Synchronization of Chaotic Fractional – Order Systems

Mahboubeh Moghaddas, Esmaeil Sameni, Nemat Changizi, Toktam Lotfi Esmaeil Sameni, Nemat Changizi, and Toktam Lotfi

***Abstract*—Synchronization of chaotic and Chen system has been done using the active sliding mode control strategy. Regarding the synchronization task as a control problem, fractional order mathematics is used to express the system and active sliding mode for synchronization. It has been shown that, not only the performance of the proposed method is satisfying with an acceptable level of control signal, but also a rather simple stability analysis is performed. The latter is usually a complicated task for nonlinear chaotic systems.**

***Index Terms*—Order derivative .**

I. Introduction

Fractional Calculus (FC) is more than 300 years old topic. A number of applications where FC has been used rapidly grows, especially during last two decades. These mathematical phenomena allow to describe a real object more accurate that the classical integer methods. The real objects are generally fractional [1], however, for many of them the fractionality is very low. The main reason for using the integer-order models was the absence of solution methods for fractional dierential equations. Recently, the fractional order linear time invariant (FOLTI) systems have attracted lots of attention in control systems society (e.g. [2]) even though fractional-order control problems were investigated as early as 1960s [3], [6]. In the fractional order controller, the fractional order integration or derivative of the output error is used for the current control force calculation. To control and synchronization of chaotic fractional-order system an active sliding mode controller (ASMC) is proposed.

The differ integral operator, represented by $0^{D_t^\alpha}$, is a combined differentiation-integration operator commonly used in fractional calculus and general calculus operator, including fractional-order and integer is defined as:

$$0^{D_t^q} = \begin{cases} \dfrac{d^q}{dt} & q > 0 \\ 1 & q = 0 \\ \displaystyle\int_0^t (d\tau)^{-q} & q < 0 \end{cases} \tag{1}$$

There are several definitions of fractional derivatives [7]. The Caputo fractional derivative of order α of a continuous function $f : R^+ \to R$ is defined as follows

$$\frac{d^q f(t)}{dt^q} = \begin{cases} \dfrac{1}{\Gamma(m-q)}\displaystyle\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} & m-1 < q < m \\ \dfrac{d^m}{dt^m} f(t) & q = m \end{cases} \tag{2}$$

II. Designing the Fractional-Orrder ASMC

Let us, consider a chaotic fractional-order description of the system as follows

$$0^{D_t^q} X_1 = A_1 X_1 + g_1(X_1) \quad 0 < q < 1 \tag{3}$$

where $X_1(t) = (x_1, x_2, x_3)^T$ are real state vector, $A_1 \in R^{3\times3}$ denotes the linear part of the system dynamics and $g_1 : R^3 \to R^3$ is nonlinear part of the system. Eq.(3) denotes the master system. Now the controller $u(t) \in R^3$ is added the slave system.

Thus:

$$0^{D_t^q} X_2 = A_2 X_2 + g_2(X_2) + u(t) \quad 0 < q < 1 \tag{4}$$

That X_2, A_2 and g_2 implies the same roles as $X_1, A_1 \; and \; g_1$ for the master system.Synchronization of the systems means finding a control signal $u(t) \in R^3$ that makes state of the slave system to evolve as the states of the master system.

Now we define errors dynamics as follows

$$0^{D_t^q} X_2 - 0^{D_t^q} X_1 = A_2 X_2 + g_2(X_2) - A_1 X_1 - g_1(X_1) + u(t) \; (5)$$

The aim is to design the controller $u(t) \in R^3$ such that:

$$\lim_{t\to\infty} \| e(t) \| = 0 \tag{6}$$

Then use with the active control designprocedure[8] $u(t)$ Change as following:

$$u(t) = H(t) - G(X_1, X_2) \tag{7}$$

whereH(t) is:

$$H(t) = Kw(t) \tag{8}$$

wherek $\in R^3$a constant is gain vector and $w(t) \in R$ is the control input that satisfies:

$$W(t) = \begin{cases} w^+(t) & s(e) \ge 0 \\ w^-(t) & s(e) < 0 \end{cases} \tag{9}$$

where $s = s(e)$ is a switching surface that describes the desired dynamics.

Constructing a sliding surface which represents a desired system dynamics and the sliding surface described as follows

$$s(e) = Ce \tag{10}$$

Manuscript received March 15, 2012; revised May 10, 2012.

Mahboubeh Moghaddas, Esmaeil Sameni, Nemat Changizi, and Toktam Lotfi are with the mahboubeh moghaddas is with Young Researchers Club, Gonabad Branch, Islamic Azad University, Gonabad, Iran (e-mail: E.ESMAEIL.SAMENI@gmail.com).

Esmaeil Sameni, Nemat Changizi, and Toktam Lotfi are with Department of control engineering Islamic Azad University, Gonabad Branch, Iran.

and$C \in R^3$is a constant vector

$$ {}_0D_t^q S = -\rho \, \mathrm{sgn}\,(s) - rs \qquad (11)$$

That sgn(0) represents the sign function .Theρ , r are gains that the sliding conditions Eq. (11) is satisfied. From Esq. (10), (11) have:

$$ {}_0D_t^q S = C\,{}_0D_t^q e = C[Ae + kw(t)] \qquad (12)$$

From Esq. (11) And (12), find control effort can be defined as:

$$ w(t) = -(CK)^{-1}[C(rI + A)e + \rho \mathrm{sgn}(s)] (13)$$

III. Numerical Simulations

The Chen system was introduced by Chen and Ueta in 1999 [9]. In this section, we consider using (ASMC) technique to obtain synchronization. This controller guarantees the synchronization two fractional orders Chen systems with the following initial conditions

$$ (x_{10}, y_{10}, z_{10}) = (1,2,1)$$

and

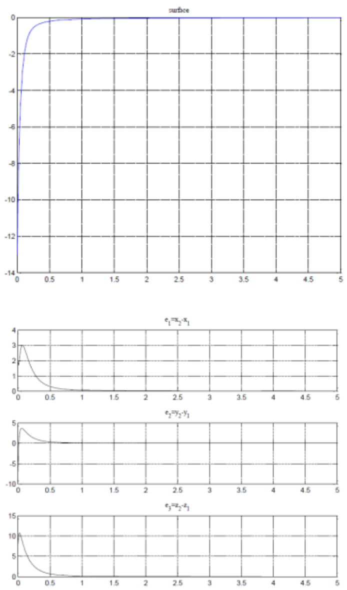
$$ (x_{20}, y_{20}, z_{20}) = (-3.8,5).$$

Consider two fractional order Chen systems as master andslave systems respectively:

$$\text{Master system} \begin{cases} {}_0D_t^q x_1 = 35(y_1 - x_1) \\ {}_0D_t^q y_1 = -7x_1 - x_1 z_1 + 28y_1. \\ {}_0D_t^q z_1 = x_1 y_1 - 3z_1 \end{cases} \qquad (15)$$

$$\text{Slave system} \begin{cases} {}_0D_t^q x_1 = 35(y_2 - x_2) \\ {}_0D_t^q y_2 = -7x_2 - x_2 z_2 + 28y_2 \, . \\ {}_0D_t^q z_2 = x_2 y_2 - 3z_2 \end{cases} (16)$$

Assume that order of the master is $q = 0.9$and order the slave is $q_=0.9$. Parameters of the controller are chosen as $k = [-3.3, -7, -5]^T$, $C = [1, -1, -1], r = 29$ and $\rho = 0.51$. Fig.1 shows the effectiveness of the proposedcontroller to synchronize two fractional-order modeledsystems.



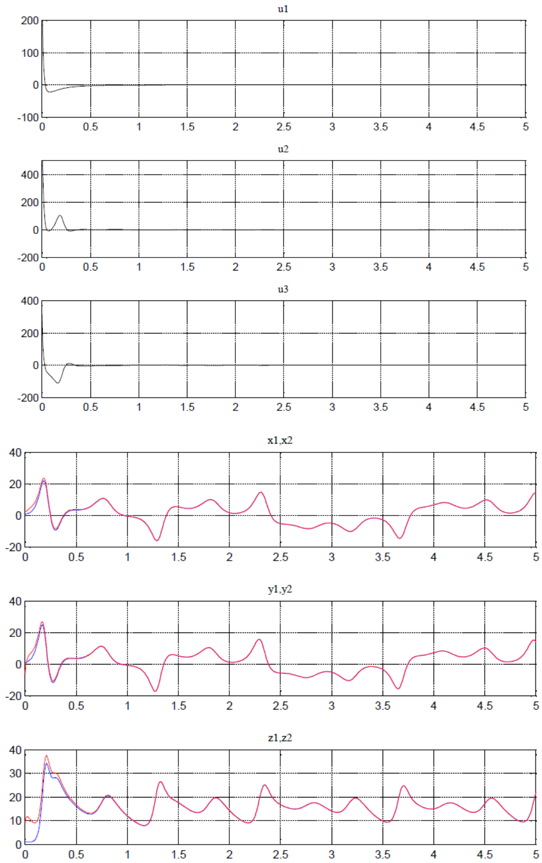


Fig. 1. Results of simulation.

IV. Conclusion

We construct a novel control law for fractional orders chaos system via active sliding mode control. Based on active control and sliding mode control methods an active sliding mode controller is derived to synchronize two Chen-Chen systems. The error system between two Chen-Chen chaotic systems is deduced using ASMC control theory.

References

[1] P. L. Duan Butzer, *Westphal U.An introduction to fractional calculus*, Singapore: World Scientific, 2000.

[2] E. Ahmed and A. S. Elgazzar, "On fractional order differential equations model for nonlocal epidemics," *Physica A*, vol. 379, pp. 607–614, 2007.

[3] W. M. Ahmad and R. El-Khazali, "Fractional-order dynamical models of Love," *Chaos Soliton Fract*, vol. 33, pp. 1367–1375, 2007.

[4] R. de Levie, "Fractals and rough electrodes," *J. Electroanal. Chem*. Vol. 281, pp. 1–21, 1990.

[5] S. Westerlund, "Dead matter has memory! (capacitor model)," *Phys. Scrip*, vol. 43, no. 2, pp. 174–179, 1991.

[6] T. Kaplan, L. J. Gray, and S. H. Liu, "Self-affine fractal model for a metal-electrolyte interface," *Phys. Rev. B* vol. 35, no. 10, pp. 5379–5381, 1987.

[7] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. S. Zhou, "The synchronization of chaotic systems," *Physics Reports*, vol. 366, pp. 1-101, 2002.

[8] I. Podlubny, *Fractional differential equations*, Academic Press, New York, 1999.

[9] G. Chen and T. Ueta, "Yet another attractor," *Int. J. Bifurcation Chaos* vol. 9, pp. 1465–1466, 1999.