Image Denoising Based on Compressed Sensing

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***Abstract*—This paper presents a new approach for image denoising based on compressed sensing. In this method, an unknown noisy image of interest is observed (sensed) through a limited number linear functional in random projection, then original image is reconstructed using the observation vector and the existed recovery algorithms such as L1_minimization. Simulation results inform this method is an efficient method for image denoising.**

***Index Terms*—Noise reduction, image processing, image denoising, compressed sensing.**

I. Introduction

Image denoising is an open problem and has received considerable attention in the literature for several decades. Most of the conventional spatial filtering techniques as the mean filter and Gaussian filter have the disadvantage of blurring the edges when reducing noise. Although the median filter can preserve edges, the fine structures are suppressed and it tends to produce regions of constant or nearly constant intensity in homogeneous image regions. The adaptive minimum mean squared error (MMSE) filter outperforms the two kinds of filters mentioned above by analyzing the local image intensity statistics. However, there is no guarantee that such a denoised image with high PSNR has acceptable visual quality.

In recent years, wavelet transform (WT) based methods have been applied to the problem of noise reduction and have been shown to outperform the traditional Wiener filter, Median filter, and modified Lee filter in terms of root mean squared error (MSE), peak signal noise ratio (PSNR) and other evaluation methods.

Using transform based methods, first step we transform image data to frequency or time-frequency domain using DFT and Wavelet respectively, then keep only some large coefficients using the properly thresholding level and throw away rest. The small number of largest coefficients that has main information of image is kept and most noise coefficients that are small will be set to zero. When we reconstruct the image from these coefficients, the noise has been reduced. However, if the noise or image's parameters change, the thresholding level and our algorithms must be amended.

In addition to mentioned above image denoising methods, we use compressive sensing [1] as a new method for image denoising [2] which eliminate both transforming and thresholding steps [3]. Compressive sensing performs sampling and compression in one step then will perform reconstructing and denoising signal simultaneously. This paper has the following structure: In Section II we introduce the Compressive sensing and reconstructing methods; in Section III we relate Compressive sensing to the image denoising problem; in Section IV we look at some numerical results and finally conclude in Section V.

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II. Compressive Sensing

Shannon Nyquist theorem presents sampling rate need at least two times faster than the signal bandwidth for fidelity signal reconstructing. Compressive sensing theory basically presents that the sparse or compressible signals can be reconstructed from a surprisingly small number of linear measurements, which provides that the measurements satisfy an incoherence property. Algebraically, for reconstructing a sparse signal $x \in R^n$ which has a few non-zero coefficients, we need $m \ll n$ linear non-adaptive measurements which are observed from:

$$y = \phi \times X \tag{1}$$

where ϕ is a $R^{m\times n}$ matrix, which is called sensing matrix and y is a R^m vector, which is called observation vector. The sampling matrix is usually treated by random selection of the entries; among the well-known random matrices are i.i.d Gaussian [2] and Rademacher [10] matrices.

The two appealing algorithmic approaches for signal recovery which are basis pursuit (L1-minimization) [1] and greedy pursuit such as Orthogonal Matching Pursuit (OMP) and iterative hard thresholding (IHT) [4] [5], have been received much attention.

Using L1-minimization approach, Candes and Tao beautifully proved [6] the signal could reconstruct precisely by solving the linear program:

$$\text{(L1): } min\ \|v\|_1 \quad subject\ to \quad y = \phi v \tag{2}$$

Which prove the sensing matrix ϕ obeys a Restricted Isometric Condition which is defined as follow: Restricted Isometric Condition for sensing matrix definition: let ϕ be a measurement matrix. ϕ satisfies Restricted Isometric Condition (RIC) if there exists a constant number $\delta_s \in (0, 1)$ as:

$$(1-\delta_s)\|v\|_2 \ \le \|\ \phi v\|_2 \le (1+\delta_s\ )\|v\|_2 \tag{3}$$

with every T-sparse vector $v \in R^n$ and |T| ≤| s |. The RIP and incoherence can be achieved with high probability simply by selecting ϕ as a random matrix [2].

III. Compressed Image Denoising Method

For image denoising, we first transform the image corrupted with noise to sparse domain using:

$$\Phi = \Psi \times (x + z) \quad (4)$$

where z is the Additive noise. Then we sample from Φ by mixing matrix $M_{m\times n}$ where M is stable and incoherence with the matrix transform:

$$\Psi\, y = M \times \Phi \ = \ M \times \Psi \times (x + z) \quad (5)$$

and $M_{m\times n} \times \Psi_{\ n\times n}$ which would be called the compressed sensing matrix A. According to the observation vector $y = A \times x$, we need to reconstruct the original image from this observation. It is known that sparsity is a basic principle in fidelity reconstruction. Also it is known the noise is not sparse in common domain. Hence most of part will be removed by compressed sensing due to recovery a just M dimensional vector of noise which is reconstructed. Also we can reconstruct the exact signal due to sparsity. Stated principle is basic idea for compressed sensing image denoising (CSID) (Fig.1) and has steps.

CSID algorithm:

- Firstly, Do sparse transform for signal $X + Z$ formed by mixing signal X and noise Z, and obtain $\Phi = \Psi \times (X + Z)$.
- Secondly, Design a $M \times N$ dimensional observation matrix $\boldsymbol{M}$ which is stable and unrelated with the transform basis Ψ, then use $\boldsymbol{M}$ to measure Φ and acquire the observation vector $Y = M \times \Phi = M \times \Psi \times (X + Z)$.
- Finally, Restore signal X' by reconstructing Y (There are many reconstruction algorithm, such as orthogonal matching pursuit method, etc.) which complete the denoising of signal X.

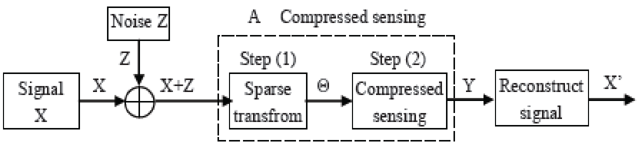

Fig. 1. Compressive sensing denoising steps

IV. Simulation and Results

Several noisy monochrome images which corrupted with Additive Gaussian White noise are sampled with Gaussian Random compressive sensing matrices, and reconstructed with the corresponding algorithms. Representative results obtained with three of these images appear in Fig. 2–5. The used procedure was as follows. First, each original image was sparsified by computing its wavelet transform (Haar) and then retained pre-determined fraction (e.g., 5%, 10%, or 15%) of its wavelet coefficients via keeping the largest and setting the rest to zero. Typically, images were distorted by these operations, especially when the number of coefficients retained was small. None of the tested images exhibited natural sparsity in this wavelet basis (or in any of a few other bases tried) below 5–10%. Fig. 2 depicts results for a 64×64 pixel synthetic image (the "Shepp-Logan phantom") commonly used as a surrogate for MRI brain images. Distortion of the original image due to the sparsifying transformation is not evident at 15% sparsity and rather severe at 10% sparsity. In Fig.2, we added AWGN noise with zero mean and variance to the 15% sparsified images. Then we measured with measurement matrix, finally we have reconstructed related image by IHT algorithm. We performed similar step for the 10% sparsified images in Fig.3.

Due to increasing the sparsity in the image, we can reconstruct the image using fewer measurements. Hence the complexity decreases. In Fig.3 we examine this algorithm for 10% sparsify image. As shown in Fig.3, we can reconstruct the image with 3200 samples.

In fig.4 we have compared IHT and OMP algorithms. Simulation results show that these algorithms have same performance but the run-time in IHT algorithm is 45 seconds and for OMP algorithm is 60 seconds, which inform IHT is faster than OMP.

Finally in Fig.5, we have compared some known classic filters and CISD algorithms. These filters and achieving PSNR have been shown in Table I. Table I results indicate the compressive sensing can remove then white nose from the image as same as the classical filter. But with noticeable difference which inform in the compressive sensing method we don't need to adapt the algorithm when the parameter of noise or signal have been changed.

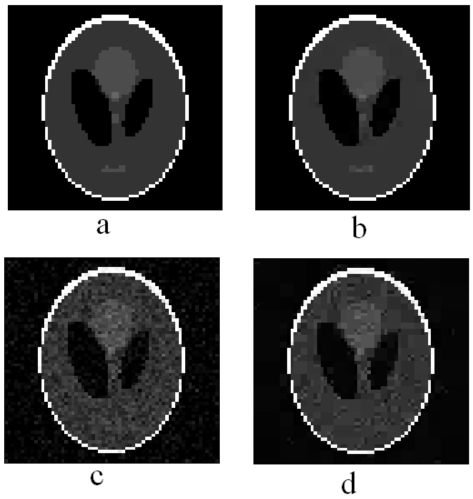

Fig. 2. a) original 64×64 phantom image,
b) %15 sparsify image
c) Noise-polluted image PSNR is 15dB
d) Denoised image using CSID algorithm, PSNR of denoised image is 24dB and number of measurements are 3800 samples.

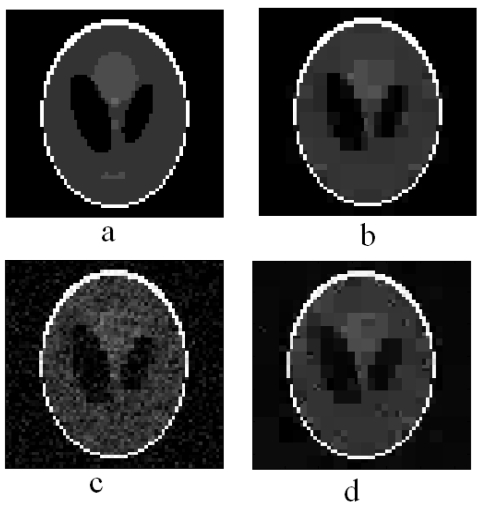

Fig. 3. a) Original 64×64 phantom image,
b) %10 sparsify image
c) Noise-polluted image PSNR is 15dB
d) Denoised image using CSID algorithm, PSNR of denoised image is 36dB and number of measurements are 3200 samples.

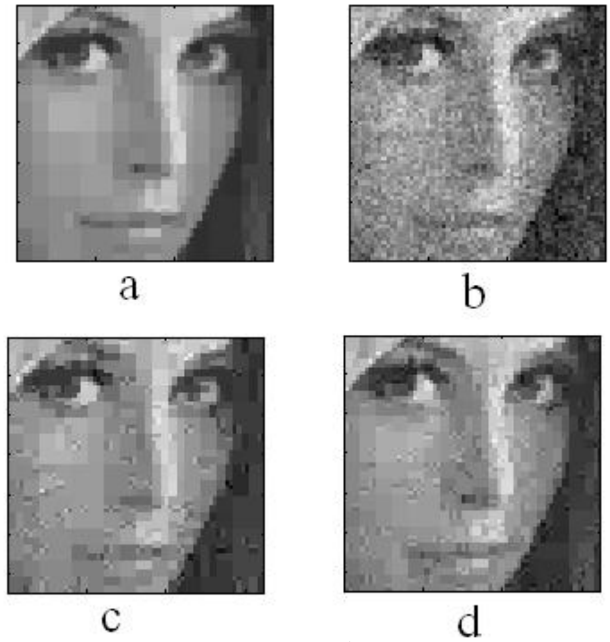

Fig. 4. a) %15 Sparsify image,
b) Noise-polluted image PSNR is 10dB
c) Denoised image, reconstructing algorithm is OMP PSNR is 29.4 dB
d) Denoised image, reconstructing algorithm is IHT PSNR is 28 dB.

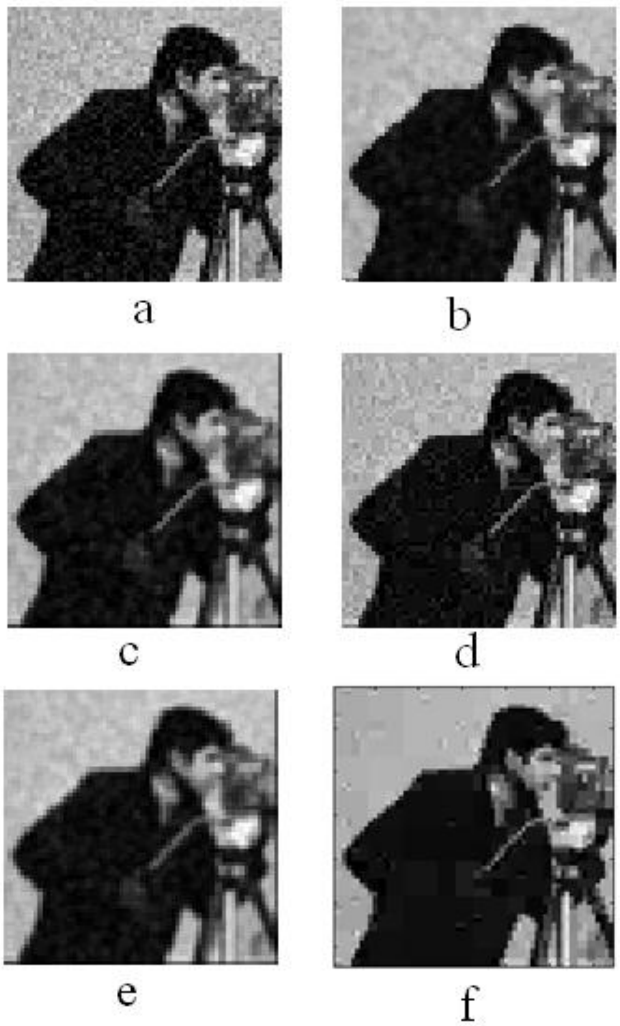

Fig. 5. a) 64×64 Noise-polluted camera man image PSNR is 15dB
b) Denoised image using Wiener filter
c) Denoised image using Median filter
d) Denoised image using Wavelet denoising
e) Denoised image using Gaussian filter
f) Denoised image using CSID method.

TABLE I: COMPARISON PSNR RESULTS OF SOME KNOWN CLASSIC FILTERS AND CISD

Image denoising Method	*PSNR (dB)*
Wiener filter	23.45 dB
Median filter	18.74 dB
Wavelet denoising	24.06 dB
Gaussian filter	19.11 dB
CSID method	23.78 dB

Then we were applied the compressive sensing algorithm for the MRI noisy image that has a PSNR equal to 10 dB. Per every step, we increased the number of measurements (or samples). As shown in Fig.6, according to increasing the number of measurements, the performance of denoising is growing. We show in Fig.7 the achieved PSNR with increasing the number of measurements. When the number of measurements is 1500, PSNR is equal to 10.3 dB for reconstructed MRI image and when the number of measurements is 3750, achieved PSNR is 29 dB.

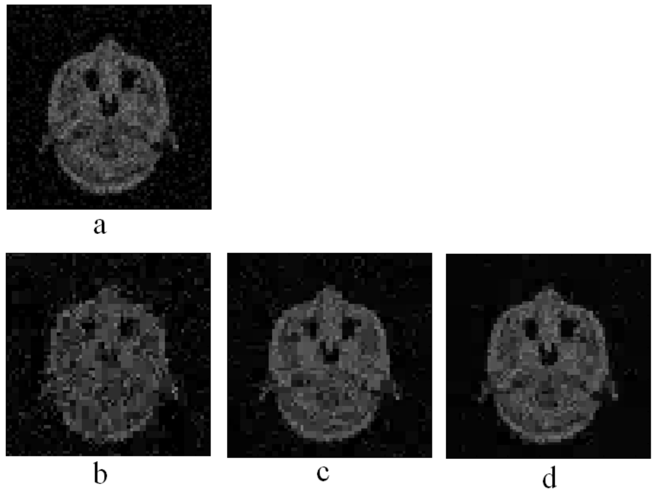

Fig. 6. Reconstructed image using OMP algorithm
a) Noise-polluted MRI image PSNR is 10dB,
b) Using 1800 samples PSNR is 14dB
c) Reconstructed image using 1800 samples PSNR is 22dB
d) Reconstructed image using 1800 samples PSNR is 28dB

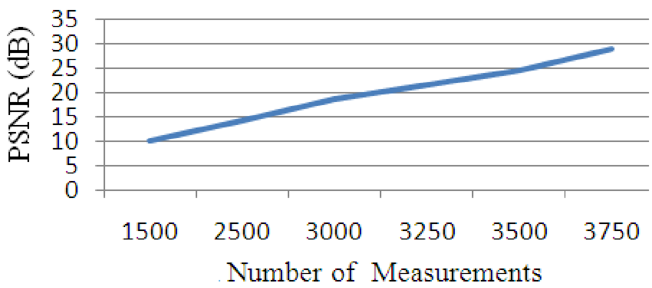

Fig. 7. Achieved PSNR with increasing the number of measurements

V. CONCLUSIONS

In this paper, we presented and simulated a new approach for image denoising based on compressed sensing. In this method, an unknown noisy image of interest is observed (sensed) through a limited number linear functional in random projection, then original image is reconstructed using the observation vector and the existed recovery algorithms such as L1_minimization. Simulation results indicate we can reduce additive Gaussian white noise from the image using compressive sensing. Sampling and compression accomplishing is one of steps of this method. Also, the reconstructing and denoising will are implementing in another step of this method. Using classical filter for image denoising, we need to redesign algorithm parameters owing to the change of signal parameters such as frequency, amplitude, etc. But in CISD algorithm, we don't need to change the algorithm parameters when the image or noise parameters have been changed. Simulation results show that the performance of compressive sensing denoising is the same as classic filter or in some occasion fairly better than those.

REFERENCES

[1] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 489–509, 2006.

[2] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[3] J. Tropp and A. Gilbert, "Signal recovery from partial information via orthogonal matching pursuit," *IEEE Trans. Inform. Theory*, vol. 53, no. 12, pp. 4655–4666, Dec. 2007.

[4] Candes, E. J.; Tao, T, "Decoding by linear programming," *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4203–4215, Dec. 2005.

[5] G. Davis, S. Mallat, and M. Avellaneda, "Greedy adaptive approximation," *J. Constr. Approx*. 13, pp. 57–98, 1997.

[6] A. C. Gilbert, M. Muthukrishnan, and M. J. Strauss, "Approximation of functions over redundant dictionaries using coherence," in *Proc. 14th Annu. ACM-SIAM Symp. Discrete Algorithms*, Baltimore, MD, Jan. 2003, pp. 243–252.

[7] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM J. Sci. Comput*, vol. 20, no. 1, pp. 33–61, 1999.

[8] T. Blumensath and M. Davies, "Iterative thresholding for sparse approximations," *Journal of Fourier Analysis and Applications* vol. *14* , no. 5, 2008, pp. 629–654.

[9] R. Baraniuk, M. Davenport, R. DeVore, and M. B. Wakin, "The johnson-lindenstrauss lemma meets compressed sensing," *http://www.dsp.rice.edu/cs/jlcs-v03.pdf*, May 2006.

[10] Lei Zhu, Yaolin Zhu, Huan Mao, and Meihua Gu, "A New Method for Sparse Signal Denoising Based on Compressed Sensing," *KAM 2009*, vol. 1, pp. 35 – 38.

[11] E. Candes and T. Tao, "Near-optimal signal recovery from random projections: universal encoding strategies," *IEEE Trans. Inform. Theory*, vol. 52, pp. 5406–5425, 2004.

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