Synchronization of Chaotic Fractional – Order Systems with Different Orders Using ASMC

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***Abstract*—In this paper the main objective is to investigate on chaotic behavior of fractional – order Chen. It has been shown that this problem could lead to synchronization of two master and slave systems with the different fractional-order. We propose a controller based on active sliding mode control (ASMC) theory to synchronize chaotic fractional-order systems in master-slave structure based on stability theorems in the fractional calculus, analysis of stability is performed for the proposed method. Finally, numerical simulations (synchronizing fractional – order Chen – Chen systems) are presented to show the effectiveness of the proposed controller.**

I. Fractional Calculus

Although many mathematicians have searched on the fractional calculus since many years ago, but is application in engineering, especially in modeling and control, does not have many antecedents. Fractional calculus is a more than 300 years old topic. Fractional calculus has been known since the early 17th century [1]. It has useful application in many fields of science like engineering, physics, mathematical biology [2], psychological and life sciences [3]. In physical chemistry, the current is proportional to the fractional derivative of the voltage when the fractional interface is put between a metal and an in ionic medium [4]. The electrode-electrotype interface is a sample of fractional-order processes because at metal-electrolyte interfaces the impedance is proportional to the non-integer order of frequency for small angular frequencies [5]. Synchronization in chaotic dynamic systems has attracted increasing attention of scientists from various research fields for its advantages in practical application [6]. Among the fractional order controllers, the fractional order active sliding mode control h (FOASMC) has been dealt more than others. In this paper, we introduce a fractional-order system chaotic*Chen*. To control and synchronization of chaotic fractional-order system an active sliding mode controller (ASMC) is proposed.

The differ integral operator, represented by $0^{D_t^\alpha}$, is a combined differentiation-integration operator commonly used in fractional calculus and general calculus operator, including fractional-order and integer is defined as:

$$0^{D_t^\alpha} = \begin{cases} \dfrac{d^\alpha}{dt} & \alpha > 0 \\ 1 & \alpha = 0 \\ \displaystyle\int_0^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \qquad (1)$$

There are several definitions of fractional derivatives [7]. The best-known one is the Riemann-*Liouvile* definition, which is given by

$$\frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma}\frac{d^n}{dt^n}\int_o^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (2)$$

where n is an integer such that$n-1<\alpha<n$,$\Gamma(0)$ is the Gamma function.

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt \qquad (3)$$

Thus, the fractional integral operator of order α can be represented by the transfer function $H(s) = \frac{1}{s^\alpha}$ in the frequency domain.

The standard definition of fractional-order calculus does not allow direct implementation of the fractional operators in time-domain simulations. An efficient method to circumvent this problem is to approximate fractional operators by using standard integer-order operators. In Ref.[8], an effective algorithm is developed to approximate fractional-order transfer functions, which has been adopted in [9] and has sufficient accuracy for time-domain implementations. In Table 1 of Ref [10], approximations for ${}^{1}/_{s^\alpha}$ with α from 0.1 to 0.9 in step 0.1 were given with errors of approximately 2 dB. We will use the ${}^{1}/_{s^{0.95}}$ approximation formula [9] in the following simulation examples.

$$\frac{1}{s^{.95}} \approx \frac{1.2831s^2 + 18.6004s + 2.0833}{1.2831s^3 + 18.4738s^2 + 2.6574s + 0.003} \qquad (4)$$

In the simulation of this paper, we use approximation method to solve the fractional-order differential equations.

II. Designing the Fractional-Orrder Active Sliding Mode Control and Analysis

Let us, consider a chaotic fractional-order description of the system as follows

$$0^{D_t^{\alpha_1}} X_1 = A_1 X_1 + g_1(X_1) \quad 0 < \alpha_1 < 1 \qquad (4)$$

where $X_1(t) = (x_1, x_2, x_3)^T$ are real state vector, $A_1 \in R^{3\times 3}$ denotes the linear part of the system dynamics and $g_1: R^3 \to R^3$ is nonlinear part of the system. Eq.(1) denotes the master system. Now the controller $u(t) \in R^3$ is

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added the slave system.
Thus:

$$ {}_0D_t^{\alpha_2} X_2 = A_2 X_2 + g_2(X_2) + u(t) \quad 0 < \alpha_2 < 1 \tag{5}$$

That X_2, A_2 and g_2 implies the same roles as X_1, A_1 *and* g_1 for the master system.Synchronization of the systems means finding a control signal $u(t) \in R^3$ that makes state of the slave system to evolve as the states of the master system.

Now we define errors dynamics as follows

$$ {}_0D_t^{\alpha_2} X_2 - {}_0D_t^{\alpha_1} X_1 = A_2 X_2 + g_2(X_2) - A_1 X_1 - g_1(X_1) + u(t) \tag{6}$$

The aim is to design the controller $u(t) \in R^3$ such that:

$$\lim_{t \to \infty} \| e(t) \| = 0 \tag{7}$$

Then use with the active control design procedure [11] $u(t)$ Change as following:

$$u(t) = H(t) - G(X_1, X_2) \tag{8}$$

where $H(t)$ is:

$$H(t) = Kw(t) \tag{9}$$

where $k \in R^3$a constant is gain vector and $w(t) \in R$ is the control input that satisfies:

$$W(t) = \begin{cases} w^+(t) & s(e) \geq 0 \\ w^-(t) & s(e) < 0 \end{cases} \tag{10}$$

where $s = s(e)$ is a switching surface that describes the desired dynamics.

Constructing a sliding surface which represents a desired system dynamics and the sliding surface described as follows

$$s(e) = Ce \tag{11}$$

and $C \in R^3$ is a constant vector

$$ {}_0D_t^{\alpha_2} S = -\rho \operatorname{sgn}(s) - rs \tag{12}$$

that sgn(0) represents the sign function .The ρ, r are gains that the sliding conditions Eq. (12) is satisfied. From Esq. (11), (12) have:

$$ {}_0D_t^{\alpha_2} S = C {}_0D_t^{\alpha_2} e = C[Ae + kw(t)] \tag{13}$$

From Esq. (12) And (13), find control effort can be defined as:

$$w(t) = -(CK)^{-1}[C(rI + A)e + \rho \operatorname{sgn}(s)] + {}_0D_t^{q_1} X_1 - {}_0D_t^{q_2} X_1 \tag{14}$$

III. Numerical Simulations

The Chen system was introduced by Chen and Ueta in 1999 [12]. In this section, we consider using (ASMC) technique to obtain synchronization. This controller guarantees the synchronization two fractional orders Chen systems with the following initial conditions

$$(x_{10}, y_{10}, z_{10}) = (1,1,1)$$

and

$$(x_{20}, y_{20}, z_{20}) = (3. -6, 9).$$

Consider two fractional order Chen systems as master and slave systems respectively:

$$\text{Master system} \begin{cases} {}_0D_t^{\alpha_1} x_1 = 35(y_1 - x_1) \\ {}_0D_t^{\alpha_1} y_1 = -7x_1 - x_1 z_1 + 28y_1. \\ {}_0D_t^{\alpha_1} z_1 = x_1 y_1 - 3z_1 \end{cases} \tag{15}$$

$$\text{Slave system} \begin{cases} {}_0D_t^{\alpha_2} x_1 = 35(y_2 - x_2) \\ {}_0D_t^{\alpha_2} y_2 = -7x_2 - x_2 z_2 + 28y_2 . \\ {}_0D_t^{\alpha_2} z_2 = x_2 y_2 - 3z_2 \end{cases} \tag{16}$$

Assume that order of the master is $\alpha_1 = 0.88$ and order the slave is $\alpha_2 = 0.9$. Parameters of the controller are chosen as $k = [-1.3, -7.8, -1]^T$, $C = [1,1,-1], r = 65$ and $\rho = 0.28$. Fig.1 shows the effectiveness of the proposed controller to synchronize two fractional-order modeled systems.

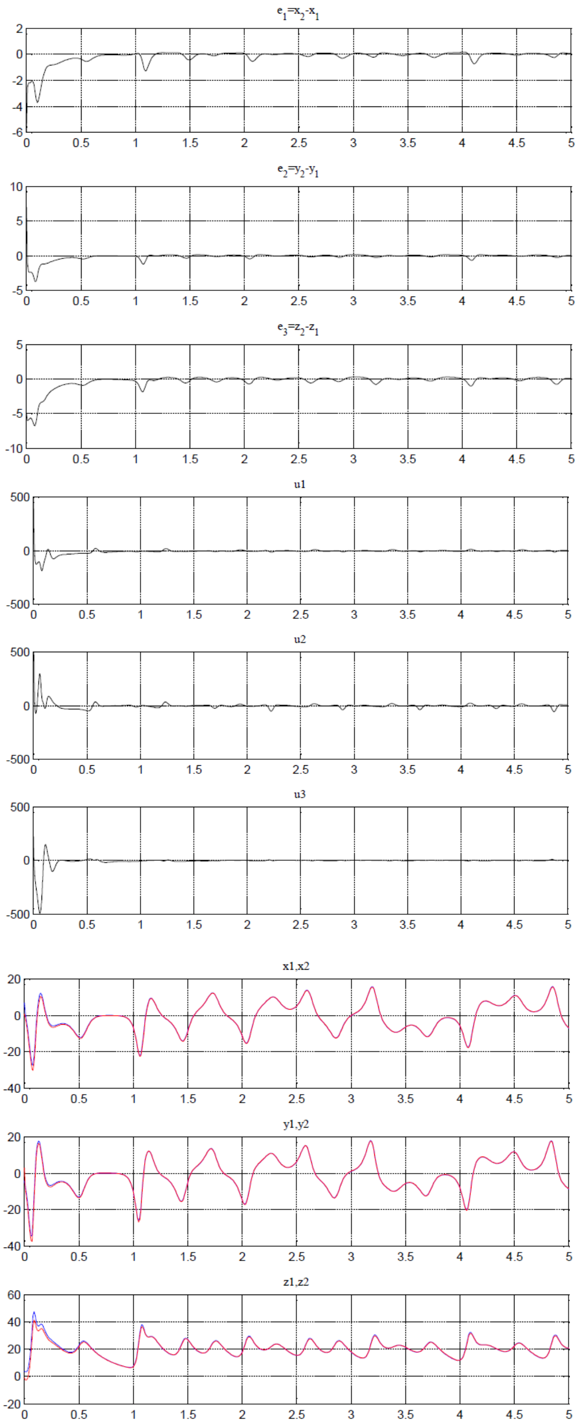

Fig. 1. Results of simulaton.

IV. Conclusion

This paper we have studied numerical methods in fractional calculus. Then, we have represented the active sliding mode control to synchronize. The control parameters $(r, k \text{ } and \text{ } c)$, the master and slave systems are synchronized. Numerical simulations show the efficiency of the proposed controller to synchronize chaotic fractional-order.

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