Prim Algorithm Approach to Improving Local Access Network in Rural Areas

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Abstract—**This paper presents an application of Prim's Algorithm to local access network in rural areas. The minimum spanning tree problem has important applications in network design which has been extensively studied in literature. The minimum spanning tree problem on a graph with edge costs and vertex profits asks for a sub tree maximizing the difference between the total cost of all edges in the sub tree and the total profits of all vertices contained in the sub tree. Minimum spanning tree problem appears in the design of utility networks (e.g. bus services, electrifications) where villages and the network connecting them have to be chosen in the most profitable way. The application of Prim's algorithms is demonstrated to the design of local access networks in a particular local government area of about 88 villages. The case study is analyzed and interesting results are obtained. The results obtained justify the need to apply this kind of algorithm for efficiency and profitability**

Index Terms—**Graph, Minimum spanning tree, optimization, Minimum cost, path.**

I. Introduction

The Minimum Spanning Tree Problem, MST problem for short, is the problem of finding a minimum spanning tree with respect to a connected graph. The literature contains several algorithmic solutions like Prim's algorithm [11] or Kruskal's algorithm [10]. Both algorithms implement the greedy method. Typically, we assume that a cost function is related to every edge and the problem is to infer a globally minimum spanning tree, which covers the initial graph. The cost function returns integer values.

The problem of finding a minimum spanning tree of a graph is an important building block for many graph algorithms, and has been extensively studied by many scientists and mathematicians. The problem has applications in the design of distributed computer and communication networks, wiring connections, transportation networks among cities, and designing pipe capacities in flow networks [8].

The Minimum Spanning Tree (MST) problem is another model of a spanning tree. A spanning tree T of G is a connected acyclic sub graph that spans all the nodes. Every spanning tree of G has n-1 arcs (where n is a number of nodes), which is a characteristic property of a tree. Also, a tree has at least two leaf nodes (i.e. nodes with degree 1), and every two nodes of a tree are connected by a unique path [4].

Given an undirected graph G = (N, A) with n = |N| nodes and m = |A| arcs and with a length or cost C_{ij} associated with each arc (i, j) ϵ A, we wish to find a model of spanning tree, called a minimum spanning tree, that has the smallest total cost (or length) of its constituent arcs, measured as the sum of costs of the arcs in the spanning tree.

The minimum spanning tree problem can be formulated as a generalization of Euclidean minimum spanning tree of a set of points in the plane (or more generally in R^n), where weight of the edge between each pair of points is the distance between those two points.

When the cities are points in the Euclidean plane, the minimum spanning tree provides a good heuristic for traveling salesman problems. The optimum traveling salesman tour is at most twice the length of the minimum spanning tree [5].

Minimum spanning trees prove important for several reasons:

1) They can be computed quickly and easily, and they create a sparse sub graph that reflects a lot about the Original graph.
2) They provide a way to identify clusters in sets of points. Deleting the long edges from a minimum Spanning tree leaves connected components that define natural clusters in the data set.
3) They can be used to give approximate solutions to hard problems such as Steiner tree and traveling Salesman.
4) As an educational tool, minimum spanning tree algorithms provide graphic evidence that greedy

Algorithms can give provably optimal solutions [3].

Two classical algorithms efficiently construct minimum spanning trees, namely Prim's and Kruskal's.

Recent advances in technology have made network design for connecting villages economically feasible. In a typical planning scenario, the input is a set of villages together with the distances between them. Cost of network is that of fuel used in traveling round the villages. Essentially the decision process faced by a profit oriented company e.g. transport companies consists of two parts: on one hand, a subset of particular villages has to be designed to connect all selected villages in cost efficient way to the existing network. The natural trade-off between maximizing the sum of profits over all selected villages from passengers and minimizing the cost of the network leads to a MST objective function. We can formulate this problem mathematically as follows.

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II. PROBLEM DEFINED

Consider a logistic bus service problem having the following features.

- A unique vehicle has to serve a set of villages exactly once during a trip.
- The driver wants to start from a particular village and ends in a specific destination.
- A minimum cost path must be determined.

The model can be formulated as follows:

Let G = (V, E, C, P) be an undirected graph with vertex and edge-weights as defined in the introduction. The minimum spanning tree problem consists of finding a connected acyclic sub graph $T = (V_T, E_T)$ of G, $V_T \subseteq V$, $E_T \subseteq E$ that maximizes the objective function.

$$C(T) = \sum_{v \in V_T} P(v) - \sum_{e \in E_T} C(e)$$

The primary focus of this paper is a construction of an efficient algorithmic implementation rather than a theoretical analysis of the problem. In the next section, we give a brief description of the algorithm. In section four, five and six we present the computational experience and finally conclusions.

III. PRIM'S ALGORITHM

Prim's algorithm is a greedy algorithm; it starts by selecting an arbitrary vertex as the root of the tree. It then grows the tree by adding a vertex that is closest (has the shortest edge to) the current tree, and adding the shortest edge from any vertex already in the tree to the new vertex. The algorithm terminates once all vertices have been added to the tree. The sum of all added edges is the cost of the MST [9].

The algorithm does so by identifying an arc (i, j) of minimum cost in the cut [S, Ŝ]. It adds arc (i, j) to the tree, node j to S, and repeats this basic step until S = N. The correctness of the algorithm follows directly from the fact that each arc that is added to the tree is contained in some minimum spanning tree with the arcs that have been selected in the previous steps [6]. Figure 1 illustrates the pseodecode of Prim's algorithm. The algorithm can start at an arbitrary root vertex of **r** and grows until the tree spans all the vertices in **v**.

```
MST-PRIM (G, w, r)
    1        Q←V[G]
    2        For each u ε Q
    3            do key[u] ← ∞
    4            key[r] ← 0
    5            ∏[r]←nil
    6            while Q ≠ Ø
    7                do  u ← EXTRACT_MIN(Q)
    8                for each v ε Adj[u]
    9                    do if v ε Q and w( u,v ) < key[v]
    10                        then ∏[v] ← u
    11                            key[v] ← w(u,v)
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Figure 1: Pseudocode of Prim's Algorithm

IV. THE CASE STUDY

As a case study a road map of a local government area was collected and digitalized into a network diagram which is a complete graph on **n** vertices. Each vertex corresponds to a village while the edges correspond to the roads connecting the villages. The edges weights are the distances between the villages. Since our input is points in the plane instead of a graph, GIS software was used to construct the complete distance graph which is an undirected graph.

The instance used Odeda Local Government map which is one of the Local Governments of Nigeria. The Local Government has 88 villages in which 96 roads connect them. The map is shown in figure 2. The distances would have been indicated on the road map but to avoid clumsiness. The Euclidean distances is weighted in meters to have a better approximation of the real distances taking into account the fact that the roads are not straight line. Table 1 shows sample of the distance in meter between two villages.

V. APPLICATION OF PRIM'S ALGORITHM TO THE CASE STUDY

TABLE 1: SAMPLE OF THE NAME OF VILLAGES AND THE DISTANCE CONNECTING THEM.

From	To	Distance in metre
Oguntoke	Oja-Ituko	2500
Oja-Ituko	Okendi-Ajana	4070
Okendi-Ajana	Oke-Agbusi	3640
Oke-Agbesi	Ijaye-Igbole	1030
Ijaye-Igbole	Oguntoke	9720
Oguntoke	Ajitadun	5700
Ajitadun	Ayoye	850
Ayoye	Adelakun	5820
Adelakun	Ariwo	4320
Ariwo	Omididun	3250
Omididun	Kajola	16110

The aims of the application of the algorithm to this case are highlighted as follows:

- It gives information on how best to provide better opportunities for people to gain access to the rural areas of the country.
- An improved network is provided within the rural areas.
- It gives information which serves as a guide for provision of social amenities like tap water, electricity, medical services (immunization programmes) etc in terms of the shortest route to take in order to make the services cost effective.
- It will help transport service providers on accessibility issues. Good transport is key to improving access to services. It underpins economic and social regeneration. However we know that over a million rural households do not have access to a car and almost half of these are in more remote

rural areas. In addition the provision of public transport in rural areas is patchy.

- It will help to analyze decision making methodologies for service delivering across urban and rural areas in order to develop good practice in policy formation and ensure the fair provision of services for rural people
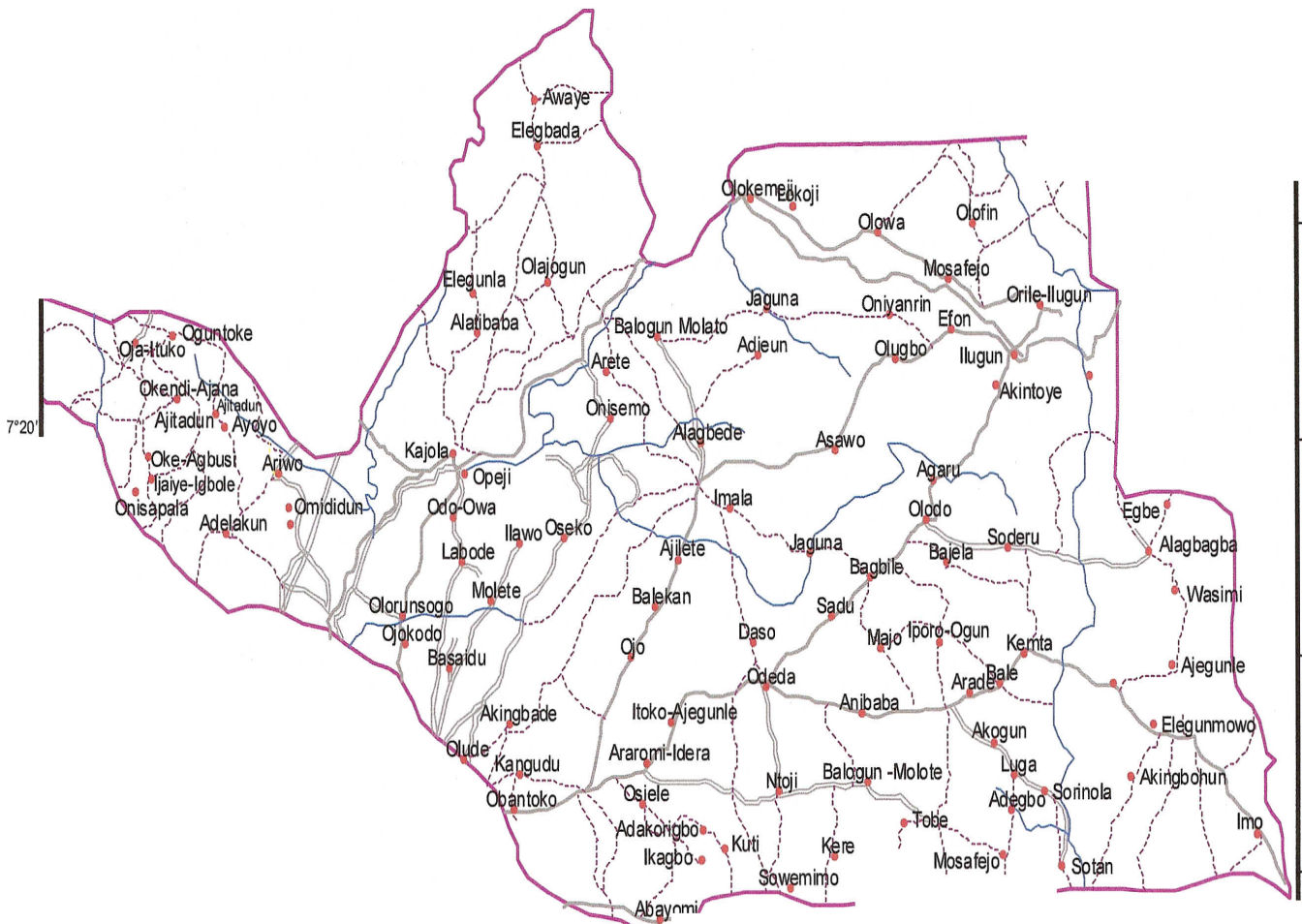

Figure 2: Map of one of Local Governments of Nigeria as a case study (Odeda)

Key

   -- Foot path.

   -- Main roads.

Oguntoke(1) OjaItuko(2) OkendiAjana(3) OkeAgbusi(4) IjayeIgbole(5) Ajitadun(6) Ayoye(7) Adelakun(8) Ariwo(9) Omididun(10) Kajola(11) Opeji(12) OdoOwa(13) Labode(14) Olorunsogo(18) Ojokodo(19) Molete(15)Ilawo(16) Basaidu(17) Alatibaba(24) Elegunnla(23) Elegbada(21) Awaye(22) Olajogun(20) Oluda(25) Akingbade(26) Oseko(27) Onisemo(28) Kangudu(29) Obantoko(30) Oja(74) Balekan(73) Ajilete(72) Alagbede(69) BalogunMolato(71) Jaguna(76) Adieun(75) Imala(68) Jaguna2(70) Daso(34)Odeda(33) ItokoAjegunle(32) AraromiIdera(31) Osiele(80) Adakongbo(82) Nitoji(52) BalogunMolote(53)Anibaba(59) Arade(43) Baale(42)Akangun(45) Luga(46) Sorinola(48) Adegbo(47) Sadu(35) Bagbile(36) Majo(37) IporoOgun(40) Kemta(44) Elegunmowo(85) Akingbohun(86) Soderu(41) Olodo(38) Agaru(67) Imo(87) Asawo(66) Olugbo(64) Efon(63) Oniyanrin(65) Olokemeji(58) Lokoji(57) Olowa(50) Mosafejo(49) OrileIlugun(60) Ilugun(61) Akitoye(62) Olodo(88) Bajela(39) Alagbagba(78) Egbe(79) Olofin(77) Tobe(55) kuti(83)Ikagbo(81) Abayomi(84) Sowemimo(54) Kere(56) Sotan(51)

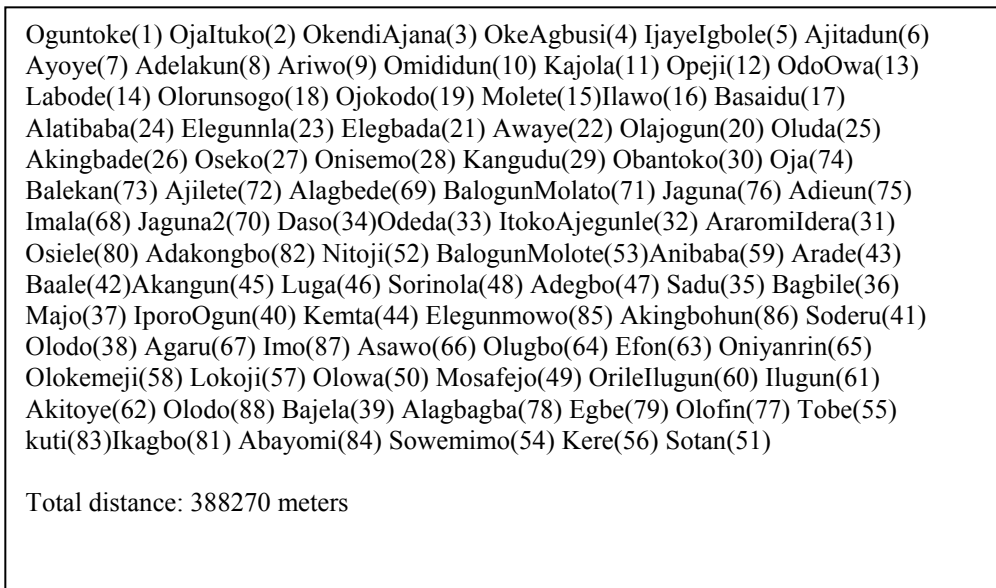Total distance: 388270 meters

Figure 3: Generation of Minimum Distance among the Villages.

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Path end-point to trace from:  Nitoji
Nitoji <-- Odeda <-- ItokoAjegunle <-- AraromiIdera <-- Obantoko
Total distance: 21960
Path end-point to trace from:  Olajogun
Olajogun <-- Kajola <-- Omididun <-- Ariwo <-- Adelakun <-- Ayoye <-- Ajitadun <--
Oguntoke
Total distance: 46730
Path end-point to trace from:  Lokoji
Lokoji <-- Olowa <-- Mosafejo <-- Adegbo <-- Luga <-- Akangun <-- Arade <-- Baale <--
IporoOgun <-- Majo <-- Bagbile <-- Sadu <-- Odeda <-- ItokoAjegunle <-- AraromiIdera
<-- Obantoko <-- Kangudu
Total distance: 60270
Path end-point to trace from:  Alatibaba
Alatibaba <-- Elegunnla <-- Elegbada <-- Olajogun <-- Kajola <-- Omididun <-- Ariwo
<-- Adelakun <-- Ayoye <-- Ajitadun <-- Oguntoke
Total distance: 64710
Path end-point to trace from:  Akitoye
Akitoye <-- Ilugun <-- OrileIlugun <-- Mosafejo <-- Adegbo <-- Luga <-- Akangun<--
Arade <-- Baale <-- IporoOgun <-- Majo <-- Bagbile <-- Sadu <-- Odeda
Total distance: 43270
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Figure 4: Generation of Minimum Distance for two specific Villages.

The information of the 88 towns and the distance among them is saved in a text file. The algorithm of the Prim's described in Figure 1 was coded in Java and run on. On executing the algorithm, it requests for the source or starting village name (node) and the program will generate all the connection between the starting village and the other villages. For example, the source village indicated by the user is village Oguntoke. A model that contains all the edges connected to each node is generated as described in figure 3. A minimum spanning tree for the whole graph is then generated together with the total sum of edge weights of the tree which is 388,270 meters.

Generating minimum spanning trees for different destination nodes/villages is also possible as shown in figure 4. A bus traveling from any of the starting villages to destination villages can follow the routes/paths generated in figure 4 for a cost effective service. Table 2 shows the effect of traveling through the routes/paths generated by the prim's algorithm on the objective function while table 3 depicts the effect of travelling through the routes without using the prim's algorithm.

Assume that the volume of fuel for traveling over 8000 meters is 1 litre for an averagely good Toyota 14 seater bus. With respect to Nigerian case study 1 litre of petrol is 70.00 Naira. Using this price for 1 litre of fuel we present the result obtained. The solution obtained is reported and compared with the result obtained before we apply the algorithm on the existing network in both table 2 and 3. For example, the value of objective function from Oguntoke to Olajogun was 554 as shown in table 3 without using Prims algorithm whereas it was 711 in table 2 where Prims algorithm was applied. The same situation applies to traveling from Obantoko to Nitoji with objective function of 507 as against 186 without using the algorithm.

TABLE 2: RESULTS OF OBJECTIVE FUNCTION USING PRIM'S ALGORITHM

A	B	C	D	E	F = (E/8000*₦70.00)	G=(A*B)	H=(G-F)
14	80.00	Oguntoke	Olajogun	46,730	40888	1120	711
14	50.00	Obantoko	Nitoji	21,960	192.15	700	507
14	130.00	Kangudu	Lokoji	60,270	527.36	1820	1292
14	130.00	Oguntoke	Alatibaba	64,710	566.21	1820	1253
14	80.00	Odeda	Akitoye	43,270	378.61	1120	741
14	60.00	Asawo	Mosafeyo	22,170	193.98	840	746

TABLE 3: RESULTS OF OBJECTIVE FUNCTION WITHOUT USING PRIM'S ALGORITHM

A	B	C	D	E	F = (E/8000*₦70.00)	G=(A*B)	H=(G-F)
14	80.00	Oguntoke	Olajogun	64,690	566	1120	554
14	50.00	Obantoko	Nitoji	58,730	513.8	700	186
14	130.00	Kangudu	Lokoji	71,520	625.8	1820	1194
14	130.00	Oguntoke	Alatibaba	85,670	749.6	1820	1070
14	80.00	Odeda	Akitoye	47,710	417.4	1120	702
14	60.00	Asawo	Mosafeyo	46,920	410.55	840	429

A = number of passengers   B = cost of traveling per head
C = starting village
D = destination village   E = total distance generated in meter   F = total edge cost
G = total profit   H = value of objective function

It can be seen that the value of the objective function in table 3 of column H is very low compared to that of table 2 where prim's algorithm was used. This means that company will make more profit with the implementation and use of the algorithm taking into account that the cost of traveling over 8000 meters is overestimated. The relative improvement of the solution proposed with respect to the existing network can be considered highly satisfactory.

VI. CONCLUSION

In the course of this work it was discovered that the algorithm is very effective in providing shortest distance between set of villages. In one way it reduces the cost of feul and time for transportation of passengers from one town to another which normally determines prices in Nigerian transportation.

Therefore, it is recommended that this paper can serve as a guide and can form further exploration into other local governments in Nigeria for effective transportation of goods and service. The method can also be explored by other organizations willing to achieve their goals with a reduced cost and effective use of their resources especially in areas of electrical installation, pipe water distribution and telecommunication, etc.

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