regarding the transmission probability of primary users and secondary users. In one case, transmission probability of both primary and secondary users are equal, i.e. (p=q), in second case transmission probability (p) of primary users is greater than the transmission probability (q) of secondary users and in third case reverse is the situation of second case, i.e. p<q. It is reflected from the graph that when p<q and fraction of primary users is within the range of 1 to 0.33, total throughput has greater values than other two cases (i.e. p=q and p>q). When number of secondary users increases from 11 to 60 (i.e. fraction of primary users decreases from 0.3125 to 0.076) and when p=q, throughput has greater values than other two cases (i.e. p>q

and $p<q$). So, as the number of secondary users increases the impact of its higher transmission probability (q) causes the total throughput to increase but when the number of secondary users cross a particular limit then this impact disappears. Because above a certain value of secondary users there will be collision between two secondary users for occupying free channels. But when fraction of primary users is closer to 1, all throughput values converge, because then there is no secondary user in the network. Then the total throughput is the throughput of primary users only. So impact of $p>q$, $p<q$ and $p=q$ have no significance.

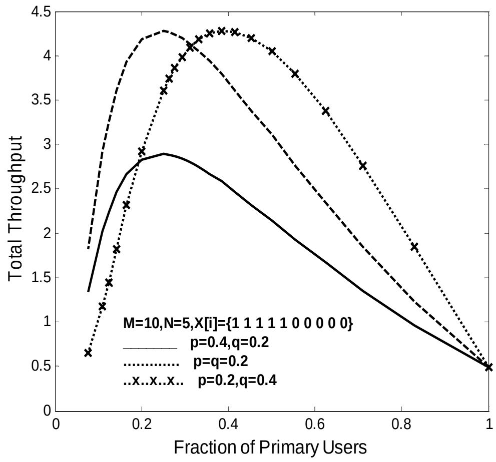

Fig. 1. Total throughput for even allocation, when $p=q$, $p>q$ and $p<q$

Fig. 2 plots total throughput with fraction of primary users for uneven allocation. Here, number of channel is 10 and number of primary users is 5. By varying the number of secondary users we get the plot. From this graph we see that this graph follows similar trend of results as the previous graph. When $p>q$, throughput value is always lower than the other two cases, i.e. p=q and $p<q$. That means if transmission probability of primary users is greater than the transmission probability of secondary users then secondary users will get lower chance of transmission. As a result throughput of secondary users' decreases. It is observed from this graph that throughput value is negative when number of secondary users is zero but this is not the case for even allocation.

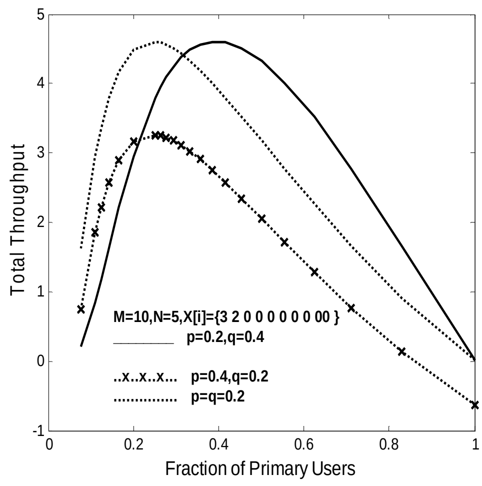

Fig. 2. Throughput for uneven allocation when $p=q$, $p>q$ and $p<q$.

Fig. 3. shows throughput variation for even allocation for different transmission probabilities of primary users when transmission probability of secondary users is kept equal to 0.2. When p is low then average per channel traffic load is low. As a result, channel can accommodate more number of secondary users. Thus, total throughput increases. Fig. 4 reflects throughput variation for even allocation for different transmission probabilities of secondary users when transmission probability of primary users is kept equal to 0.2. From Fig. 5, throughput variation for different number of primary users is revealed. Here, transmission probabilities of both primary and secondary users are kept fixed.

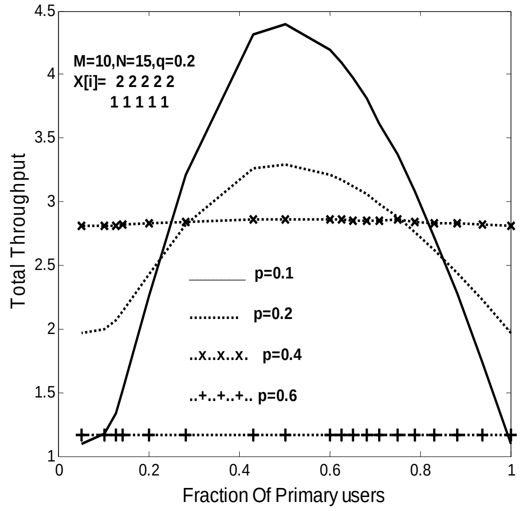

Fig. 3. Throughput for even allocation by varying p when q is fixed

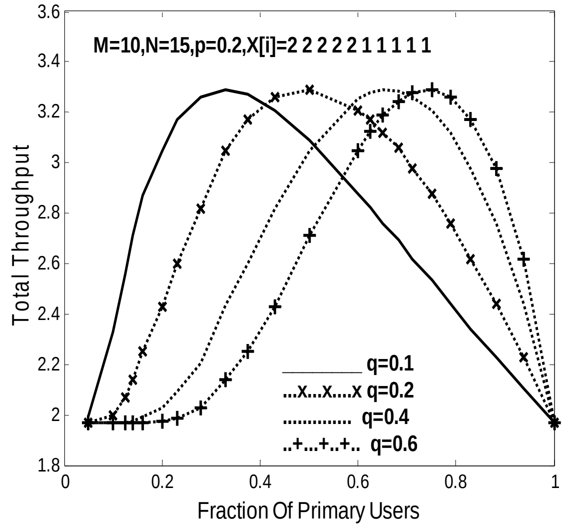

Fig. 4. Throughput for even allocation by varying q when p is fixed

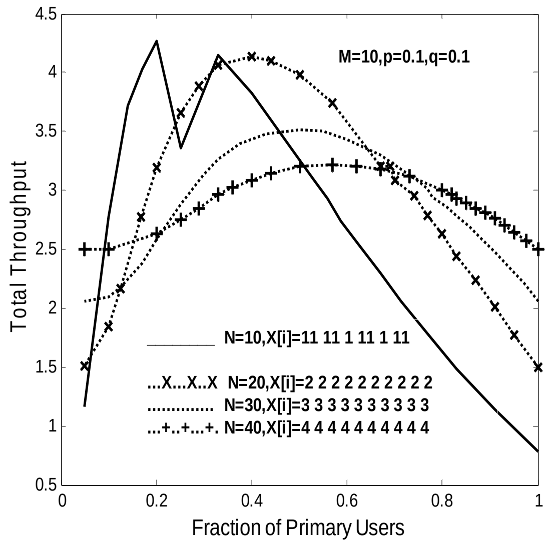

Fig. 5. Throughput variation for different allocation of primary users

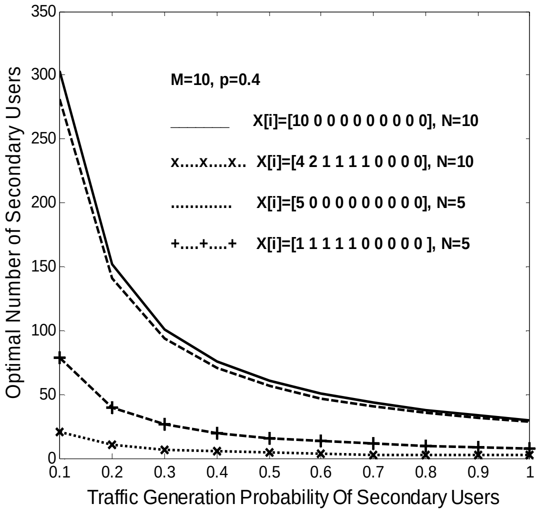

Fig. 6. Optimal number of secondary users vs. traffic generation probability of secondary users

Fig. 6 plots the variation of Optimal number of secondary users vs. traffic generation probability of secondary users where traffic generation probability of primary users is kept fixed at 0.4.Here, number of channel is equal to 10 and number of primary users are kept equal to 10 and 5 respectively. It is seen from the graph that as the traffic generation probability of secondary users increases, the optimal number of secondary users decreases. It is verified from equation 11 that when q increases then optimal number of secondary user decreases. When M=10, N=10 and q=0.1, then optimal number of secondary users is 303.A less number of secondary users means channel resource is underutilized and greater number causes collision between secondary users. Also, we may notice that for N=10, if we consider the allocation as [10 0 0 0 0 0 0 0 0 0], then more number of secondary users can be allocated than the allocation as [4 2 1 1 1 1 0 0 0 0]. Because more channels are free more number of secondary users can be accommodated. Figure 3 gets higher throughput for even allocation. When transmission probability of secondary users is fixed then by varying transmission probability of primary users we get better result than [15]. Fig. 6 plots optimal number of secondary users' vs. traffic generation probability of secondary users. For M=10, p=0.4, N=10, q=0.1 and for allocation [10 0 0 0 0 0 0 0 0 0] we get higher number of optimal secondary users (=303) than [15].

V. Conclusions

In this paper we consider fixed allocation of users in different channels. Our objective of this paper is to find out optimal number of secondary users to maximize total throughput of both primary and secondary users. As the throughput of primary users does not depend on number of secondary users, our aim is to find the optimal number of secondary users to maximize throughput of secondary users. The main contribution of this paper is that we are able to determine the expression of probability mass function of the distribution of channels by applying discrete convolution method. Assuming Poisson distribution we evaluate optimal number of secondary users and it is seen that there is a relation between transmission probability of secondary users and number of secondary users. We consider packet transmission delay of secondary users, which was not considered in [15]. Moreover we consider different transmission probabilities of primary and secondary users and thus extend the model discussed in [15]. As a future work, we want to design the relation between free channels and active secondary users so that total throughput of both primary and secondary users can be maximized.

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