Performance Evaluation of LDPC and Turbocoded OFDM System in Nakagami-M Fading

S. K. Singh, N. Sood, and A. K. Sharma

***Abstract*—In this paper, the performance analysis of OFDM based system using turbo and LDPC coding technique is presented under Nakagami-m fading channel. Nakagami-m is very recent and useful tool to evaluate the channel performance for OFDM system. The optimum value of parameter m = 1.4 is used in this paper. Here comparative analysis of turbo coding and LDPC coding schemes shows that error performance of LDPC is much better as compared to turbo codes. Both coding scheme revels the enhancement in BER performance in Nakagami-m channel.**

***Index Terms*—COFDM, nakagami-m fading, turbo codes, low density parity codes.**

I. Introduction

The population of the world is increasing rampantly due to which there is a lot of demand for high data rate multimedia communication. This has placed extreme strain on the bandwidth. In a wireless system the major sources of distortion are inter symbol interference (ISI) and multipath fading. Earlier in FDM equalizer are used to combat system impairment. These equalizers increase the complexity of the system. So the best approach is multicarrier system one of which is OFDM. In an OFDM system the stream data is divided into several low rate data stream known as subcarrier. The numbers of subchannel are as such so that the symbol duration is greater than the delay spread. To overcome ISI, guard period is inserted into OFDM symbol. Therefore OFDM system is compatible for mitigating ISI and multipath fading.

Rayleigh and Rician fast fading channels have been already studied in depth for OFDM system [1], [2]. Later on many researcher contributed to enhance the reported BER using different coding schemes [3-6]. Nakagami-m distribution is another useful and important model [7] to characterize the fading channel. It moreover contain Rayleigh (*m*=1) as well as AWGN (*m*=∞). Kang et al. [8] modeled the OFDM-BPSK system with frequency selective Nakagami-m fading channel. The work is further enhanced by Zheng et al. [9] by presenting asymptotic BER performance of OFDM system in frequency selective Nakagami-m fading channel. The threshold *m=1.4* was calculated. Turbo codes provide high gain over uncoded transmission in the presence of Nakagami-m fading [10].

Forward error correcting codes not only improve BER but also increase the spectral efficiency. Turbo codes make the system more robust to multipath fading and reduce guard period [3]. So our motivation behind this paper is to study the performance of turbo coded and LDPC coded OFDM system using flat fading Nakagami-m channel.

This paper is organized as follows: In Section II, OFDM system model is described. In Section III, Nakagami-m channel is explained. In Section IV, channel coding schemes are explained. In Section V, the analysis of simulated results of OFDM system is presented. Finally the paper is concluded in Section VI.

II. OFDM Model

A Complex base band OFDM signal with N subcarriers, is expressed as

$$s(t) = \sum_{k=0}^{N-1} D_i e^{j2\pi k f_0 t} \qquad 0 \le t \le T \tag{1}$$

For each OFDM symbol, the modulated data sequences are denoted by $D(0), D(1), \ldots\ldots. D(N-1)$. Here, f_0 denote the sub-carriers spacing and is set to $f_0 = 1/T$ the condition of orthogonality. After IFFT, the time-domain OFDM signal can be expressed as

$$S(n) = \frac{1}{N} \sum_{k=0}^{N-1} D_i e^{\frac{j2\pi k f_0 n}{N}} \tag{2}$$

After IFFT, the modulated signal is up-converted to carrier frequency f_C and then the following signal is produced and transmitted through channel:

$$x(t) = \mathrm{Re}\left\{ \sum_{k=0}^{N-1} D_i e^{j2\pi k (f_0 + f_C) t} \right\} \qquad 0 \le t \le T \tag{3}$$

$x(t)$ represents the final OFDM signal in which sub-carriers shall undergo a flat fading channel.

III. Nakagami-m Fading Channel

Nakagami-m fading distribution has gained a lot of attention in the modeling of physical fading radio channels [12]. Nakagami-m is more flexible and it can model fading condition from worst to moderate. The reason behind taking this distribution is its good fit to empirical fading data. Due to free parameter it provides more flexibility. Nakagami- m fading distribution function is given by [7]

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$$p_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{mr^2}{\Omega}\right), r \geq 0 \qquad (4)$$

where, $\Gamma(.)$ is the Gamma function, $\Omega = \overline{r^2}$ is the average power, m is fading parameter and r is Nakagami distribution envelope. Since, Nakagami distribution encompasses Scattered, reflected and direct components of the original transmitted signal, it can be generated using the envelope of the both random signal processes $r_{nlos}(t)$ for non line- of- sight envelope i.e. Rician and $r_{los}(t)$ for line-of-sight i.e. Rayleigh as per the following expression[13]

$$r(t) = |r_{nlos}(t)| \exp(1-m) + |r_{los}(t)|.(1-\exp(1-m)) \qquad (5)$$

So, this value of $r(t)$ is used as envelope of Nakagami-m channel.

In this paper, the sub-channel spacing $\left(f_0 = 1/T\right)$ is chosen so that the produced parallel fading sub-channels have flat fading characteristics. In flat fading environment, the base-band signal at the input of receiver $y(t)$ is as described as follows [11]:

$$y(t) = x(t) * r(t) + n(t) \qquad (6)$$

where, $x(t)$ denotes the base-band transmitted signal, $r(t)$ is the Nakagami-m channel envelope and $n(t)$ is the additive white Gaussian noise with zero mean.

IV. Forward Error Correcting Codes

A. Turbo Codes

By the invention of turbo codes in 1993 [14], a number of turbo coded system exploiting the powerful error correcting capability have been developed. Many factors affect the performance of turbo codes. Using **a** longer interleaver size can improve greatly the performance of the turbo-coded systems. In addition, the performance of turbo-coded systems gets improved **as** the constraint length increases. Parallel concatenated codes, as they are also known, can be implemented by using either block codes (PCBC) or convolutional codes (PCCC).

B. Turbo Encoder

The encoder for a turbo code is parallel concatenated convolutional code. As shown in Fig. 1. The binary input data sequence is represented by $u = (u1 \ldots\ldots uN)$. The input sequence is passed into the input of a recursive encoder, *RSC Encoder* 1 and a coded bit stream, **c1** is generated. The data sequence,**u** is then interleaved. The bits from the interlever, $u1$ are out in a pseudo-random manner.

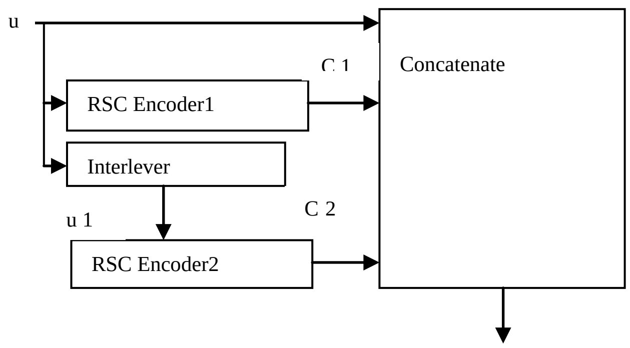

Fig. 1. Turbo encoder.

The interleaved date sequence is passed to a second recursive encoder *RSC Encoder 2*, and a second coded bit stream, $c2$ is generated. The code sequence, $c1$, $c2$ and input data u is then punctured and multiplexed. Which consisting of systematic code word. The interleaver takes each incoming block of N data bits and rearranges them into pseudo-random fashion in order to give patterns that have high weight. The puncturer periodically deletes the selected bits to reduce coding overhead. Deletion of parity bits is recommended.

C. Turbo Decoder

The standard decoding process is iterative as can be seen in Fig. 2. In Maximum a posteriori probability (MAP) technique, three different types of soft inputs are available for each decoder.

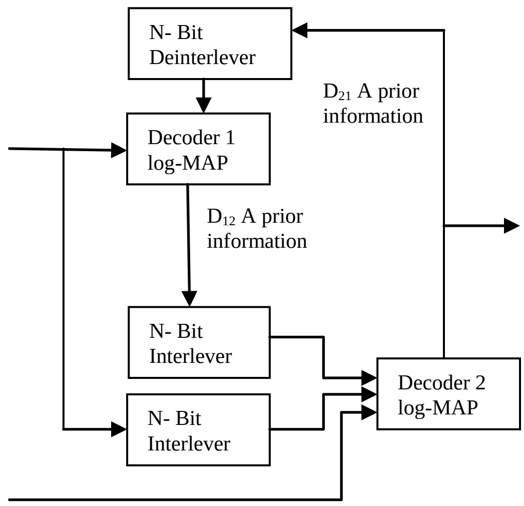

Fig. 2. Turbo decoder.

The un-coded information symbols, the redundant information resulting from first RSC code, A priori (extrinsic) information, which is the estimate of the information sequence obtained from the first decoding. In general, a symbol-by-symbol MAP algorithm is optimal for state estimation of a Markov process. MAP algorithms for turbo decoding calculate the logarithm of the ratio of APP of each information bit being one to the APP of the bit being zero. The decoder decides output=1 if probability, P (output=1|y| > P(output=0|y|) and it decides output =0 otherwise where y is the received codeword.

D. LDPC Code

LDPC codes were discovered in 1962 by Gallager [15] and rediscovered by Richardson and Urbanke[16]. The matrix representation of LDPC codes hold small number of "1" in each row and column, i.e. $W_C << n$ and $W_r << m$ for a dimension *mxn* parity matrix. The parity check matrix is shown in Fig. 3. This can provide large minimum distance of the code. However such a circumstance results a large parity check matrix. There are *m* check nodes (c-nodes; number of parity bits) and *n* variable nodes (v-nodes; number of bits in a codeword). LDPC codes are said to be regular if it is constant for every column, and $W_r = W_C\,(n/m)$. If the parity matrix *H* is low density but the number of "1" in each row or column are not constant, the code is said to be an irregular one. Here we consider irregular LDPC codes introduced in [17].

$$H = \begin{bmatrix} 1111000000 \\ 1000111000 \\ 0100100110 \\ 0010010101 \\ 0001001011 \end{bmatrix}$$

Fig. 3. Parity check matrix.

E. LDPC Encoder

The relation between code word matrix and parity check matrix is given by

$$CH^T = 0 \tag{7}$$

where *C* is a codeword matrix and *H* is a parity check matrix for *(mxn)* matrix.

$$P = mH_1^T + \left(H_2^T\right)^{-1} \tag{8}$$

The task of the encoder is then to compute the parity matrix *P* that can be directly appended to the message to produce the codeword.

F. LDPC Decoder

There are several methods used in decoding of the LDPC codes. Each method was derived individually. These are Believe Propagation (BP), Sum-Product (SP), and Message Passing (MP). The graph contains *m* check nodes and *n* variable notes. Check node   is connected to a variable node   if the element of *H* is a "1".

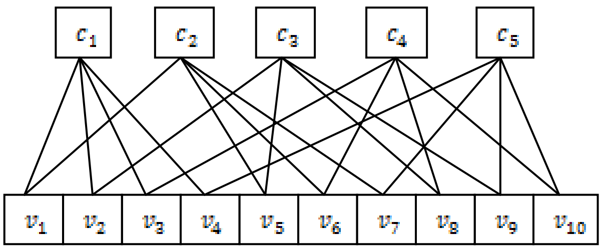

Fig. 4. Tanner graph of H matrix.

The message transfer between   and   in sum product algorithm. In each passage the log likelihood ratio (*LLR*) is recorded for is probability of its likely symbol. The decoder goes through 5 steps which are as follows:

- The Compute the initial value of $L(q_{ij})$ transmitted from the variable node *i* to check node *j*; for all *i*; $1 \le i \le n$.

$$L(q_{ij}) = L(v_i) = \frac{2y_i}{\sigma^2} = LLLr_i$$

Wher $L(v_i)$ denotes log likelihood ratio, σ^2 denotes derivation of white noise.

- Compute $L(r_{ij})$ transmitted from the check node *j* to variable node *i*; for all *i*; $1 \le i \le n$
- Modify $L(q_{ij})$ and used it as the data transmitted from the variable node *i* to check node *j*; for all *i*; $1 \le i \le n$.
- Compute the soft output.

$$L(Q_i) = L(v_i) + \sum_{j \in c_i} L(r_{ji})$$

- The soft output obtained in step 4 is then used in the hard decision as, $v_i = 1$ if $L(Q_i) < 0$, otherwise $v_i = 0$.

G. Coded OFDM Model

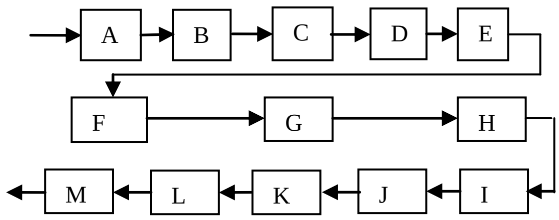

Fig. 5. Coded OFDM block model.

A = FEC encoder, *B* = Serial to Parallel convertor, *C* = BPSK Modulator, *D* = IFFT, *E* = Parallel to Serial convertor, *F* = Cyclic Prefix, *G* = Channel, *H* = Cyclic Prefix Removal, *I* = Serial to Parallel convertor, *J* = FFT, *K* = *BPSK* Demodulator, *L* = Parallel to Serial convertor, *M* = Decoder

V. Results and Discussions

In this section, BER performance of Turbo codes and LDPC codes using different fading distribution has been analyzed. OFDM-BPSK system simulated in MATLAB™ (R2009b) environment using total number of subcarrier is 400, IFFT/FFT length is 1024 and guard interval is 256. The figures demonstrate that the performance of uncoded OFDM improves with FEC, large improvement seen with increase number of iteration. The following analysis is done at constant BER of 0.005 so that coding schemes as well as fading distribution can be compared.

The result obtained under the influence of Nakagami-m distribution for turbo and LDPC codes are as follows.

A. Turbo Codes

Fig. 6 shows the BER performance of uncoded and turbo coded OFDM system as a function of SNR. We use BPSK modulation and   code rate turbo encoder. The decoding algorithm is log- MAP. The graphs are obtained at iteration number 1, 3 and 5. As shown in Fig. 6 the iteration 3 and 5 have almost the same performance. To achieve the target BER of 0.005, the reported SNR of uncoded OFDM system is 7.6dB with turbo coding scheme at fifth iteration same BER has been achieved at comparatively lower SNR of 2.4dB. So turbo coding scheme allows working at lower SNR.

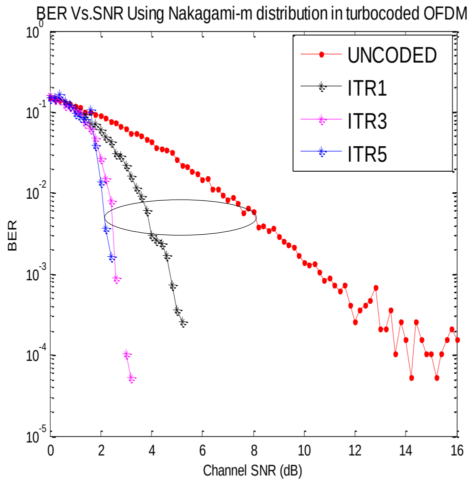

Fig. 6. BER vs SNR using nakagami-m distribution in Turbo coded OFDM.

B. LDPC Codes

In LDPC the 0.5 code rate is taken with hard decision decoding. The H is (324×64800). The BER for LDPC coding scheme has been presented in Fig. 7.

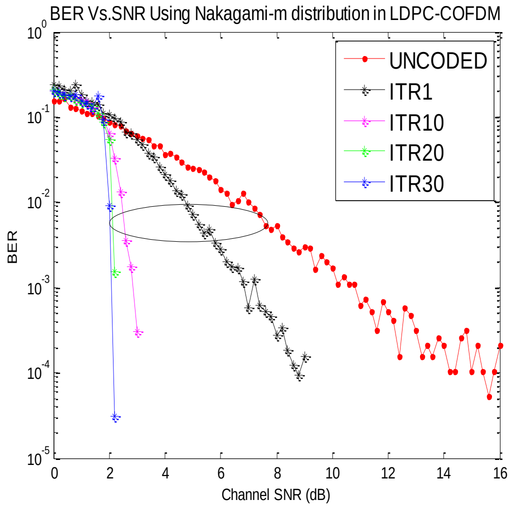

Fig. 7. BER vs SNR using nakagami-m distribution in LDPC coded OFDM.

Iteration for LDPC coding has been varied from 1 to 30. Significant result for the coded OFDM has been achieved using iteration 30. The reported SNR at target BER of 0.005 is 2.08dB. However, there is no significant improvement after 20 iteration.

VI. Conclusions

It can be concluded from the results that both coding schemes have enhance the error performance for lower SNR values. Both coding schemes perform better at SNR ≥ 1dB for both distributions. The performance is worst when the value of SNR is below 1dB. However the LDPC coded OFDM is superior to the turbo coded OFDM. The reason for that improvement for these channels could be explained by the decoding algorithm. In LDPC sum product algorithm exchange likelihoods among frequency and time direction. Both coding schemes show different BER behavior in Nakagami-m channel. However a lot of time is consumed in simulating turbo code due to complexity in decoding.

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