Study of Intelligent Optimization Methods Applied in the Fractional Fourier Transform

Wei Hongkai, Cai Zhiming, Wang Pingbo and Fu Yinfeng

Abstract—**In order to overcome the inefficiency shortcoming of traditional step-based searching method for extremum seeking in two-dimensional fractional Fourier domain, some typical intelligent optimization methods such as genetic algorithms, continuous ant colony algorithm, particle swarm optimization and chaos optimization method are introduced and applied successfully in fractional Fourier transform. To accelerate the convergence further, three optimization methods containing two improved chaos optimization methods and another hybrid method combing chaos optimization and Quasi-Newton method are proposed. The performances of the proposed optimization methods are verified by comparing with step-based method and other intelligent optimization methods based on simulation. Results show that the presented hybrid optimization algorithm is much more preferable considering computation efficiency, precision and resolution in all the above mentioned optimization methods.**

***Index Terms*—the fractional Fourier transform; genetic algorithms; continuous ant colony algorithm; particle swarm optimization; chaos optimization algorithm; Quasi-Newton algorithm; extremum seeking**

I. Introduction

Traditional Fourier transform, which is used to deal with stationary signals, can't demonstrate the time-variation characteristic of non-stationary signals. And the fractional Fourier transform (FRFT) [1]-[2] is developed recently to resolve the problem of Fourier transform. As one kind of non-stationary signals, linear frequency modulation (LFM) signal is widely used in radar, sonar and communication systems. So how to detect LFM echo signal degraded by noise is an important question and attracting more and more attention [3]-[5]. FRFT is suitable for dealing with chirp signal due to its orthonormal chirped basis. LFM signal can be concentrated in the proper fractional Fourier domain. Usually the detection and parameter estimation of LFM signal is accomplished by step-based searching method [6]-[8] in two-dimensional fractional Fourier domain using this concentrated characteristic. But the step-based seeking method is poor efficient especially when the precision is highly expected. It is significative and necessary extremely to develop some other optimization methods to decrease the time-cost of classical step-based searching method in fractional Fourier domain.

It shows that LFM signal transformed with FRFT may appear some different peak values in fractional Fourier domain but only the maximum of them conformable with LFM signal need to be found out. Intelligent optimization methods [9]-[15] such as genetic algorithms (GA), continuous ant colony algorithm (CACA), particle swarm optimization (PSO) and chaos optimization method (COA) are heuristic global searching techniques. In this paper, the intelligent optimization methods are introduced and applied to the problem of searching maximum value in two-dimension fractional Fourier domain. Some computation results show that the COA takes less iterative numbers than the former three global optimization methods in continuous function optimization problems [15]. But the traditional COA has the deficiency of costing much time to get the maximum value at the late process of iteration, which affects the rapidity of convergence. To overcome this limitation the two improved chaos optimization methods and another three hybrid algorithm combing COA and Quasi-Newton method are presented and compared with step-based searching method and the former intelligent optimization methods. Simulation results show that the performances of the three proposed optimization methods are better than that of the other five optimization methods and it seems that the proposed hybrid optimization method has the best performance in all the above mentioned optimization methods. This paper is organized as followed: The mathematical description of the fractional Fourier domain optimization problem is given in Section II. Intelligent optimization methods applied in fractional Fourier domain are introduced and summarized in Section III. Three proposed optimization methods based on COA is presented in Section IV in detail. Some simulation results are described and analyzed in Section V. conclusions are given in Section VI.

II. Problem Dsecription

The fractional Fourier transform of signal x(t) is defined as:

$$\begin{aligned} X_p(u) &= \int_{-\infty}^{+\infty} K_p(u,t)x(t)dt \\ &= \int_{-\infty}^{+\infty} A_\alpha e^{i\pi(u^2\cot\alpha - 2ut\csc\alpha + t^2\cot\alpha)} x(t)dt \end{aligned} \quad (1)$$

where:

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$$A_\alpha = \frac{e^{(-i\pi \operatorname{sgn}(\sin\alpha)/4 + i\alpha/2)}}{|\sin\alpha|^{1/2}}, \quad \alpha = \frac{\pi}{2} p, \ 0 < |p| < 2 \quad (2)$$

The kernel of FRFT approaches $K_p(u,t) = \delta(u-t)$ for $p=0$ and $K_p(u,t) = \delta(u+t)$ for $p = \pm 2$.

In practice, signal x(t) is always represented in a series of discrete values after sampling and A/D transformation. Corresponding, the fast discrete fractional Fourier transform algorithm is described as:

$$X_p(u) = \sum_{n=-N}^{N} \frac{A_\alpha}{2\Delta x} e^{i\pi\left(u^2\cot\alpha - \frac{2un\csc\alpha}{2\Delta x} + \frac{n^2\cot\alpha}{(2\Delta x)^2}\right)} x\left(\frac{n}{2\Delta x}\right) \quad (3)$$

So the optimization problem of Extremum Seeking for LFM signal in fractional Fourier domain can de expressed as following:

Look for proper (p_0, u_0) to make the target function $f(p,u) = |X_p(u)|^2$ maximum. That is:

$$(p_0, u_0) = \underset{(p,u)}{\arg\max}\left(|X_p(u)|^2\right) \quad (4)$$

We can see from (3) and (4) that the fractional domain optimization target function is multi-dimensional and non-linear exponential complicated function. Moreover, the discrete signals especially LFM signal transformed by FRFT, the target function is non-convex, and as a result, many local peak values may exist besides the global maximum value in fractional Fourier plane, which increases the difficulty of Extremum searching further.

III. Intelligent Optimization Methods

A. Genetic Algorithms (GA)

Genetic Algorithms is a population-based approach widely used to the solution of different optimization problems. The basic idea of GA approach arises from the thought of evolution and the diagram of GA is illustrated in Fig. 1.

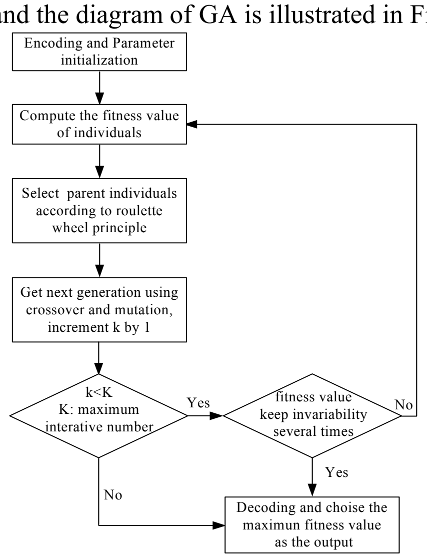

Figure 1. Diagram of genetic algorithm

B. Continuous Ant Colony Algorithm (CACA)

Continuous Ant Colony Algorithm is developed from the basic Ant Colony Algorithm to solve the problem of continuous function optimization. Each ant release pheromone proportion to function value in the process of searching. Ant adjusts its behavior by pheromone and is prone to select the position exists more pheromone. The outline of CACA is given below:

(1) Set maximum iterative number K, pheromone, and select several feasible solutions randomly as initialized ants.

(2) Compute the function value of each ant and update local and global pheromone.

(3) Ants select next solution according to pheromone information.

(4) Termination criterion: if k< K, then k=k+1 and go back to (2). Otherwise, stop.

C. Particle Swarm Optimization (PSO)

Particle Swarm Optimization, inspired by the social behavior of swarms of birds and fish schools, is one of the artificial lives or multiple agents' type techniques. PSO exploits a swarm of particles probing promising regions of the D-dimension search space with adaptable velocity. Each particle changes its position according to the best position it encountered and the best position attained by all particles. The update formula of velocity and position is stated by (5) and (6):

$$v_i^{k+1} = w_i v_i^k + c_1 a(p_i - x_i^k) + c_2 b(p_g - x_i^k) \quad (5)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (6)$$

where:

v_i^k : Velocity vector of particle i at iteration *k*.

v_i^{k+1} : Modified velocity of particle i at next iteration *k*+1.

x_i^k : Positioning vector of particle i at iteration *k*.

a, b : Random number between 0 and 1.

p_i : Best position found by particle *i*

p_g : Best position found by particle swarm separately.

c_1, c_2 : Positive constants.

w_i : Weight function for velocity of particle *i*.

x_i^{k+1} : modified position of particle *i* at next iteration *k*+1.

D. Chaos Optimization Algorithm(COA)

Chaos is an universal phenomenon occurs in non-linear and deterministic systems. And chaotic motion has the feature of ergodicity that is Chaotic movement can go through all the sates of a certain space. The chaos optimization method proposed by Li and Wang [15] hunts the feasible solution based on the ergodicity characteristic of chaotic variant. The schedule of COA is demonstrated as followings:

Step 1) Produce chaotic sequences using chaotic evolution equation (7). Note that the interval of chaotic sequences is between 0 and 1.

$$\begin{cases} y_{n+1} = \mu y_n (1 - y_n) \\ 0 \le y_1 \le 1 \end{cases}, n = 1, 2, \cdots \quad (7)$$

Step 2) Map the chaotic sequences to variable interval by way of carrier wave according to the particular resolved problem.

Step 3) Compare the function value of each chaotic sequence and pick out the best value as the output when the function value keep invariant and doesn't not increment anymore.

IV. Three Improved Methods Based On Chaos Optimization Algorithms

The conventional chaos optimization algorithm has the shortcoming of inefficiency in the late iteration, and the three improved optimization algorithms are presented to overcome that disadvantage.

A. The First Improved Chaos Optimization Algorithm(ICOA1)

COA has two main shortages. Firstly chaotic sequences are formed using certain chaotic map. The logistic map signified by (7) is usually adopted in COA. But the distribution of chaotic sequences produced by logistic map is non-uniform leading to the slow constringency. The unlimited fold map remarked by (8) is uniform map with which we replace logistic map to accelerate the rate of convergence. The distribution of the two maps is demonstrated in fig. 2. Secondly the number of chaotic sequences is always set to large in order to ensure find the maximum value of multimodal function. The convergence of COA is affected and related with the initialized value. So we reduce the number of chaotic sequences and execute several times of the COA with different initialized value. This is the first proposed COA.

$$\begin{cases} y_{n+1} = \sin(2/y_n) \\ -1 \le y_1 \le 1 \end{cases}, n = 1, 2, \cdots \quad (8)$$

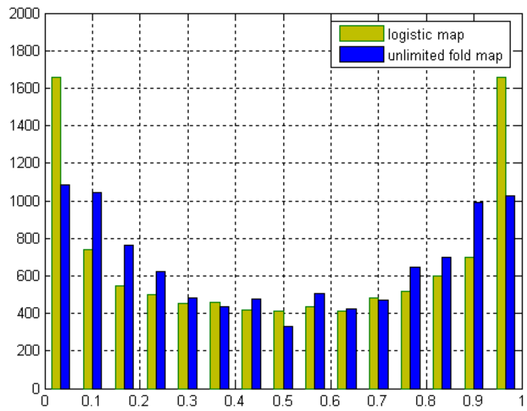

Figure 2. Distribution of logistic map and unlimited fold map

B. The second improved Chaos Optimization Algorithm(ICOA2)

Like the first improved COA, the second improved COA also reduces the number of chaotic sequences to enhance the efficiency but in another way apart from using unlimited fold map instead of logistic map. Usually after several iterations the evaluated value is near the true maximum value. Therefore we use the former results of COA as the initial value of the next chaotic sequences of COA. The truth value can be searched after a number of times. The diagram of first proposed CAO is illustrated in fig. 3. Note that the diagram of the second proposed method is similar to the first proposed method except that the iteration of COA is based on the former searching value in the second proposed method.

C. The Hybrid Optimization Algorithm(HOA)

Although our two proposed improved chaos optimization methods especially the second one enhance the convergence rate of the traditional chaos optimization algorithm in a certain extent, the proposed algorithms only use the function value of signal with different fractional Fourier domain in the procedure of iterations affecting the efficiency of the algorithm. The Quasi-Newton method [16]-[17], which overcomes the drawbacks of Newton method for maintaining the positive-definiteness property of Hessian matrix and solving its inverse matrix, has the speed of superlinear when its initial value is nearly the peak value due to its full use of the objective function value and gradient information. Usually the COA can find the rough region of possible solution after several iterations. So it's feasible to combine the COA and Quasi-Newton method as hybrid algorithm in order to ensure both the global solution and the fast convergence ability. The diagram of the hybrid algorithm is demonstrated in fig. 4.

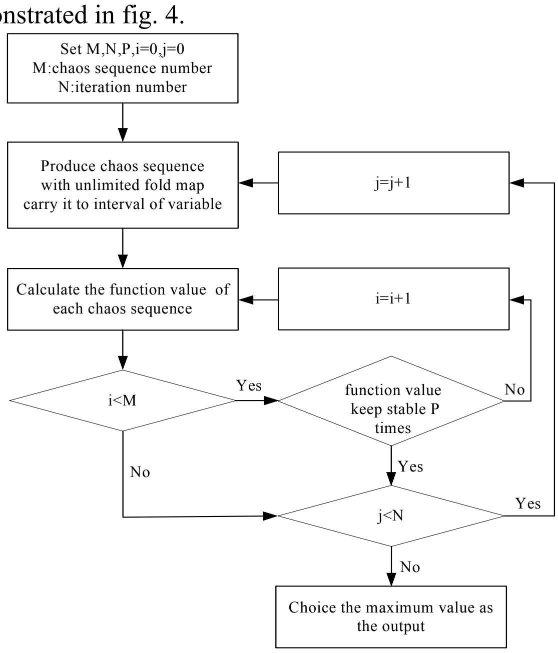

Figure 3. Diagram of the first ICOA

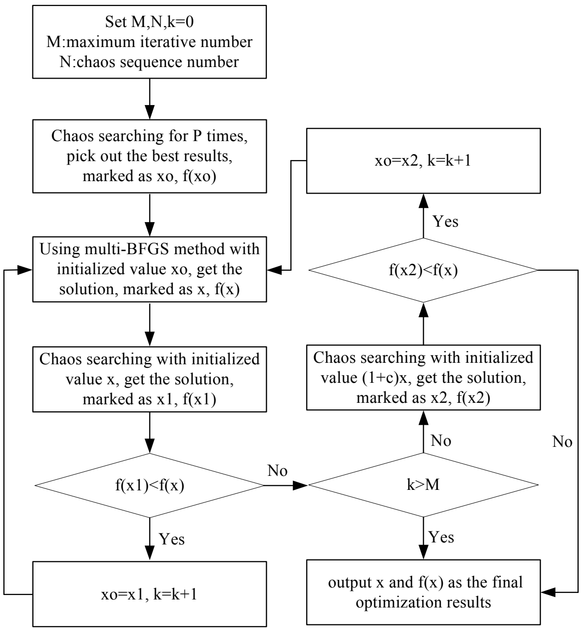

Figure 4. Diagram of the hybrid algorithm

V. SIMULATION RESULTS

We compare the performances of the above mentioned optimization methods from the following examples.

Example 1. $x(t)=e^{i2\pi f_0 t+i\pi\mu t^2}+w(t)\,, t\in[0.T]$.

Where the pulse width T is 0.1s, the frequency band is 100-300Hz, the sampling frequency is 2000Hz, w(t) is Gaussian white noise, and the signal noise ratio is -5dB.

The comparison of different optimization methods is described in Table I.

TABLE I. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR LFM SIGNAL AND GAUSSIAN NOISE

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**	**time (s)**
Step-based (0.001)	(1.0635,1.4072)	(1.063,1.4496)	17.2
Step-based (0.0005)		(1.0635,1.4496)	34.4
GA		(1.0633,1.4496)	29.8
CACA		(1.0633,1.4496)	21.5
PSO		(1.0632,1.4496)	11.4
COA		(1.0634,1.4496)	8.8
ICOA1		(1.0633,1.4496)	5.9
ICOA2		(1.0633,1.4496)	4.8
HOA		(1.0634,1.4496)	3.3

Example 2. $x(t)=e^{i2\pi f_0 t+i\pi\mu t^2}+w(t)\,, t\in[0.T]$.

Where the pulse width T is 0.3s, the frequency band is 100-300Hz, the sampling frequency is 2000Hz, w(t) is Gaussian white noise, and the signal noise ratio is -5dB.
The comparison of different optimization methods is described in Table II.

TABLE II. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR LFM SIGNAL AND GAUSSIAN NOISE

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**	**time (s)**
Step-based (0.001)	(1.0635,1.4981)	(1.064,2.4291)	37.2
Step-based (0.0005)		(1.0635,1.5250)	76.0
GA		(1.0637,2.4291)	47.5
CACA		(1.0637,2.4291)	45.8
PSO		(1.0636,2.4291)	26.9
COA		(1.0637,2.4291)	19.6
ICOA1		(1.0636,2.4291)	13.4
ICOA2		(1.0636,2.4291)	10.6
HOA		(1.0635,2.4291)	7.0

Example 3. $x(t)=e^{i2\pi f_0 t+i\pi\mu t^2}+w(t)\,, t\in[0.T]$.

Where the pulse width T is 0.5s, the frequency band is 100-300Hz, the sampling frequency is 2000Hz, w(t) is Gaussian white noise, and the signal noise ratio is -5dB.

The comparison of different optimization methods is described in Table III.

TABLE III. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR LFM SIGNAL AND GAUSSIAN NOISE

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**	**time (s)**
Step-based (0.001)	(1.0635,3.1466)	(1.064,3.1465)	91.5
Step-based (0.0005)		(1.0635,3.1465)	136.8
GA		(1.0636,3.1465)	117.6
CACA		(1.0637,3.1465)	138.9
PSO		(1.0637,3.1465)	65.8
COA		(1.0637,3.1465)	48.6
ICOA1		(1.0637,3.1465)	33.4
ICOA2		(1.0636,3.1465)	25.5
HOA		(1.0636,3.1465)	14.9

Example 4. $x(t)=e^{i2\pi f_0 t+i\pi\mu t^2}+w(t)\,, t\in[0.T]$.

Where the pulse width T is 0.7s, the frequency band is 100-300Hz, the sampling frequency is 2000Hz, w(t) is Gaussian white noise, and the signal noise ratio is -5dB.
The comparison of different optimization methods is described in Table IV.

TABLE IV. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR LFM SIGNAL AND GAUSSIAN NOISE

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**	**time (s)**
Step-based (0.001)	(1.0635,3.7231)	(1.064,3.1465)	142.9
Step-based (0.0005)		(1.0635,3.1465)	284.5
GA		(1.0636,3.7283)	183.1
CACA		(1.0637,3.7283)	198.9
PSO		(1.0637, 3.7283)	101.6
COA		(1.0637, 3.7283)	74.4
ICOA1		(1.0637, 3.7283)	51.1
ICOA2		(1.0636, 3.7283)	40.1
HOA		(1.0637, 3.7283)	25.2

Example 5. $x(t)=r(t)\,, t\in[0.T]$.

Where the pulse width of transmitted signal T is 0.432s, the frequency band of the LFM signal is 650-850Hz, the sampling frequency is 5000Hz, r(t) is reverberation containing echo signal detected by matched filter. The echo target appears in the distance of 5.2 kilometer approximately. As is illustrated in fig. 5.

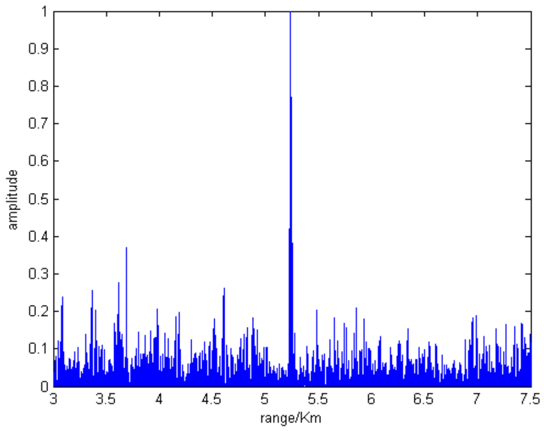

Figure 5. Detection of matched filter

The comparison of different optimization methods is described Table V.

TABLE V. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR LFM ECHO SIGNAL IN REVERBERATION BACKGROUND

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**	**time (s)**
Step-based (0.001)	(1.0255,6.9472)	(1.024,6.8530)	168.3
Step-based (0.0005)		(1.0235,6.8530)	339.7
GA		(1.0237,6.8530)	252.4
CACA		(1.0237,6.8530)	236.0
PSO		(1.0237,6.8530)	112.5
COA		(1.0237,6.8530)	87.3
ICOA1		(1.0237,6.8530)	60.2
ICOA2		(1.0237,6.8530)	44.9
HOA		(1.0237,6.8530)	25.1

Example 6. $x(t) = r(t)\,, t \in [0.T]$.

Where the pulse width of transmitted signal T is 0.432s, the frequency band of the LFM signal is 650-850Hz, the sampling frequency is 5000Hz, r(t) is reverberation containing echo signal detected by matched filter. The echo target appears in the distance of 4.1 kilometer approximately. As is illustrated in fig. 6.

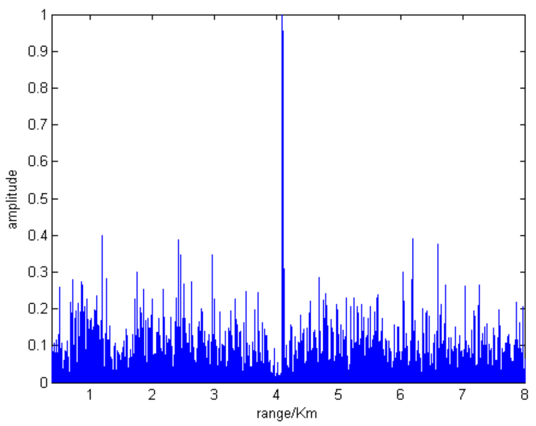

Figure 6. Detection of matched filter

The comparison of different optimization methods is described Table VI.

TABLE VI. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR LFM ECHO SIGNAL IN REVERBERATION BACKGROUND

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**	**time (s)**
Step-based (0.001)	(1.0255,6.9658)	(1.026,6.9606)	157.3
Step-based (0.0005)		(1.0255,6.9606)	314.9
GA		(1.0257,6.9606)	216.0
CACA		(1.0257,6.9606)	217.3
PSO		(1.0256,6.9606)	115.1
COA		(1.0258,6.9606)	81.6
ICOA1		(1.0257,6.9606)	55.5
ICOA2		(1.0256,6.9606)	41.9
HOA		(1.0256,6.9606)	23.1

As can be seen from table I to VI that the theoretical and estimated values are a little different due to the effects of both discrete sampling and noise background. The time of all the mentioned optimization methods is related directly with the pulse width of the signal, which can be seen obviously from table I to IV with the same parameters except pulse width. This is because the computation time of fast discrete FRFT adopted in this paper is $O(N \log N)$, N is the time-bandwidth product. The elapsed time of step-based optimization method is in proportional with step. When the step is 0.001, it takes shorter time than that of GA and CACA, but as the increase of the step such as 0.0005, it costs much time that of GA and CACA. The estimated errors of the global optimization methods are not more than 0.0002 and could be regarded as the same approximately. So the performance of GA is almost the same as that of CACA but is worse than that of PSO considering both accuracy and speed. And the performance of COA is best in all the above optimization methods except our proposed methods due to the ergodicity property of each chaotic sequence and the use of second carrier technique that is the second search nearby the first searching results. The iteration ratios of the three proposed optimization methods are better than that of COA. Performance of ICOA2 is better than ICOA1 because it make full use of the former information. And the performance of the proposed hybrid algorithm is the best in all the mentioned optimization methods due to its adopting Quasi-Newton method and using the information of objective function sufficiently such as the function value and gradient information.

Example 7. $x(t) = \sum_{m=1}^{3} a_m e^{i2\pi f_{0m} t + i\pi\mu_m t^2} + w(t)\,, t \in [0.T]$.

Where the pulse width T is 0.2s, the frequency bands of three LFM signals are 98-300Hz, 350-500Hz and 600-806Hz separately, the sampling frequency is 4000Hz, the amplitudes a_1=1.0, a_2=2.99 and a_3=3, w(t) is Gaussian noise, and the signal noise ratio is -5dB.

The comparison of different optimization methods is described in Table VII.

TABLE VII. OPTIMIZATION RESULTS OF VARIOUS SEARCHING METHODS FOR THREE LFM SIGNAL AND GAUSSIAN NOISE

Optimization methods	**Theoretical value (u_0, p_0)**	**Estimated value (u^*,p^*)**
Step-based (0.001)	(1.0328,4.9644)	(1.033,4.9674)

Step-based (0.0001)		(1.0329,4.9674)
GA		(1.0326,4.9674)
CACA		(1.0329,4.9674)
PSO		(1.0328,4.9674)
COA		(1.0329,4.9674)
ICOA1		(1.0329,4.9674)
ICOA2		(1.0329,4.9674)
HOA		(1.0329,4.9674)

The true peak values of three LFM signals in fractional Fourier domain demonstrated in Fig. 7 are (1.0321, 1.4054), (1.0239, 3.0031) and (1.0328, 4.9644) separately. And the function value of (1.0328, 4.9644) is maximum which is a little bigger that of (1.0239, 3.0031). As can be seen from Table VII that all the intelligent optimization methods and our proposed algorithms can find the true global maximum value although there exits two almost same values. The resolution of the first and third LFM signal is no less than 0.0001. It is the reason that the step-based method with step at least 0.0001 can distinguish the two different LFM signals. The intelligent optimization methods and our proposed algorithms can distinguish different LFM signals due to their searching mechanism with precision less than 0.0001.

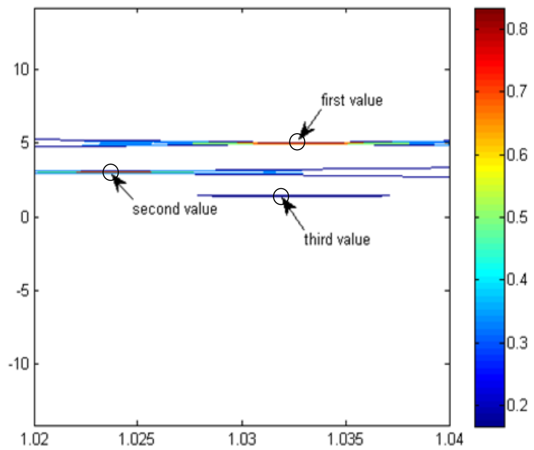

Figure 7. Counter map of three LFM signals in fractional Fourier domain

VI. Conclusions

Signals transformed with the fractional transform can be demonstrated in multi-value objective function and the maximum of them needed to be found out. The intelligent optimization methods are global heuristic searching algorithm. In this paper, we first introduce some intelligent optimization algorithms such as GA, CACA, PSO and COA in fractional Fourier transform for extreme searching to resolve the problem of the common used step-based method which is time consuming especially when the precision is highly desired. Then we present two improved chaos optimization algorithms in order to enhance the iterative speed. The two proposed improved chaos optimization algorithms only use the value of the objective function. So the third hybrid algorithm combing COA and Quasi-Newton method is proposed in order to accelerate the convergence rate further. Simulation results show that performances of the three proposed optimization method are better than that of step-based method and some other intelligent optimization methods that is GA, CACA and PSO. And it concludes that the proposed hybrid optimization algorithm is much more preferable balancing computation efficiency and precision.

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