Intuitionistic Fuzzy Interval Information System

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***Abstract*—The notion of fuzzy subsets was introduced to model imprecision, ambiguity and uncertainty. Further it was generalized to intuitionistic fuzzy subsets and interval valued intuitionistic fuzzy subsets for their usefulness and applicability. In recent years, several ranking approaches based on dominance relations have been developed, in which a dominance degree and an entire dominance degree are employed in information system. Even though the theory of fuzzy sets paved a way to decision making from qualitative information, it can not be used to apply in problem with a qualitative information with lack of knowledge. So in this paper an intuitionistic fuzzy interval information system (IFIIS) is introduced and an approach of ranking from IFIIS based on the dominance degree is introduced and studied by illustrating an example.**

***Index Terms*—Interval valued intuitionistic fuzzy sets, information system, new novel accuracy score, dominance degree.**

I. Introduction

Zadeh introduced the concept of fuzzy sets, which has been a mathematical model to solve with imprecision, ambiguity and uncertainty [1]. Theory of fuzzy sets has also developed its own measures of qualitative information, which finds application in areas such as management, medicine and meteorology. Even though the theory of fuzzy sets paved a way to model qualitative information, it can not be used to apply in problem with a qualitative information with lack of knowledge. So it was generalized to intuitionistic fuzzy subsets by Atanassov and further generalized to interval valued intuitionistic fuzzy sets by Atanassov and Gargov [2] - [4].

In last decades, rough set theory introduced by Pawlak has an important role in the field of decision making analysis [5] - [6]. In decision making analysis, interval data is an important class of data, and generalized form of single-valued data. Qian et al. proposed a ranking approach for all objects based on dominance classes and the entire dominance degree [7].

In fuzzy information system, objects are evaluated by a set of values in the unit interval or linguistic terms [8] - [9]. Even though the theory of fuzzy sets paved a way to decision making from qualitative information, it can not be used to apply in problem with a qualitative information with lack of knowledge [8] - [9]. In information system, comparison of data plays a main role to define a dominance degree. A new novel accuracy score is defined to rank intuitionistic interval values which rank completely for comparable intuitionistic fuzzy intervals in [10]. So in this paper, an intuitionistic fuzzy interval information system is defined and a ranking algorithm based the dominance degree using this new novel accuracy score is given.

This paper is organized as follows. Section 2 briefly reviews the definition of interval valued intuitionistic fuzzy sets and interval information system. In Section 3, an intuitionistic fuzzy interval information system and fuzzy dominance degree are defined and some of its properties are studied. In section 4, an algorithm for ranking of objects in interval valued intuitionistic fuzzy information system is given and it is illustrated in an example. Finally the conclusions are drawn in section 5.

II. Preliminaries

Here we give a brief review of preliminaries.

Definition 2.1 [4]: Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. Let $X(\neq \Phi)$ be a given set. An interval valued intuitionistic fuzzy set in X is an expression given by $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$, where $\mu_A : X \to D[0,1], \gamma_A : X \to D[0,1]$ with the condition.

$$0 < sup_x \mu_A(x) + sup_x \gamma_A(x) \le 1$$

Conveniently, an interval valued intuitionistic fuzzy set in X is given by

$$A=\{\langle x,[\mu_{A_L}(x),\mu_{A_U}(x)],[\gamma_{A_L}(x),\gamma_{A_U}(x)]\rangle \mid x\in X\},$$

with $0<\mu_{A_U}(x)+\gamma_{A_U}(x)\le 1$, $\mu_{A_L}(x)\ge 0,\ \gamma_{A_L}(x)\ge 0.$

For each element x we can compute the unknown degree (hesitancy degree) of belongingness of $x\in X$ in A as follows $\Pi_A(x)=[1-\mu_{A_U}(x)-\gamma_{A_U}(x), 1-\mu_{A_L}(x)-\gamma_{A_L}(x)]$. We will denote the set of all the IVIFSs in X by IVIFS(X). An IVIFS value is denoted by $A=([a,b],[c,d])$ for convenince.

Definition 2.2 [4]: Let A,B $\in$ IVIFS(X). A subset relation is defined by $A\subset B \Leftrightarrow \mu_{A_L}(x)\le\mu_{B_L}(x)$, $\mu_{A_U}(x)\le\mu_{B_U}(x)$ and $\gamma_{A_L}(x)\ge\gamma_{B_L}(x)$, $\gamma_{A_U}(x)\ge\gamma_{B_U}(x)$, for every $x\in X$.

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Definition 2.3 [4]: The equality of two IVIFS is defined by $A = B \Leftrightarrow A \subset B$ and $B \subset A$.

Definition 2.4 [10]: Let $A = ([a,b],[c,d])$ be an interval valued intuitionistic fuzzy number, a new novel accuracy function L of an interval-valued intuitionistic fuzzy value, based on the unknown degree is defined by $L(A) = \dfrac{a+b-d(1-b)-c(1-a)}{2}$.

Definition 2.5 [10]: Let $A_1 = ([a_1,b_1],[c_1,d_1])$ and $A_2 = ([a_2,b_2],[c_2,d_2])$ be two IVIFSs. Then $A_1 \le A_2$ if $L(A_1) \le L(A_2)$.

Definition 2.6 [9]: An information system (IS) is a quadruple $S = (U, AT, V, f)$ where U is a finite non-empty set of objects and AT is a finite non-empty set of attributes, $V = \cup_{a \in AT} V_a$ where V_a is a domain of attribute a, and $f: U \times AT \to V$ is a total function such that $f(x,a) \in V_a$ for every $a \in AT, x \in U$, called an information function. An information system is called a fuzzy information system (FIS) if V_a is a set of values in the unit interval or linguistic terms.

III. Intuitionistic Fuzzy Interval Information System (IFIIS)

In this section intuitionistic fuzzy interval information system, fuzzy dominance relation, entire dominance degree are introduced and studied.

Definition 3.1: An information system is called intuitionistic fuzzy interval information system if V_a is a set of interval valued intuitionistic fuzzy numbers.

We denote $f(x,a) \in V_a$ by $f(x,a) = ([\mu_{a_L}(x), \mu_{a_U}(x)],[\nu_{a_L}(x), \nu_{a_U}(x)])$ where $\mu_{a_L}(x), \mu_{a_U}(x), \nu_{a_L}(x), \nu_{a_U}(x) \in [0, 1]$.

Example 3.1: An Interval valued intuitionistic fuzzy information system with $U = \{x_1, x_2, ... x_{10}\}$ and $AT = \{a_1, a_2, ... a_5\}$ is given in table I.

Definition 3.2: Let $a \in AT$ be a criterion. Let $x, y \in U$. Then $x >_a y$ means that x is better than (outranks) y with respect to the criterion a, ie. $L(x) > L(y)$ with respect to the criterion a. Also $x =_a y$ means that x is equally good as y ie. L(x) = L(y) with respect to the criterion a.

Definition 3.3: Let $S = (U, AT, V, f)$ be an IFIIS and $A \subseteq AT$. Let $B_A(x,y) = \{a \in A \mid x >_a y\}$ and let $C_A(x,y) = \{a \in A \mid x =_a y\}$. The fuzzy dominance relation $R_A(x,y): U \times U \to [0,1]$ between two objects x and y is defined by

$$R_A(x,y) = \frac{|B_A(x,y)| + |C_A(x,y)|/2}{|A|}.$$

TABLE I: An Intuitionistic Fuzzy Interval Information System.

	a_1	a_2	a_3	a_4	a_5
x_1	(0.2,0.4)	([0,0.2], [0,.2])	(0.4,0.4)	(0.2,0.4)	([0.2,0.4, [0,0.2])
x_2	([0,0.2], [0.2,0.4])	(0, [0.4,0.6])	([0.4,0.6, [0,0.2])	(0,0.6)	(0.2,0.8)
x_3	([0,0.2], [0.2,0.4])	(0, [0.4,0.6])	([0.2,0.4, [0.4,0.6])	(0.4, [0.2,0.4])	(0.2, [0.4,0.8])
x_4	(0, [0,0.6])	(0,1)	([0.2,0.4, [0.4,0.6])	(0.2,0.6)	(0.4,0.2)
x_5	(0.6,0.4)	([0.4,0.6, [0.2,0.4])	(0.8,0)	([0.2,0.4, [0,0.4])	([0.4,0.6, [0,0.2])
x_6	([0,0.2], [0.2,0.4])	([0.2,0.4, [0,0.4])	([0.2,0.6, [0,0.2])	([0.2,0.4, [0.4,0.6])	([0.4,0.6, [0,0.2])
x_7	(0.2,0.8)	(0.2,0.6)	(0.4,0.4)	(0.2.0.8)	(0.4,0.2)
x_8	([0.2,0.4, [0,0.6])	([0.4,0.6, [0,0.2])	([0.6,0.8, [0,0.2])	([0,0.4], [0,0.2])	(0.8,0.2)
x_9	([0.2,0.4, [.4,0.6])	(0.4,0.4)	([0.4,0.6, [0,0.2])	([0,0.4], [0.2,0.6])	(0.6,0.2)
x_{10}	(0.4, [0.2,0.6])	(0.4,0.6)	(1,0)	(0.4,0.2)	(1,0)

Property 3.1: $R_A(x,y)$ has the following properties

1) $0 \le R_A(x,y) \le 1$.
2) $R_A(x,x) = 1/2$.
3) $R_A(x,y) + R_A(y,x) = 1$.
4) If $x \ge_a y$ for all $a \in A$ then $R_A(x,z) \ge R_A(y,z)$.
5) If $x <_a y$ for all $a \in A$ then $R_A(z,x) < R_A(z,y)$.

Proof: Here we prove only (3). The rest of the properties are similar using definition.

$$3. R_A(x,y) + R_A(y,x) = \frac{|\{a \in A \mid x >_a y\}| + \dfrac{|\{a \in A \mid x =_a y\}|}{2} + |\{a \in A \mid y >_a x\}| + \dfrac{|\{a \in A \mid y =_a x\}|}{2}}{|A|}$$

$$= \frac{|\{a \in A \mid x >_a y \ or \ x =_a y \ or \ x <_a y\}|}{|A|} = 1$$

Definition 3.4: Let $S = (U, AT, V, f)$ be an IFIIS and $A \subseteq AT$. The entire dominance degree of each object is defined as $R_A(x_i) = \dfrac{1}{|U|}\sum_{j=1}^{|U|} R_A(x_i, x_j)$.

IV. Algorithm for Ranking Objects in IFIIS

Let $S = (U, AT, V, f)$ be an intuitionistic fuzzy interval information system.

A. Algorithm

1) Find the new novel fuzzy score $L(f(x,a))$ using definition 2.4, $\forall a \in A \ (A \subseteq AT)$ and $\forall x \in U$.

2) Enumerate $B_A(x,y)$ using $B_A(x,y)=\{a\in A \mid x >_a y\}$ and $C_A(x,y)$ using $C_A(x,y)=\{a\in A \mid x =_a y\}$.

3) Calculate the fuzzy dominance relation $R_A(x,y)$ using definition 3.3.

4) Calculate the entire Dominance degree $R_A(x_i)$ of each object as in definition 3.4.

5) The objects are ranked using entire dominance degree. The larger the value of $R_A(x_i)$, the better is the object.

B. *Numerical Illustration*

In this section, the algorithm is illustrated for the example 3.1. By step 1, the new novel fuzzy score $L(f(x,a))$ using definition 2.4, for all $a\in A\ (A\subseteq AT)$ and for all $x\in U$ is found.

Calculate the fuzzy dominance relation $R_A(x,y)$ and it is tabulated in table II. For example, $B_A(x_1, x_2) = \{a_1, a_2, a_4, a_5\}$ and $C_A(x_1,x_2) = \{\}$ and hence $R_A(x_1,x_2) = 4/5 = 0.8$.

TABLE II: FUZZY DOMINANCE RELATION BETWEEN TWO ALTERNATIVES $R_A(X, Y)$.

R_A	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	0.5	0.8	0.8	0.8	0	0.2	0.7	0	0	0
x_2	0.2	0.5	0.4	0.6	0	0.3	0.4	0	0.1	0
x_3	0.2	0.6	0.5	0.7	0.2	0.3	0.4	0.2	0.2	0
x_4	0.2	0.4	0.3	0.5	0	0	0.5	0	0	0
x_5	1	1	0.8	1	0.5	0.9	1	0.6	0.8	0.4
x_6	0.8	0.7	0.7	1	0.1	0.5	1	0	0.4	0.2
x_7	0.3	0.6	0.6	0.5	0	0	0.5	0	0	0
x_8	1	1	0.8	1	0.4	1	1	0.5	1	0.2
x_9	1	0.9	0.8	1	0.2	0.6	1	0	0.5	0.2
x_{10}	1	1	1	1	0.6	0.8	1	0.8	0.8	0.5

Now the entire dominance degree of each object $R_A(x_i)$ is found by definition 3.4. For example,

$$R_A(x_1)=\frac{1}{10}(R_A(x_1,x_1)+R_A(x_1,x_2)+\ldots R_A(x_1,x_{10}))=0.38$$

and $R_A(x_i)$ is given by

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
R_A	.38	.25	.33	.19	.8	.54	.25	.79	.62	.85

So by step 5, x_{10} is selected as the best object from the IFIIS.

V. CONCLUSION

Since the selection of best object from available objects based on criteria is widely useful in all fields of Engineering, Management Science and Medicine, in this paper an intuitionistic fuzzy interval information system (IFIIS) has been studied and an approach of ranking from IFIIS based on the dominance degree is introduced and studied by illustrating an example.

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