International Journal of Computer Theory and Engineering, Vol. 2, No. 2 April, 2010
1793-8201

A Family of Covering Rough Sets Based Algorithm for Reduction of Attributes

Nguyen Duc Thuan, *Member,* IACSIT

Abstract— **Attribute reduction of an information system is a key problem in rough set theory and its application. It has been proven that finding the minimal reduct of an information system is a NP-hard problem. Main reason of causing NP-hard is combination problem. In this paper, we theoretically study covering rough sets and propose an attribute reduction algorithm of decision systems. It based on results of Chen Degang et al in consistent and inconsistent covering decision system. The time complexity of this algorithm is O(|Δ||U|2). Two illustrative examples are provided that shows the application potential of the algorithm.**

Index Terms— **Attribute Reduction, Covering Decision System, Covering Rough Sets, Consistent and Inconsistent Decision System.**

I. Introduction

Rough set theory is a mathematical tool to deal with vagueness and uncertainty of imprecise data. The theory introduced by Pawlak in 1982 has been developed and found applications in the fields of decision analysis, data analysis, pattern recognition, machine learning, and knowledge discovery in databases.

In Pawlak original rough set theory, partition or equivalence (indiscernibility) relation is a primitive concept. However, equivalence relations are too restrictive for many applications. One response to this has been to relax the equivalence relation to similarity relation or tolerance relation and others. Another response has been to relax the partition to a cover and obtain the covering rough sets. Hence, the covering generalized rough set theory is a model with promising potential for applications to datamining, we address some basic problems in this theory, one of them is attribute reduction problem.

In [1], Cheng Degang et al. have defined consistent and inconsistent covering decision system and their attribute reduction. They gave an algorithm to compute all the reducts of decision systems. Their method based on discernibility matrix. But, in rough set theory, it has been proved that finding all the reduct of information systems or decision tables is NP-complete problem. Hence, sometime we only need to find an attribute reduction. In this paper, using some results of Chen Degang et al, we propose an algorithm which is finding a minimal attribute reduct information decision system.

The remainder of this paper is structured as follows. In section 2 briefly introduces some relevant concepts and results. In this section, we also present. Section 3, we present a new algorithm and two theorems (2.5, 2.6) as a base for it. Two illustrative examples are provided that shows the application potential of the algorithm in section 4. At last, the paper is concluded with a summarization in section 5.

II. Some Relevant Concepts And Results

In this section, we first recall the concept of a cover and then review the existing research on covering rough sets of Cheng Degang et al.

One kind of suitable data set for covering rough sets is the information systems that some objects have multiple attribute values for a given attribute. This kind of data set is available when some objects have multiselections of attribute values for a given attribute. So we have to list all the possible attribute values. One example of this kind of data set is the combination of several information systems. For illustrative purposes, we can review an interesting example in [1].

Example 2.1 ([1]) Let us consider the problem of evaluating credit card applicants. Suppose $U = \{x_1 ..., x_9\}$ is a set of nine applicants, E= {education; salary} is a set of two attributes, the values of ''education'' are {best; better; good}, and the values of ''salary'' are {high; middle; low}. We have three specialists {A, B, C} to evaluate the attribute values for these applicants. It is possible that their evaluation results to the same attribute values may not be the same, listed below:

For attribute ''education''

A: best $=\{x_1, x_4, x_5, x_7\}$, better= $\{x_2, x_8\}$, good= $\{x_3, x_6, x_9\}$

B: best $=\{x_1, x_2, x_4, x_7, x_8\}$, better=$\{x_5\}$, good= $\{x_3, x_6, x_9\}$

C: best $=\{x_1, x_4, x_7\}$, better=$\{x_2, x_8\}$, good=$\{x_3, x_5, x_6, x_9\}$

For attribute ''salary''

A: high $=\{x_1, x_2, x_3\}$, middle $= \{x_4, x_5, x_6, x_7, x_8\}$, low=$\{x_9\}$

B: high $=\{x_1, x_2, x_3\}$, middle $= \{x_4, x_5, x_6, x_7\}$, low= $\{x_8, x_9\}$

C: high $= \{x_1, x_2, x_3\}$, middle $= \{x_4, x_5, x_6, x_8\}$, low= $\{x_7, x_9\}$

Suppose the evaluations given by these specialists are of the same importance. If we want to combine these evaluations without losing information, we should union the evaluations given by each specialist for every attribute value as shown in Table 1. This classification is not a partition, but a cover, which reflects a kind of uncertainty caused by the differences in interpretation of data.

Thuan Nguyen Duc was born in Hue, Vietnam, 1962

Brief Biographical History:

-1985, Bachelor in Mathematics- Lecturer of Hue University, Vietnam.

-1998, Master in Information Technology, Lecturer of Nha Trang University, Vietnam.

-2006, Ph.D Student in Institute of Information Technology, Vietnamese academy of Science and Technology.

Current research: Rough set, Datamining, Distributed Database...

TABLE I. CLASSIFICATION BY EVALUATION OF ALL THREE SPECIALISTS

Salary	Education		
	Best	Better	Good
High	$\{x_1, x_2\}$	$\{x_2\}$	$\{x_3\}$
Middle	$\{x_4, x_5, x_7, x_8\}$	$\{x_5, x_8\}$	$\{x_5, x_6\}$
Low	$\{x_7, x_8\}$	$\{x_8\}$	$\{x_9\}$

A. Covering rough sets and induced covers

Definition 2.1 Let U be a universe of discourse, C a family of subsets of U. C is called a cover of U if no subset in C is empty and $\cup C = U$.

Definition 2.2 Let $C = \{C_1, C_2..., C_n\}$ be a cover of U. For every $x\in U$, let $C_x = \cap\{C_j: C_j \in C, x\in C_j\}$. $Cov(C) = \{C_x: x\in U\}$ is then also a cover of U. We call it induced over of C.

Definition 2.3 Let $\Delta= \{C_i: i=1, m\}$ be a family of covers of U. For every $x\in U$, let $\Delta x= \cap\{C_{ix}: C_{ix}\in Cov(C_i), x\in C_{ix}\}$ then $Cov(\Delta) = \{\Delta_x: x\in U\}$ is also a cover of U. We call it the induced cover of Δ.

Clearly Δ_x is the intersection of all the elements in every C_i including x, so for every $x\in U$, Δ_x is the minimal set in Cov(Δ) including x. If every cover in Δ is an attribute, then $\Delta_x= \cap\{C_{ix}: C_{ix}\in Cov(C_i), x\in C_{ix}\}$ means the relation among C_{ix} is a conjunction. Cov(Δ) can be viewed as the intersection of covers inΔ. If every cover in Δ is a partition, then Cov(Δ) is also a partition and Δ_x is the equivalence class including x. For every x, y ∈ U, if $y \in \Delta_x$, then $\Delta_x \supseteq \Delta_y$, so if $y \in\Delta_x$ and $x \in\Delta_y$, then $\Delta_x=\Delta_y$. Every element in Cov(Δ) can not be written as the union of other elements in Cov(Δ). We employ an example to illustrate the practical meaning of C_x and Δ_x.

Example 2.2 ([1]) In Example 2.1 if let $\Delta= \{C_1, C_2\}$, where C_1 denotes the attribute ''education'' and C_2 denotes the attribute ''salary'', then

$C_1= \{C_{11}=\{x_1, x_2, x_4, x_5, x_7, x_8\}$ (best), $C_{12}=\{x_2, x_5, x_8\}$ (better), $C_{13} = \{x_3, x_5, x_6, x_9\}$ (good)}

$C_2= \{C_{21}=\{x_1, x_2, x_3\}$ (high), $C_{22}=\{x_4, x_3, x_6, x_7, x_8\}$ (middle), $C_{23}= \{x_7, x_8, x_9\}$ (lower)}

We have $C_1x_5 = \{x_5\} = C_{11}\cap C_{12}\cap C_{13}$, which implies the possible description of x_5 is {(best ∨ better ∨ good} according to attribute ''education''. $\Delta_{x8} = (C_{11} \cap C_{12}) \cap (C_{22} \cap C_{23})$ which implies the possible description of x_8 is {(best ∨ better) ∧ (middle ∨ lower)}.

For every $X \subseteq U$, the lower and upper approximation of X with respect to Cov(Δ) are defined as follows:

$$\underline{\Delta}(X) = \cup\{\Delta_x : \Delta_x \subseteq X\},$$

$$\overline{\Delta}(X) = \cup\{\Delta_x : \Delta_x \cap X \neq \varnothing\}$$

The positive, negative and boundary regions of X relative to Δ are computed using the following formulas respectively:

$$POS_\Delta(X) = \underline{\Delta}(X), NEG_\Delta(U - \overline{\Delta}(X),$$

$$BN_\Delta(X) = \overline{\Delta}(X) - \underline{\Delta}(X)$$

Clearly in Cov(Δ), Δ_x is the minimal description of object x.

B. Attribute reduction of consistent and inconsistent decision systems

Definition 2.4 Let $\Delta = \{C_i: i=1,..m\}$ be a family of covers of U, D is a decision attribute, U/D is a decision partition on U. If for $\forall x\in U$, $\exists D_j \in U/D$ such that $\Delta x \subseteq D_j$, then decision system (U,Δ,D) is called a consistent covering decision system, and denoted as $Cov(\Delta)\leq U/D$. Otherwise, (U,Δ,D) is called an inconsistent covering decision system. The positive region of D relative to Δ is defined as

$$POS_\Delta(D) = \bigcup_{X\in U/D} \underline{\Delta}(X)$$

Remark 2.1 Let D={d}, then d(x) is a decision function d: U → Vd of the universe U into value set Vd. For every $x_i, x_j \in U$, if $\Delta_{xi} \subseteq \Delta_{xj}$, then $d(x_i) = d([x_i]_D) = d(\Delta_{xi}) = d(\Delta_{xj}) = d(x_j) = d([x_j]_D)$. If $d(\Delta_{xi}) \neq d(\Delta_{xj})$, then $\Delta_{xi} \cap \Delta_{xj} = \varnothing$, i.e $\Delta_{xi} \not\subset \Delta_{xj}$ and $\Delta_{xj} \not\subset \Delta_{xi}$.

Definition 2.5 Let (U,Δ, D= {d}) be a consistent covering decision system. For $C_i \in \Delta$, if $Cov(\Delta\text{-}\{C_i\}) \leq U/D$, then C_i is called superfluous relative to D in Δ, otherwise C_i is called indispensable relative to D in Δ. For every $P \subseteq \Delta$ satisfying $Cov(P) \leq U/D$, if every element in P is indispensable, i.e., for every $C_i \in P$, $Cov(\Delta\text{-}\{C_i\}) \leq U/D$ is not true, then P is called a reduct of D relative to D, relative reduct in short. The collection of all the indispensable elements in D is called the core of Δ relative to D, denoted as CoreD(Δ). The relative reduct of a consistent covering decision system is the minimal set of conditional covers (attributes) to ensure every decision rule still consistent. For a single cover C_i, we present some equivalence conditions to judge whether it is indispensable.

Definition 2.6 Suppose U is a finite universe and $\Delta = \{C_i: i=1,..m\}$ be a family of covers of U, $C_i \in\Delta$, D is a decision attribute relative Δ on U and d: U → V_d is the decision function V_d defined as $d(x) = [x]_D$. (U,Δ,D) is an inconsistent covering decision system, i.e., $POS_\Delta(D)\neq U$. If $POS_\Delta(D)=POS_{\Delta\text{-}\{Ci\}}(D)$, then C_i is superfluous relative to D in Δ. Otherwise C_i is indispensable relative to D in Δ. For every $P \subseteq \Delta$, if every element in P is indispensable relative to D, and $POS_\Delta(D)=POS_P(D)$, then P is a reduct of $POS_\Delta(D)=POS_{\Delta\text{-}\{Ci\}}(D)$ relative to D, called relative reduct in short. The collection of all the indispensable elements relative to D in Δ is the core of Δ relative to D, denoted by $Core_D(\Delta)$.

C. Some results of Chang et al

Theorem 2.1 ([1]) Supposing U is a finite universe and $\Delta = \{C_i: i=1,..m\}$ be a family of covers of U, the following statements hold:

1) $\Delta_x = \Delta_y$ if and only if for every $C_i \in \Delta$ we have $C_{ix} = C_{iy}$.
2) $\Delta_x \supset \Delta_y$ if and only if for every $C_i \in \Delta$ we have $C_{ix} \supseteq C_{iy}$ and there is a $C_i \in \Delta$ such that $C_{i0\,x} \supset C_{i0\,y}$.
3) $\Delta_x \not\subset \Delta_y$ and $\Delta_y \not\subset \Delta_x$ hold if and only if there are C_i, $C_j \in \Delta$ such that $C_{ix} \subset C_{iy}$ and $C_{jx} \supset C_{jy}$ or there is a $C_{i0} \in \Delta$ such that $C_{i0\,x} \not\subset C_{i0\,y}$ and $C_{i0\,y} \not\subset C_{i0\,x}$.

Theorem 2.2 ([1]) Suppose $Cov(\Delta)\leq U/D$, $C_i \in\Delta$, C_i is then indispensable, i.e., $Cov(\Delta\text{-}\{C_i\}) \leq U/D$ is not true if and only if there is at least a pair of $x_i, x_j \in U$ satisfying $d(D_{xi})\neq d(D_{xj})$, of which the original relation with respect to Δ changes after C_i is deleted from Δ.

Theorem 2.3 ([1]) Suppose $Cov(\Delta) \le U/D, P \subseteq \Delta$, then $Cov(P) \le U/D$ if and only if for $x_i, x_j \in U$ satisfying $d(\Delta_{xi}) \ne d(\Delta_{xj})$, the relation between x_i and x_j with respect to Δ is equivalent to their relation with respect to P, i.e., $\Delta_{xi} \not\subset \Delta_{xj}$ and $\Delta_{xj} \not\subset \Delta_{xi} \Leftrightarrow P_{xi} \not\subset P_{xj}, P_{xj} \not\subset P_{xi}$.

Theorem 2.4 ([1]) Inconsistent covering decision system $(U,\Delta, D = \{d\})$ have the following properties:

1) For $\forall x_i \in U$, if $\Delta_{xi} \subset POS_\Delta(D)$, then $\Delta_{xi} \subseteq [x_i]_D$; if $\Delta_{xi} \not\subset POS_\Delta(D)$, then for $\forall x_k \in U$, $\Delta_{xi} \subseteq [x_k]_D$ is not true.
2) For any $P \subseteq \Delta$, $POS_P(D) = POS_\Delta(D)$ if and only if $\underline{P}(X) = \underline{\Delta}(X)$ for $\forall X \in U/D$.
3) For any $P \subseteq \Delta$, $POS_P(D) = POS_\Delta(D)$ if and only if $\forall x_i \in U$, $\Delta xi \subseteq [x_i]_D \Leftrightarrow Pxi \subseteq [x_i]_D$.

III. Algorithm Of Attribute Reduction

In this section, we propose a new algorithm of attribute reduction. Theorem 2.5 and 2.6 are theorical foundation for our proposing. This algorithm finds an approximately minimal reduct.

A. Two theorems as a base for new algorithm

Theorem 2.5 Let $(U,\Delta,D=\{d\})$ be a covering decision system. $P \subseteq \Delta$, then we have:

a. $(U,\Delta,D=\{d\})$ is a consistent covering decision system when it holds:

$$\sum_{x \in U} \frac{|\Delta_x \cap [x]_D|}{|\Delta_x|} = |U|$$

b. Suppose $Cov(\Delta) \le U/D$, $C_i \in \Delta$, C_i is then indispensable, i.e., $Cov(\Delta-\{C_i\}) \le U/D$ is true if and only if

$$\sum_{xi \in U} \sum_{xj \in U} \left|(\Delta_{xi} \cap \Delta_{xj} \cup (P_{xi} \cap P_{xj})\right| \left|d(\Delta_{xi}) - d(\Delta_{xj})\right| = 0$$

Where $Cov(\Delta-\{C_i\})=\{P_x : x \in U\}$, $Cov(\Delta)= \{\Delta_x : x \in U\}$

Proof:

a. By define of a consistent covering decision system, clearly for every $x \in U$, $\Delta_x \subseteq [x]_D$ is always true, thus we have

$$|\Delta_x \cap [x]_D| = |\Delta_x|$$

i.e

$$\sum_{x \in U} \frac{|\Delta_x \cap [x]_D|}{|\Delta_x|} = |U|$$

b. Let $Cov(\Delta-\{C_i\})=\{P_x : x \in U\} = Cov(P)$, $Cov(\Delta)= \{\Delta_x : x \in U$, by theorem 2.3, P is a reduct or Ci is indispensable, for x_i, $x_j \in U$ satisfying $d(\Delta_{xi}) \ne d(\Delta_{xj})$, the relation between x_i and x_j with respect to Δ is equivalent to their relation with respect to P, i.e., $\Delta_{xi} \not\subset \Delta_{xj}$ and $\Delta_{xj} \not\subset \Delta_{xi} \Leftrightarrow P_{xi} \not\subset P_{xj}, P_{xj} \not\subset P_{xi}$. Follow remark 2.1, If $d(\Delta_{xi}) \ne d(\Delta_{xj})$, then $\Delta_{xi} \cap \Delta_{xj} = \varnothing$, i.e

$$\left|(\Delta_{xi} \cap \Delta_{xj}) \cup (P_{xi} \cap P_{xj})\right| = 0$$

If $x_i, x_j \in U$ satisfying $d(\Delta_{xi}) = d(\Delta_{xj})$ then

$$\left|d(\Delta_{xi}) - d(\Delta_{xj})\right| = 0$$

In other words, it holds:

$$\sum_{xi \in U} \sum_{xj \in U} \left|(\Delta_{xi} \cap \Delta_{xj} \cup (P_{xi} \cap P_{xj})\right| \left|d(\Delta)_{xi} - d(\Delta)_{xj}\right| = 0$$

This completes the proof.

Theorem 2.6 Let $(U,\Delta,D=\{d\})$ be an inconsistent covering decision system. $P \subseteq \Delta$, $POS_P(D) = POS_\Delta(D)$ if and only if $\forall xi \in U$,

$$\sum_{xi \in U} \left[\left| \frac{\Delta_{xi} \cap [x_i]_D}{\Delta_{xi}} \right| - \left| \frac{P_{xi} \cap [x_i]_D}{P_{xi}} \right| \right] = 0$$

Proof:

By theorem 2.4, from third condition $\forall x_i \in U$, $\Delta_{xi} \subseteq [xi]_D \Leftrightarrow P_{xi} \subseteq [xi]_D$ i.e $\forall x_i \in U$,

$$|\Delta_{xi} \cap [x]_D| = |\Delta_{xi}| \Leftrightarrow |P_{xi} \cap [x]_D| = |P_{xi}|$$

In other words, we have theorem above.

B. Algorithm of attribute reduction in covering decision system:

Input: A covering decision system
$S= (U,\Delta, D= \{d\})$

Output: One product RD of Δ.

Method

Step 1: Compute

$$CI = \sum_{x \in U} \frac{|\Delta_x \cap [x]_D|}{|\Delta_x|}$$

Step 2: If CI = |U| {S is a consistent covering decision system} then goto Step 3 else goto Step 5.

Step 3: Compute

$$\Delta_x, d(\Delta_x), \forall x \in U$$

Step 4: Begin
For each $C_i \in \Delta$ do
if

$$\sum_{xi \in U} \sum_{xj \in U} \left|(\Delta_{xi} \cap \Delta_{xj} \cup (P_{xi} \cap P_{xj})\right| \left|d(\Delta_{xi}) - d(\Delta_{xj})\right| = 0$$

{Where Δ - $\{C_i\}$ = $\{Px : x \in U\}$}
then $\Delta := \Delta - \{C_i\}$;
Endfor;
goto Step 6.
End;

Step 5: Begin
For each $C_i \in \Delta$ do
if

$$\sum_{xi \in U} \left[\left| \frac{\Delta_{xi} \cap [x_i]_D}{\Delta_{xi}} \right| - \left| \frac{P_{xi} \cap [x_i]_D}{P_{xi}} \right| \right] = 0$$

then $\Delta := \Delta - \{C_i\}$;
{Where Δ - $\{C_i\}$ = $\{Px : x \in U\}$}
Endfor;
End;

Step 6: RD=Δ; the algorithm terminates.

By using this algorithm, the time complexity to find one reduct is polynomial.

At the first step, the time complexity to compute CI is O(|U|).

At the step 2, the time complexity is O(1).

At the step 3, the time complexity is O(|U|).

At the step 4, the time complexity to compute $\sum\sum()$ is $O(|U|^2)$, from i=1..|Δ|, thus the time complexity of this step is $O(|\Delta||U|^2)$.

At the step 5, the time complexity is the same as step 4. It is $O(|\Delta||U|^2)$.

At the step 6, the time complexity is O(1).

Thus the time complexity of this algorithm is $O(|\Delta||U|^2)$ (Where we ignore the time complexity for computing Δ_{xi}, P_{xi}, i= 1..|Δ|).

IV. Illustrative Examples

A. Example for a consistent covering decision system

Suppose $U = \{x_1, x_2, .., x_9\}$, $\Delta = \{C_i, i=1..4\}$, and
$C_1=\{\{x_1, x_2, x_4, x_5, x_7, x_8\},\{x_2, x_3, x_5, x_6, x_8, x_9\}\}$,
$C_2=\{\{x_1, x_2, x_3, x_4, x_5, x_6\},\{x_4, x_5, x_6, x_7, x_8, x_9\}\}$,
$C_3=\{\{x_1, x_2, x_3\},\{x_4, x_5, x_6, x_7, x_8, x_9\},\{x_8, x_9\}\}$,
$C_4=\{\{x_1, x_2, x_4, x_5\},\{x_2, x_3, x_5, x_6\},\{x_7, x_8\},\{x_5, x_6, x_8, x_9\}\}$
$U/D=\{\{x_1, x_2, x_3\}, \{x_4, x_5, x_6\}, \{x_7, x_8, x_9\}\}$
where, $\Delta_i=\Delta_{xi}$, P_i is P_{xi} (for short)

Step 1:
$\Delta_1=\{x_1, x_2\}$, $\Delta_2=\{x_2\}$, $\Delta_3=\{x_2, x_3\}$,
we have $d(\Delta_1) = d(\Delta_2) = d(\Delta_3) = 1$,
because $\Delta_1, \Delta_2, \Delta_3 \subseteq \{x_1, x2, x3\}$,
$\Delta_4=\{x_4, x_5\}$, $\Delta 5=\{x_5\}$, $\Delta 6=\{x_5, x_6\}$,
we have $d(\Delta_4) = d(\Delta_5) = d(\Delta_6) = 2$,
because $\Delta_4, \Delta_5, \Delta_6 \subseteq \{x_4, x_5, x_6\}$,
$\Delta_7=\{x_7, x_8\}$, $\Delta_8=\{x_8\}$, $\Delta_9=\{x_8, x_9\}$,
we have $d(\Delta_7) = d(\Delta_8) = d(\Delta_9) = 3$,
because $\Delta_7, \Delta_8, \Delta_9 \subseteq \{x_7,x_8, x_9\}$
$CI = 9 \Rightarrow S$ is consistent system.

Step 2:
$P - \{C_1\}$:
$P_1=\{x_1, x_2\}$, $P_2=\{x_2\}$, $P_3=\{x_2, x_3\}$,
$P_4=\{x_4,x_5\}$, $P_5=\{x_5\}$, $P_6=\{x_5, x_6\}$,
$P_7=\{x_7, x_8\}$, $P_8=\{x_8\}$, $P_9=\{x_8, x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|=0$$

$\Delta=\Delta - \{C_1\} = \{C_2, C_3, C_4\}$.

Step 3:
$P=\Delta - \{C_2\}$
$P_1=\{x_1, x_2\}$, $P_2=\{x_2\}$, $P_3=\{x_2, x_3\}$,
$P_4=\{x_4, x_5\}$, $P_5=\{x_5\}$, $P_6=\{x_5, x_6\}$,
$P_7=\{x_7, x_8\}$, $P_8=\{x_8\}$, $P_9=\{x_8, x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|=0$$

$\Delta=\Delta - \{C_2\} = \{C_3, C_4\}$

Step 4:
$P= \Delta - \{C_3\}$:
$P_1=\{x_1, x_2, x_4, x_5\}$, $P_2=\{x_2\}$, $P_3=\{x_2, x_3, x_5, x_6\}$,
$P_4=\{x_4, x_5\}$, $P_5=\{x_5\}$, $P_6=\{x_5, x_6\}$,
$P_7=\{x_4, x_5, x_7, x_8\}$, $P_8=\{x_5, x_8\}$, $P_9=\{x_5, x_6, x_8, x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|\neq 0$$

(we can see $(\Delta_1\cap\Delta_4)=\emptyset$, but $(P_1\cap P_4)\neq\emptyset$, $|d(\Delta_1)-d(\Delta_4)|\neq 0$)
$\Delta= \{C_3,C_4\}$.

Step 5:
$P= \Delta - \{C_4\}$
$P_1=\{x_1, x_2, x_3\}$, $P_2=\{x_1, x_2, x_3\}$, $P_3=\{x_1, x_2, x_3\}$,
$P_4=\{x_4, x_5, x_6, x_7, x_8, x_9\}$, $P_5=\{x_4, x_5, x_6, x_7, x_8, x_9\}$,
$P_6=\{x_4,x_5,x_6,x_7,x_8,x_9\}$
$P_7=\{x_7, x_8, x_9\}$, $P_8=\{x_7, x_8, x_9\}$, $P_9=\{x_7, x_8, x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|\neq 0$$

(we can see $(\Delta_6\cap\Delta_7) =\emptyset$, but $(P_6\cap P_7)\neq\emptyset$, $|d(\Delta_6)-d(\Delta_7)|\neq 0$)
$\Delta= \{C_3,C_4\}$.

Step 6:
$RD= \{C_3,C_4\}$ is a reduct. i.e. attributes with respect to C_1, C_2 are deleted.

B. Example for a inconsistent covering decision system

Suppose $U=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10}\}$ và $=\{C_i, i=1..4\}$
$C_1=\{\{x_1,x_2,x_3,x_4,x_6,x_7,x_8,x_9,x_{10}\},\{x_3,x_4,x_6,x_7\},\{x_3,x_4,x_5,x_6,x_7\}\}$
$C_2=\{\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\},\{x_6,x_7,x_8,x_9\},\{x_{10}\}\}$
$C_3=\{\{x_1,x_2,x_3,x_6,x_8,x_9,x_{10}\},\{x_2,x_3,x_4,x_5,x_6,x_7,x_9\}\}$
$C_4=\{\{x_1,x_2,x_3,x_6\},\{x_2,x_3,x_4,x_5,x_6,x_7\},\{x_6,x_8,x_9,x_{10}\},\{x_6,x_7,x_9\}\}$
$U/D=\{\{x_1,x_2,x_3,x_6\}, \{x_4,x_5,x_7\}, \{x_8,x_9,x_{10}\}\}$

Step 1:
$\Delta_1=\{x_1,x_2,x_3,x_6\}$; $\Delta_2=\{x_2,x_3,x_6\}$; $\Delta_3=\{x_3,x_6\}$;
$\Delta_4=\{x_3,x_4,x_6,x_7\}$; $\Delta_5=\{x_3,x_4,x_5,x_6,x_7\}$;$\Delta_6=\{x_6\}$;
$\Delta_7=\{x_6,x_7\}$; $\Delta_8=\{x_6,x_8,x_9\}$; $\Delta_9=\{x_6,x_9\}$; $\Delta_{10}=\{x_{10}\}$;
$CI \neq 9 \Rightarrow S$ is an inconsistent system.

Step 2: $P - \{C_1\}$:
$P_1=\{x_1,x_2,x_3,x_6\}$; $P_2=P_3=\{x_2,x_3,x_6\}$;
$P_4=P_5=\{x_2,x_3,x_4,x_5,x_6,x_7\}$;
$P_6=\{x_6\}$; $P_7=\{x_6,x_7\}$; $P_8=\{x_6,x_8,x_9\}$;
$P_9=\{x_6,x_9\}$; $P_{10}=\{x_{10}\}$;

$$\sum_{xi\in U}\left[\left|\frac{\Delta_{xi}\cap[x_i]_D}{\Delta_{xi}}\right|-\left|\frac{P_{xi}\cap[x_i]_D}{P_{xi}}\right|\right]=0$$

$\Delta=\Delta - \{C_1\}=\{C_2,C_3,C_4\}$. C_1 is dispensable.

Step 3: $P - \{C_2\}$
$P_1=\{x_1,x_2,x_3,x_6\}$; $P_2=P_3=\{x_2,x_3,x_6\}$;
$P_4=P_5=\{x_2,x_3,x_4,x_5,x_6,x_7\}$; $P_6=\{x_6\}$;
$P_7=\{x_2,x_3,x_4,x_5,x_6,x_7\}$; $P_8=\{x_6,x_8,x_9, x_{10}\}$;
$P_9=\{x_6,x_9\}$; $P_{10}=\{x_6,x_8,x_9, x_{10}\}$

$$\sum_{xi\in U}\left[\frac{|\Delta_{xi}\cap[x_i]_D|}{|\Delta_{xi}|}-\frac{|P_{xi}\cap[x_i]_D|}{|P_{xi}|}\right]\neq 0$$

C_2 is in dispensable. $\Delta=\{C_2,C_3,C_4\}$.

Step 4: $P - \{C_3\}$
$P_1=\{x_1,x_2,x_3,x_6\}$; $P_2=P_3=\{x_2,x_3,x_6\}$;
$P_4=P_5=\{x_2,x_3,x_4,x_5,x_6,x_7\}$; $P_6=\{x_6\}$;
$P_7=\{x_6,x_7\}$; $P_8=P_9=\{x_6,x_8,x_9\}$; $P_{10}=\{x_{10}\}$

$$\sum_{xi\in U}\left[\left|\frac{\Delta_{xi}\cap[x_i]_D}{\Delta_{xi}}\right|-\left|\frac{P_{xi}\cap[x_i]_D}{P_{xi}}\right|\right]=0$$

$\Delta=\Delta - \{C_3\}=\{C_2,C_4\}$. C_3 is dispensable

Step 5: $P - \{C_4\}$
$P_1= P_2=P_3= P_4=P_5=\{x_1, x_2,x_3,x_4,x_5,x_6,x_7\}$
$P_6= P_7=\{x_6,x_7\}$; $P_8=P_9=\{x_6, x_7,x_8,x_9\}$; $P_{10}=\{x_{10}\}$

$$\sum_{xi\in U}\left[\frac{|\Delta_{xi}\cap[x_i]_D|}{|\Delta_{xi}|}-\frac{|P_{xi}\cap[x_i]_D|}{|P_{xi}|}\right]\neq 0$$

C_4 is in dispensable. $\Delta=\{C_2,C_4\}$.

Step 6:
$RD= \{C_2,C_4\}$ is a reduct. i.e. attributes with respect to C_1, C_3 are deleted.

TABLE II. COMPARISION WITH RESULTS OF CHEN DEGANG ET AL

Algorithm of Chen Degang et al	New Algorithm
Example 1	
Red(Δ) = {{C_3, C_4}, {C_2, C_3}}	RD= {C_3,C_4}
Example 2	
Red(Δ) = {{C_2, C_4}, {C_2, C_3}}	RD= {C_2,C_4}

Note: Where Red(Δ) = Collection all reducts of Δ; RD is a reduct of Δ

V. CONCLUSIONS

In this paper, we propose a new attribute reduction algorithm. It based on results of Chen Degang et al in consistent and inconsistent covering decision system. The time complexity of this algorithm is $O(|\Delta||U|^2)$. Compare with the results of Cheng Degang's the algorithm, our result is compatible (Table II). In next time, we will experiment with UCI databases and compare with different algorithms. We also study algorithms which are developed from the theory of traditional rough sets.

REFERENCES

[1] Chen Degang, Wang Changzhong, Hu Quinghua, "A new approach to attribute reduction of consistent and inconsistent covering rough sets", Information Sciences, 177 (2007) 3500-3518

[2] Guo-Yin Wang, Jun Zhao, Jiu-Jiang An, Yu Wu, "Theoretical study on attribute reduction of rough set theory: Comparision of algebra and information views", Proceedings of the Third IEEE International Conference on Cognitive Informatics (ICCI'04), 2004

[3] Ruizhi Wang, Duoqian Miao, Guirong Hu, "Descernibility Matrix Based Algorithm for Reduction of Attributes", IAT Workshops 2006: 477- 480

[4] Yiyu Yao, Yan Zhao, Jue Wang, "On Reduct Construction Algorithms, Rough sets and Knowledge Technology", First International Conference, RSKT2006, Proceedings, LNAI 4062, pp 207-304, 2006

[5] Zhi Kong, Liqun Gao, Qingli Wang, Lifu Wang, Yang Li, "Uncertainty Measurement Based on General relation", In 2008 American Control Conference, Westin Sealtle Hotel, Seatle, Washington, USA, FrA102.3:3650-3653, June 11-13,2008,