VMathLab, a 3D-Game-Like System for College Math Learning Enhancement

Yonggao Yang, Lin Li, and Kiranmai Bellam

Abstract—**Mathematics is a challenging subject to many American students at all levels, from K12 to college. In this article, we first address the issues encountered in the traditional classroom and explore the approaches to ease the problems. Then, we present a 3D-game-like math teaching and learning system—*VMathLab* which was developed as a supplemental tool to enhance students' mathematics learning. *VMathLab* consists of a number of learning modules covering various mathematics topics, including college algebra, trigonometry, and calculus I. Each module is an individual virtual reality environment where students can follow tutorials and solve mathematics problems while playing 3D games. As an on-line system, *VMathLab* enables students to collaborate in team work and learn from each other. Moreover, the system can record students' performance and generate statistical data for instructors to do analysis. Experimental results and student surveys through pilot teaching show that the system is effective.**

Index Terms—**College algebra, calculus I, gaming, learning modules, trigonometry, virtual reality**

I. INTRODUCTION

Traditional mathematics teaching in America faces a serious challenge. Students are losing interest in mathematics and their performance is far below that of students in other industrialized countries [1]. Many students have to retake mathematics courses several times before getting a pass grade. Some students even have to give up their engineering dreams due to the failure of completing the mathematics requirement of the Science, Technology, Engineering, and Mathematics (STEM) degree plans. This leads to a sharp decline of the number of engineering degrees awarded in America, while that number is increasing in other nations. Studies show that "math is boring" and "math is difficult" top the reasons of students' losing interest in mathematics. A crucial reason of STEM students losing enthusiasm in math is that they cannot see the connection between math and real world engineering when they are learning math in traditional classrooms [2]. A recent survey shows that 81% interviewed students would like to know how people in music, sports, video games, and science and engineering use mathematics [3]. Thus, how to effectively engage students and how to bridge math learning and science/engineering problem become the key to solve the problem.

Mathematics topics are often abstractive and untouchable which contribute a lot to the high failure rate of students. Mastery of these concepts requires students to build *mental models* on phenomena which incorporate invisible factors representing intangible items and abstractions [1], [4], [5]. Meanwhile, students living in digital age are active learners. Traditional math teaching is inefficient due to the lack of multiple ways to represent information using interactive text, image, sound and video to engage a broad range of learners.

To address these issues, we developed a new college math teaching/learning system—*VMathLab* as a supplement tool to enhance students' math learning. Our strategy is to construct 3D game-like learning modules using virtual reality and computer graphics technologies to make abstract and non-intuitive math concepts "visible", "touchable", and therefore, easy to understand. Through many real world cases and gaming elements, we not only make math learning entertaining, but also bridge mathematics and engineering by allowing students to see how mathematics is used in solving real life problems. As an open structured platform, the *VMathLab* system can be easily expanded to incorporate other course topics, such as Statistics, Discrete Math, etc.

The rest of the paper is organized as follows: Section 2 describes the advantages of using virtual reality and gaming technologies in math learning. Section 3 presents the whole picture of the *VMathLab* system. Section 4 gives a closer look at a few learning modules and provides the details of design and implementation. Section 5 concludes the article.

II. GAMING AND VIRTUAL REALITY TECHNOLOGIES IN MATH LEARNING ENHANCEMENT

3D virtual learning systems have been widely used to facilitate in teaching intricate concepts [4], [6], [7]. Such an environment possesses unique features that traditional classroom teaching does not have. It can: (1) *help learners understand complex, abstract, and non-intuitive concepts and systems*. The ability to work with abstract and multidimensional phenomena is critical in mathematics learning. It is imperative to find ways to help learners recognize patterns, understand physical processes, move among different frames of reference, and control dynamic models that may contain intangible information. Traditional math concepts displaying through computer screens or book illustrations are two-dimensional. It is hard for the learners to imagine or observe objects from different angles. Using 3D graphics and simulation technologies, a virtual reality learning system can create and visualize 3D representations of data, events, and concepts that have no physical form in real world. (2) *Enhance the learning by making the experience game-like, while still retaining the underlying content*. A virtual learning environment is engaging, entertaining, and interactive. Due to its multiple ways of information delivery and impressive learning procedure,

students can master complex knowledge faster and retain them longer when they are actively involved in constructing the knowledge through learning-by-doing. (3) *Enable us to create a better environment for those who have difficulty in learning from traditional classrooms*. Study shows that students struggling in math courses typically have excessive absences and eventually give up on themselves as learners. They are tired of the learning style in traditional classroom and hard to be motivated even by experienced teachers using inquire-based pedagogy. Virtual learning systems with intuitive contents and activities resemble the entertainment and communication media can reengage them by providing a new teaching approach.

In recent years, many educators have explored approaches of using virtual environment for teaching renovation. For example, "teaching in Second Life" has shown great potential for encouraging students and proved to be effective in knowledge delivery [8], [9]. Compared with others' work, our project, supported by the university and external funding sources, aims to create a special educational environment[1] to address the critical math teaching issues discussed in previous section and investigate new teaching methods.

III. VMathLab—A Game-Like Virtual Learning Environment

A. General Architecture

The entire *VMathlab* is designed as a 3D gaming system, consisting of a universal "Entrance" and individual learning modules/games. The modules are categorized according to different mathematics topics (e.g. Trigonometry, College Algebra, Calculus I, etc). Each module is a relatively independent virtual world targeting on a specific math concept with two major portions, *tutorial* and *exercise*. A logical architecture of *VMathLab* is as shown in Table I.

TABLE I: LEARNING MODULES IN VMATHLAB.

		Fundamental Identities
		One Variable Equation
	Algebra	Quadratic Equation
		Parallel and Perpendicular World
		……
VMathLab		Angles and Measures
(Entrance)		Sine, Cosine, and Ferris Wheel
	Trigonometry	Sum and Difference of Identities
		Height of Water Tower
		……
		Secant and Tangent
		Limit
	Calculus I	Rate of Change and Derivative
		Min and Max
		……

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[1] "Second life" is a general 3D virtual world environment, not specially designed for education

After logging into the system, the game players, students, first see the "Entrance" which is a 3D abandoned virtual city. Accompanied by typical gaming background music, the mission is narrated and its content is slowly rolling up from the bottom, saying "*This beautiful city has been conquered by mathematics monsters. Your mission is to defeat the monsters by taking mathematics tutoring and solving mathematics questions to bring people back to the city. Do you want to be that hero?*" As shown in Fig. 1, the "empty" streets are dotted with "math posters" that serve as the pathway leading to the learning modules. A video clip is linked to each poster providing details of the associated math topic. The players can move mouse cursor on or away to start or stop movie playing. Standing aside of the poster is a teacher avatar who is designated to answer students' questions. If a poster is clicked, the player will enter the specific learning module scenarios.
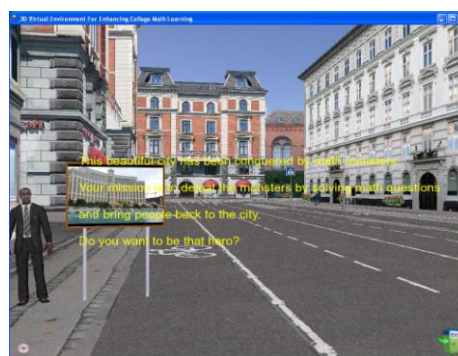

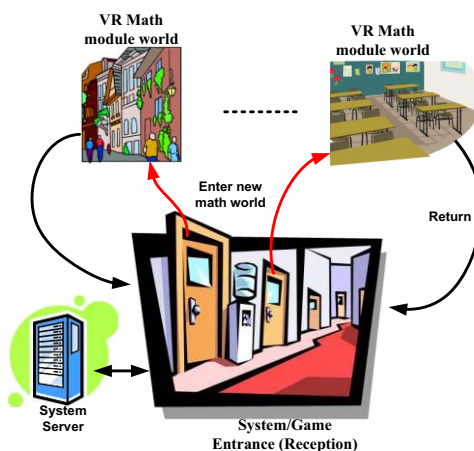Fig. 1. Snapshot of the "Entrance" module.

Fig. 2. *VMathLab* system structure

Players can navigate freely in the city and find posters linking to the math modules that they are required to visit. They can also teleport to a specific poster for quick access to the module without "roaming" all the way to it. With each learning module scenario, student can take tutoring and do exercises in a game-like environment. Upon finishing the work, the students can leave the module world and return to the "Entrance" by clicking a "Home" button at right-bottom corner of the screen. Based on the player's performance, he can rescue a certain number of people avatars and bring them back to the "ruined" city. The design is depicted in Fig. 2.

B. VMathLab Server

A *VMathLab* server is built to administer system configuration and store 3D scene files. The server allows

instructor to easily remove, hide, or add math modules without changing any source code or updating client program. The module configuration file, moduleConfig.txt, lists all modules embedded in the system. Each module has a log with the following fields directing the system how to work with it: (i) hide or visible; (ii) linked poster in the "Entrance"; (iii) main file name; (iv) the image file displayed on the poster; and (v) the movie clip file played on the poster. Administrator/instructors can re-configure the log to make the environment fit new teaching demands. If there are any module file changes, clients (student computes) will be automatically notified to download it and update the virtual environment to the latest version.

The server is also responsible for account management and works as the mediator to facilitate collaboration among students (e.g. user authentication, real time chatting, etc), as shown in Fig. 4. Through the server, instructors can monitor students' behaviors in real time, such as who is in the system, who is doing what activities, which learning module is visited, what math exercises/questions are attempted and what the students' answers are, etc. A student can be "kicked" out of the system if he violates the class and system regulation. Accordingly, students' performance will be recorded for grading and analysis purposes. Instructors can use these data to evaluate students' understanding, investigate new module functions, and therefore, **to** improve teaching effectiveness.
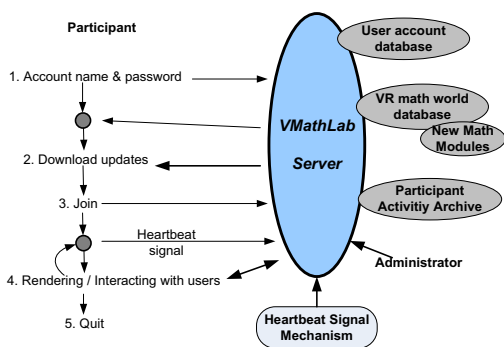

Fig. 3. Communication between server and clients.

The server has a "Heartbeat Mechanism" to synchronize the clients with the system server. If a client leaves the *VMathLab* system abnormally, the heartbeat mechanism can quickly detect and report **it** to the administrator.

C. Math Learning Modules

The *VMathLab* system is an on-going project. During the past two years, we have developed more than 40 learning modules, covering a wide variety of college math topics. As stated, all modules are game-like. Students can interact (play) with the 3D objects and manipulate the animation, which sets students free from the traditional classroom. The snapshots depicted in Fig. 5 provide a glimpse of the virtual learning environment.

Fig. 5(a) is a 3D virtual classroom with teacher and student avatars, giving students a comfortable learning environment to discuss, review and solve questions. Fig. 5(b) is the "Fundamental Identities" module for College Algebra. As the key for fast and precise calculation relies on memorizing frequently used identities, the module uses a duck shooting game to help students build firm memory on the common identities. For instance, when the player sees a duck carrying

the $a^3 + 3a^2b + 3ab^2 + b^3$ passing the stage, he/she needs to click the right "bullet" ─ identiy factor $(a+b)^3$ at the bottom of the screen to hit the duck and earn scores. Students can set the duck's walking speed to test how familiar they are with the fundamental identities.

Fig. 5(c) is a snapshot of the "Sine, Cosine, and Ferris Wheel", which is designed to explain trigonometry functions. In traditional classroom, students usually have troubles to imagine a case with periodical angle change and how trigonometry functions are represented. This module uses 3D game environment to bridge "Sine" and "Cosine" concepts to a real life phenomenon. For example, while the wheel rotates, the height of a tourist's seat with time reference varies following a sine/cosine curve. Students can manipulate the rotating speed and observe the period change of the function.

(a) Virtual classroom

(b) Fundamental identities

(c) Sine, Cosine, and Ferris Wheel

(d) Cannonball and projectile

Fig. 4. Snapshots of college mathematics modules.

Fig. 5(d) is a snapshot of the "Cannonball and projectile" module. Under the 3D virtual environment, students can instruct avatars, adjust the shooting angles, and initialize the cannonball speed. When the ball is flying, a projectile will depicts the ball's traveling track. The speed and angle of its velocity and the horizontal and vertical components will be displayed in real time. The maximum height reached and the maximum travel distance will be highlighted in the end.

These gaming components and interacting media enable students to comprehend math concepts easily and connect the related knowledge to our daily life.

IV. USE MATHEMATICS TO SOLVE ENGINEERING PROBLEMS

In mathematics, engineering problems are always word problems. Solving word problems is an essential and challenging task to STEM students. A virtual learning environment is ideal to foster students' interests and cultivate their engineering thinking in that it can exhibit an engineering problem in 3D space, provide multiple perspectives of objects, enhance students' understanding through interaction, and guide students in correct deduction. This section presents three learning cases in *VMathLab* for solving engineering questions with appropriate math knowledge.

Case Study 1: Explore the relationship between driving speed and acceleration. The related math concept to this problem includes "Linear Equations", "Factors", "Quadratic Equations", "Rate of Change", etc.

Scenario: Alice and Bob frequently drive on TX-290 highway that connects Houston and Prairie View.

Type 1 Questions (with the constant driving speed): At 10:00am, Alice leaves Houston and heads to Prairie View at 65 miles per hour. Five minutes later, Bob leaves Prairie View and drives to Houston at 70 miles per hour. If the distance between Houston and Prairie View is 35 miles, at what time will Alice and Bob meet on TX-290? How far away from Houston is their encountering place? If both persons drive from Houston to Prairie View, Alice's speed is 60 miles per hour, Bob's speed is 70 miles per hour, and Bob leaves three minutes after Alice leaves, at what time and at what place will Bob catch up Alice on TX-290?

Type 2 Questions (with variable driving): at 10:00 am, Alice leaves Houston and drives to Prairie View at 65 miles per hour. After five minutes, she changes her driving speed to 70 miles per hour and then maintains that speed; at 10:10 am, Bob leaves Prairie View and drives to Houston at 70 miles per hour. Four minutes later, he changes his driving speed to 60 miles per hour and maintains the speed. If the distance between Houston and Prairie View is 45 miles, at what time will Alice and Bob come across each other on TX- 290? How far away is their encountering position from Houston? If Bob's initial speed is 30 miles per hour and he constantly accelerates his speed by five miles/hour^2, when and where will they see each other?

Animation and Interaction: the module uses 3D animation to illustrate the questions and make the scene and situation virtually alive (Fig. 5). Valid parameters such as velocity, distance, and starting time will be randomly generated by computer. The driving time and distance are rendered on screen and updated in real time while the cars are moving. The animation can be paused at any time so that students can verify the results and see whether their understanding is correct. Audio is added to increase the game

Fig. 5. Explore the travel problem.

-like atmosphere. The module also assists students in variable setting and math equation deduction. Students can observe the 3D scene and animation from different viewpoint, including bird's-eye-view, driver's view, ground view, and so forth.

Case Study 2: Measure the height of water tower on Prairie View AandM University campus. Climbing to the top of the tower and using a long tape measure to find the height is risky and unpractical. Instead, we can use alternative approaches, either the characteristics of similar triangles or angle measurement with a protractor, to accomplish this task. The math topics related to this engineering problem include "Equation Set", "Trigonometry Functions", etc.

When entering this module, students will see the virtual scene consisting of a water tower, a park chair, some trees, two virtual students and a virtual teacher. First, the teacher presents the question to students. Then he explains the principles and engineering approaches used to accomplish this task (e.g. how to use a protractor to measure elevation angle). After that, the system animates the procedures of solving the problem in 3D space. Finally, students will be allowed to freely navigate in the 3D space and solve randomly popped out questions related to the concept.

In this case (Fig. 6), the math theory employed is the trigonometry functions on two triangles sharing the same height: in a sunny day, two students stand at different distances to the water towers. The standing spots and the tower are in the same line. If x denotes the distance from the girl to the tower base, l denotes the distance between the two students, and θ_1 and θ_2 denote the elevation angles from the two standing spots, we can find the tower height y by solving the two trigonometric function based equations:

$$\tan \theta_1 = \frac{y}{x}, \quad \tan \theta_2 = \frac{y}{l + x}$$
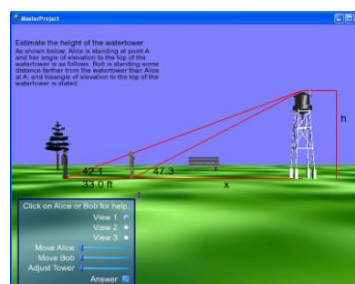

Fig. 6. Measure the height of a water tower.

Through interaction, students can adjust the parameters such as tower height and student spots to do exercises and consolidate their understanding. They can also switch to another scene which guides them to learn how to find the tower height by using the principles of similar triangle. Likewise, the module assists students in math deduction and supports multiple viewing angles.

Case Study 3: Find the cone volume. A conical paper cup can be made by cutting a piece of round paper on the radius and overlapping the paper by an angle (Fig. 7). The math topics related to this engineering problem include "Volume Calculation", "Critical Points", "First and Second Order of Derivative", "Maximum and Minimum", etc.

When entering this module, students first see the virtual scene of the paper, scissors, and a water bucket. The software will first demonstrate how the cone volume would change with different overlapping angles. This is only to give students basic feelings and no any calculation numbers are shown. Students are prompted to recall the how to calculate area, arc length, volume, derivative, and critical points. Then, a virtual teacher will explain the principles and give step-by-step instructions. For example, in this case, if R denotes the radius of the round paper and α denotes the overlapping angle, we can deduct the height and bottom radius of the cone as:

$$h = \sqrt{R^2 - r^2} \;\; , \; r = (2\pi R - \alpha R)/2\pi$$

and therefore, find the volume by:

$$v = \frac{1}{3}\pi r^2 h = \frac{R^3}{24\pi^2} \cdot (2\pi - \alpha)^2 \cdot \sqrt{4\pi\alpha - \alpha^2}$$

After that, following the concepts of critical points and derivatives, we can calculate $v'(\alpha)$, $v''(\alpha)$ and find the maximum volume of the cone. The system animates the problem solving procedure in 3D animation. A plot will be draw concurrently for comparison purpose. Finally, students will be asked to solve similar problems to strengthen understanding (e.g. find the carton volume and maximum, the carton is constructed using a piece of rectangle shaped paper).
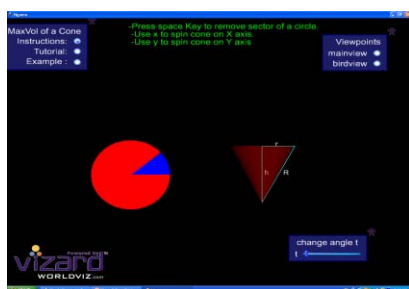

Fig. 7. Volume of the cone.

V. Conclusion

In this paper, we presented an innovative virtual reality math learning system─*VMathLab* developed by using 3D computer game technology. The system is designed to engage students, improve their learning effectiveness, and provide a new teaching methodology.

During the past two semesters, we experimentally tested *VMathLab* in College Algebra and Trigonometry courses Surveys show very positive feedback. More than 90% of students feel excited about this learning tool, and 80%

students believe the modules can enhance their math learning. The common questions students asked are like "*Why did not we use the modules earlier, like from the beginning of the semester?*" and "*Can we have more such mathematics learning modules?*" It truly encourages us to put more effort along this direction. Currently, we are refining the existing modules based on the students' feedback, and we plan to use it broadly in math teaching. We are also working on new learning modules to accommodate other math courses, such as Statistics and Calculus II.

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