Network Flow Modeling: An Approach to Allocating Contractors to Public Works

Arogundade O. Tale, Xiaoguang Yang, and S. Onimole

Abstract—**This paper attempt to make decision on the best way a division can allocate teams to contractors, both experience and inexperience. The contractors will consecutively be assigned to their district to handle some public Work. All of these need to be achieved at the cheapest possible way. With these in mind, graph theory was employed in designing a network flow to model the problem. The model was validated with some random cost of allocating contractors to public work. The model was solved using the Hungarian Method, an assignment problem solution algorithm. The solution was implemented with java programming language. With the solution we have another decision making tool.**

Index Terms—**Allocation, assignment, graph theory, minimum cost, network flow.**

I. Introduction

Network modeling is an aspect of Operations Research (OR). This subject matter is considered both as an art and a science [5]. The art lies in the ability to illustrate the concepts efficient and scares in a well-defined mathematical model of a given situation. The sciences consists the derivation of computational techniques for solving such models. OR can therefore be referred to as a scientific tool that can be employed for solving managerial problem involving decision making. When an Operational problem arises in a firm, an Operation Research Analyst is employed to work with Operations Team to solve such problem.

The importance of Models in OR cannot be over emphasized as this forms the bases for solution method. If the Problem is not well modeled, there will be unavoidable uncertainty in the solution technique, and the resulting problem is very much likely to be erroneous or impossible. The technique used most prominently for solving OR model is Linear Programming. This technique is design for models with strict linear objective and constraint functions [7]. Problems can also be model as network with Network Programming. This research work employees Network Flow Programming for modeling the problem and Assignment/Allocation algorithm for solving the Problem.

In network programming problem, the objective is mainly to move some entity from one point to another through a network efficiently, using the expensive transmission facilities effectively.

The OR models are design to optimize a specific objective criterion subject to a set of constraint, the quality of the resulting solution (or problem) depends on the accuracy of the model representing the real system. The result validates the model and if there are surprises in the result, the model is considered poor and better one has to be prepared. When a valid model is obtained, it is implemented as a tool introduced into the (an organization) system to solve (managerial) problem.

II. History of Network Flow

Network flow, is a problem domain that lies at the boundary between several fields of inquiry, including applied mathematics, computer science, engineering, management, and operations research. The field has a rich and long tradition, tracing its root back to the work of Gustav Kirchhoff and other early pioneer of electrical engineering and mechanics who first systematically analyzed electric circuits. Most of the early works were descriptive in nature, answering such questions as: If we apply a set of voltages to a given network, what will be the resulting current flow? Nowadays we will ask questions like: If we have an alternative way to use a network, which alternative will be most cost-effective? Our intellectual heritage for answering such questions is much more recent and can be traced to the late 1940s to early 1950s.

This period is just about the end of World War II. It was when the research and practitioner communities simultaneously developed optimization as an independent field of enquiry and launched the computer revolution, leading to the powerful instruments we know today for performing scientific and managerial computation [4].

In the past few years, researchers [8, 9] have made contribution to the design and analysis of network flow algorithms with improved worst-case performance guarantees at an explosive, almost dizzying pace. Moreover, these contributions were very surprising throughout the 1950s, 1960s and 1970s; network flow had evolved into rather mature field, so much so that most researchers and practitioner communities believed that further innovations would be hard to come by. As it turned out, nothing could be further from the truth.

The yearly problem of publicly owned corporation of allocating work to private contractors is addressed in [2]. The authors describe the formulation and solution of the problem as an optimization model (an integer programme0. He pointed out that the formulated model can be regarded as a minimum cost network flow model.

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III. Assignment Problem

This involves scheduling workers to job on a one-to-one basis but more generally, they involve permutation of a set of objects. The situation can be illustrated by assigning workers of varying degree of skill to jobs. The job that happens to match a Worker's skill cost less than that in which the worker is not as skilful. The number of operators is presumed equal to number of jobs. This condition can be guaranteed by creating dummy workers or jobs.

The data of the assignment problem consist of two equal size sets $N1$ and $N2$ (i.e. $|N1| = |N2|$), a collection of pairs $A \subseteq N1 \times N2$ representing possible assignments, and a cost c_{ij} associated with each element $(i, j) \in A$. In the assignment problem, the aim is to pair, at minimum possible cost, each object in N1 with exactly one object in $N2$. Examples of the assignment problem include assigning people to projects, jobs to machines, and tenants to apartments. The assignment problem is a minimum cost flow problem in a network G = ($N1 \in N2$, A) with $b(i) = 1\ \forall\ i \in N1$, $b(i) = -1,\ \forall\ i \in N2$, and $u_{ij} = 1\ \forall\ (i, j) \in A$.

IV. Minimum Cost Flow Problem

The minimum cost flow problems are the most fundamental of all network flow model. It is about finding the cheapest possible way of sending a certain amount of flow through a flow network. The problems are easy to state e.g. determine the least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes. Minimum cost flow problem can be perceive as a linear program, and therefore can be solved using linear programming methodologies. Below are the set of notation and some underlying definitions that are necessary for the task.

Given directional network graph $G = (N, A)$ with cost c_{ij} and capacity u_{ij} associated with every arc $(i, j) \in A$. Associated with each node $i \in$ N is a number $b(i)$ which indicates its supply or demand depending on whether $b(i) > 0$ or $b(i) < 0$. Hence, the minimum cost flow problem can be stated as follows:

$$Minimize Z(x) = \sum_{(i,j)\in A} c_{ij} x_{ij}$$

$$Subjectto:$$

$$\sum_{(j:(i,j)\in A)} x_{ij} - \sum_{(j:(j,i)\in A)} x_{ji} = b(i)\ foralli \in N$$

$$0 \le x_{ij} \le u_{ij}\ forall(i, j) \in A$$

Let C denote the largest magnitude of any arc cost. Also, let U denote the largest magnitude of any supply/demand or finite arc capacity. Furthermore, assuming that the lower bounds l_{ij} on arc flow are all zero hence $l_{ij} \le x_{ij} \le u_{ij}$, and the supply/demand at the nodes satisfy the condition: $\sum_{i\in N} b(i) = 0$ Than the minimum cost flow problem has a feasible solution, given that all arc as a nonnegative cost.

V. Definition of the Problem

This Paper considers the scenario below as a problem that needed to be solved as network flow problem:

Each division of n divisions of a company subcontracts work to private contractors. The work is of several different types and is done by teams, each of which is capable of doing all types of work. One of these divisions is divided into several districts: the j_{th} district requires r_j teams. The contractors are of two types: experience and inexperience. Each contractor i quotes a price c_{ij} to have a team conduct the work in district j. The objective is to allocate the work in the district to various contactors, satisfying the following conditions: (1) each district j has r_j assigned team; (2) the division contracts with contractor no more than u_i teams, the maximum number of teams it can supply; and (3) each district has at least one experienced contractor assigned to it. The problem is to be formulated as minimum cost flow problem for a division with three districts, and with two experienced and two inexperience contactors [1].

To allocate works to the three (3) districts in a division to various contactors, satisfying the following conditions:

- Each district j has r_j assigned team;
- The division contracts with contractor no more than u_i teams, the maximum number of teams it can supply; and
- Each district has at least one experienced contractor assigned to it.

This can be expressed mathematically as a linear programming problem:

Objective:

$$Minimize Z(r) = \sum_{ij\in A} C_{ij} r_{ij}$$

$$Subjectto: 0 < r_j \le u_i$$

$$CE \in D, 0 < CE$$

where CE is an experience contractor and D is a district.

Using a network flow to have contractor i with team u_{ij} conduct the work in district j at a price c_{ij} i.e as shown in Fig. 1.

Fig.1. The Basic network flow model for the problem in question

With the intent of optimizing cost for the assignment of the contractors to the districts using a minimum cost flow to implemented the model.

VI. Segments of Operation Research Work

An OR project cannot be conducted and controlled by the OR Analyst alone, although he may be an expert on the modeling and model solution technology. He cannot possibly be an expert in all the areas where OR problem arises. In this wise, an OR team should normally include: members of the organization directly responsible for the functions in which problem exist as well as for the execution and implementation of the recommended solution [6].

The major phases through which the OR team would proceed in order to effect an OR work is in 5 levels. These are discussed below:

- **Definition of the problem**: This involves defining the possibility of the problem under investigation. The result of the investigation is to identify three foremost elements of the decision problem namely: description of the decision alternative; determination of the objective of the work; and specification of the limitations under which the modeled system operates.
- **Model Construction**: Such model should specify quantitative expressions for the objective and the constraints of the problem in terms of its decision variable.
- **Solving the Model:** This entails the use of a well-defined optimization algorithm for the mathematical model.
- **Model validation**: The model is tested to see if it represents the system. One of the methods used for testing the validity of a model is to compare its performance with some historical data. Simulations are also used as an independent tool for verifying the output of the model.
- **Implementation of the final result**: This involves the translation of the result into detail operating instruction issued in an understandable form to the individuals that will administer and operate the recommended system.

A. Model Construction

To design a network flow for the problem using graph theory, each district node is Split into two nodes, one of which requires an experienced contractor.
The problem can be represented as a graph G.
where:

$$G = \left(NCE \cup NCI \cup NDE \cup NDI \cup ND \cup \{s\}, A \right)$$

Give that NCE contains one node for each experienced contractor, NCI contains one node for every inexperienced contractor, NDE, NDI and ND each contains a copy of the node *j* for every district *j*. Each node $j \in NDE$ has a demand of one unit and has an incoming arc (*i*, *j*) of cost c_{ij} and unit capacity from the node representing the experienced contractor *i*. A positive flow on the arc (*i*, *j*) denotes that contractor i has been assigned to district j. Each node $j \in NDI$ has a demand of zero unit, has an incoming arc (*i*, *j*) of unit capacity, and a cost of cij units from the node representing the inexperienced contractor i. The supply of the source node is $\sum_{j \in ND} r_j$

A minimum cost flow in the resulting network will give an optimal assignment of the contractors to the districts. (see Fig. 2.)

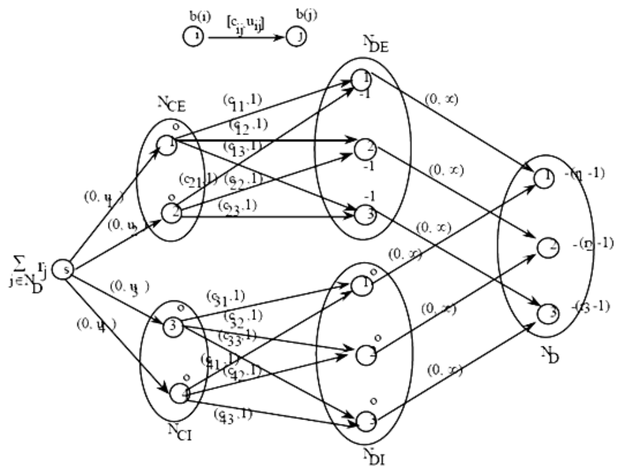

Fig. 2. A Full Network Flows Model for the problem

B. Model solution

The model is a network flow model, but the solution to the model can be computed using assignment problem methodology, which is to find the best Contractor for the Public Work. In the stated problem, one of the main objectives is the allocation of contractors to public work. If the above model is divided onto two, with the Experience Contractors and Inexperience Contractors separated. We have:

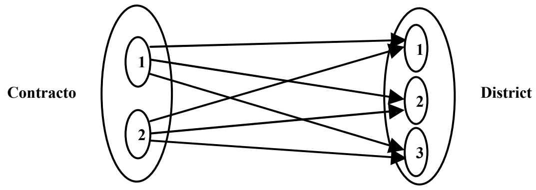

Fig.3. The network flow model represented as an Assignment Problem Model

With the assumption that the Division will not allocate Teams to both the Experience Contractors and Inexperience Contractors beyond its total capacity. Since each District must have at least one Experience Contractor allocated to it. It can also be assumed that, if each district is allocated two Contractors, one will be Experience and the other will either be Experience or Inexperience Contractor.

The general assignment model will work only with assigning Contractors i to Public Work j (where *i*, *j*= 1, 2, 3, , n) at cost c_{ij}. From the model in Fig. 3 it can be presumed that there is loss in generality because the number of contractors does not match the District. However, a fictitious contractor or District can be added to balance things out. In this case there is no restriction on the maximum number of contractors that can be allocated to each district, so a dummy contractor d is added to give a generalized assignment problem model in Fig. 4.

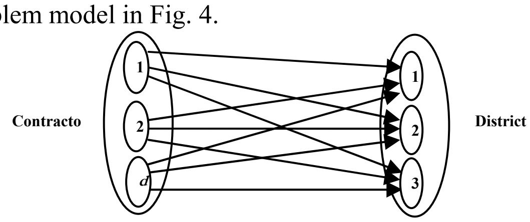

Fig.4. A generalized Assignment Problem model for the Network flow Model

With d in place for a dummy contractor, the assignment model can now be represented with the table below without loss of generality.

TABLE 1: A TABULAR REPRESENTATION OF THE ASSIGNMENT MODEL

Contractors \ Public Work	1	2	…	n
1	C_{11}	C_{12}	…	C_{1n}
2	C_{12}	C_{22}	…	C_{2n}
⋮	⋮	⋮	⋮	⋮
n	C_{n1}	C_{n2}	…	C_{nn}

C. Model Validation

The Modeled Assignment will be validated by Hungarian Method. For this the cost c_{ij} to have contractor i handle a public work j will be assumed. Then the following steps will be followed:

Step 1. From the cost matrix, identify each row's minimum, and subtract it from all the entries of the row.

Step 2. For the matrix resulting from step 1, identify each column's minimum, and subtract it from all the entries of the column.

Step 3. Identify the optimal solution as feasible assignment associated with the zero elements of the matrix obtained in step 2.

Using the assignment model, the cost can be randomly supplied to validate the mode. See example in table 2 below.

TABLE 2: AN EXAMPLE TO VALIDATE THE ASSIGNMENT MODEL

contactors \ Public Work	1	2	3	Row minimum
1	1 5	10	9	9
2	9	11	10	9
d	10	12	8	8

If step 1. & step 2 is applied to the above table; it will result in table 3 below.

TABLE 3: SOLUTION TO THE ASSIGNMENT PROBLEM

contactors \ Public Work	1	2	3
1	6	0	0
2	0	5	1
d	2	3	0

The cell with underlined zeros entries provides the optimum solution. The result gave an optimal assignment of the contractors to the public work at the district.

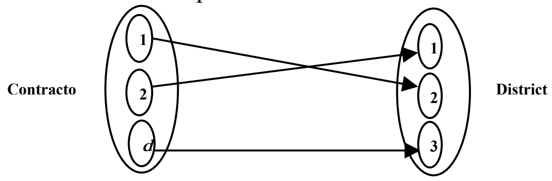

Fig.5. The Optimal Assignment of Contractor to the Public works

Hence, the objective function is satisfied:

$$MinimizeZ(r) = \sum_{ij\in A} c_{ij} r_{ij} = 9+10+8$$

= 27 (minimum cost for the allocation problem)

D. Implementation

Java Programming language was employed for the implementation of the solution to the model of the Problem. The codes for the implementation of this project and its applet interface are listed are explained below.

There are three main structures, namely Driver.java, dr.html and Solver2.java. The class Driver is an Applet program. Dr.html is an applet viewer by which Driver.java can be put on view. Driver.java calls Solver2.java as a function to solve major iterative matrix problem in the program [3]. The program gives a final network flow diagram for allocating the Contractors to the Public Work. With cost table used, an optimized allocation of Contractors to the Public Work is obtained.

VII. CONCLUSIONS AND FUTURE WORK

The problem in this project and most of the problems Operations Research tackles are cluttered and complex, often entailing considerable uncertainty. Operations Research can use advanced quantitative methods, modeling, problem structuring, simulation and other analytical techniques to examine assumptions, facilitate an in -depth understanding and decide on practical action.

The stand alone program written for the implementation of the solution in this paper requires an IDE or command prompt for its execution. Knowing that Operations Research also has its connection with computer science, and that operations researchers use computer programs (software) for implementation, the programme can be modified and packaged into an application (software) which, can be executed without IDE or command prompt.

The management of an organization may seek a very wide range of operational improvements - for example, greater efficiency, better customer service, higher quality or lower cost. Whatever the business engineering aim, Operations Research should be able to offer the flexibility and adaptability to provide objective help. Since the scope of the project is limited to allocation problem, more work can and should be carried out to make this work adaptable to more Operations Research problems.

REFERENCES

[1] Revinda K. Ahuja, Thomas L. Magnanti, James B. Orlin. Network flows: theory, algorithm and applications. Prentice Hall, Upper Saddle River, New Jersey, 1993.

[2] Cheshire, M., K. I. M. McKinnon, and H. P. Williams. 1984. The efficient allocation of private contractors to public works. Journal of the operational research quarterly 35, 705-709

[3] Microsoft® Encarta® 2008. © 1993-2007 Microsoft Corporation. All rights reserved.

[4] M.V. Kirby, Operational Research in War and Peace: The British experience from the 1930 to 1970,ImperialCollegePress,2003. http://en.wikipedia.org/wiki/Operations_research

[5] M.S. Sochi, "What about the 'O' in O.R.?" OR/MS Today, December, 2007, p. 12,

[6] Frederick S. Hillier and Gerald J. Lieberman, Introduction to Operations Research, McGraw-Hill: Boston MA. Eighth edition. International edition, 2005.
[7] Handy A. Tasha, Operations Research: An Introduction, Prentice Hall. Eighth edition, 2006.
[8] Wayne Winston, Operations Research: Applications and Algorithms. Duxbury Press; 4th. Edition, 2003.
[9] Robert J. Vanderbilt, Linear Programming; Foundations and Extensions, 3rd. edition, Intl' Series in Operations Research and Management Science, Vol. 114, Springer Vera, 2008.

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