International Journal of Computer Theory and Engineering, Vol. 1, No. 4, October, 2009
1793-8201

# Fuzzy Mathematical Model for Utilization of the Resources

Manoj Kumar Jain, A. K. Dalela and Sandeep Kumar Tiwari

***Abstract*—In current era, overcrowding has become a problem to control customer traffic and their demand. Every day, customer is demanding new verity of service from service provider and they are not satisfied with services which are getting from any service provider. Resource planning is a critical task for management. It identifies how much resources are needed to match future customer traffic demand. It directly affects customer satisfaction and revenues. In this paper, we present Fuzzy mathematical model for utilization of the resources, which helps alleviate a big problem that management and marketing executive's face– making optimal resource investment decisions and their capacity utilization [3],[4] and [7].**

**This paper helps, the prediction process by presenting actual customer traffic usage from a business perspective, in a form that is useful to both management and marketing executives. We show that information from this model can be used to: (1) quantify revenue earned on each unit of branch,(2) quantify return-on-investment on performing a resource upgrade, and (3) quantify the loss due to customer dissatisfaction when a unit of branch is not upgraded. We also illustrate how these formulations based on business information can be used to improve resource planning decisions [1], [2], [4], and [16].**

**Effective Resource Management increases profitability by optimizing utilization and minimizing bench time of branch resources, and generates goodwill and loyalty among your staff that translates to competitive advantages in recruiting and retaining the best talent.**

***Index Terms*—**ATF: Acceptable total flow

Notation:

- $I$ - a set of time intervals,
- $J$ - a set of customer arrival ,
- $K$ - a set of customer departure ,
- $a^j_i$ - arrival demand through the *j*-th customer during the *i*-th time interval,
- $d^k_i$- departure demand through the *k*-th customer during the *i*-th time interval,
- $X^j_i$- queue at the *j*-th customer arrival during the *i*-th time interval,
- $Y^k_i$ - queue at the *k*-th customer departure during the *i*-th time interval,
- $w^j_i$- flow through the *j*-th customer arrival during the *i*-th time interval,
- $z^k_i$ - flow through the *k*-th customer departure during the *i*-th time interval.

School of Studies in Mathematics, Vikram University, Ujjain. M.P, INDIA,
Department of Mathematics, Govt Science College, Jabalpur, M. P., INDIA,
School of Studies in Mathematics, Vikram University, Ujjain. M..P., INDIA

## I. INTRODUCTION

The explosive increase in the number of customer as well as in volume of services poses significant challenges to the management and, by extension, to the service providers. Service providers are faced with two challenges today. *(1) They want larger number of customers in order to increase revenues. (2) They want to provide better service to customer efficiently*. Resource planning plays a crucial role in helping service providers tackle these challenges. Resource planning is the process of predicting tomorrow's needs and preparing for them today. Resource planning involves combining marketing information and traffic analysis results to predict future resource requirements [11], [12], and [16].

Intelligent resource planning can result in enormous cost savings and increased customer satisfaction. The importance of resource planning cannot be over-emphasized. Resource planning for service provider requires good understanding of customer traffic growth. The overall traffic growth depends on the number of new customer on the branch as well as the demand increase per customer. Predicting these growth factors determines the decision on the investment size for upgrading resources. Currently, there is no standardized process to combine both growth factors. Prediction of traffic growth and intelligent decision making can be greatly facilitated by correlating customer data with business relevant information [8], [10] and [11]. Such information can be classified into:

1. Customer service information: Considering service of individual customer (or set of customer segments) reveals more information than considering the aggregated services of all subscribers [20].
2. Value of the customer to the business: The revenue and costs of an individual customer are important in evaluating the value of an investment. Clearly, an investment is worth while only if the revenue per customer outweighs the costs incurred per customer [2], [9] and [10].
3. Competition in the market: Since competition in different geographical regions can impact customer loyalty, it is important to consider its impact before making costly investment decisions.

By associating such business information with customer traffic data, the service provider can make better decisions about investments. However, today's resource planning for the branch does not incorporate this kind of information. The dominant reason is that the data collection and analysis process is not in place. The current process suffers from the problem that the customer volume overwhelms conventional management systems. Moreover, marketing and engineering disciplines of the service provider business lack a common vision. Marketing managers concentrate only on customer numbers and ignore the traffic-volume aspects of the business, while Management concentrate only on volumes of business and ignore the customer aspect of the business. Each discipline views data in isolation, which results in myopic decision making [17], [19], and [20].

A holistic view of the data is necessary for informed resource investment decisions and their utilization. In this paper, we present, fuzzy mathematical model for utilization of the resources that overcomes these lacunae. We show how, this model can be used to combine marketing, revenue, and management aspects of a service provider's business in order to make efficient resource planning decisions. In this paper, we only focus on utilization of resource planning for regional branch network. Our solutions are directly applicable to resource planning in backbone networks [6], [15], and [17]. Highlight a section that you want to designate with a certain style, then select the appropriate name on the style menu. The style will adjust your fonts and line spacing. **Do not change the font sizes or line spacing to squeeze more text into a limited number of pages.** Use italics for emphasis; do not underline.

To insert images in *Word,* position the cursor at the insertion point and either use Insert | Picture | From File or copy the image to the Windows clipboard and then Edit | Paste Special | Picture (with "Float over text" unchecked).

INTERNATIONAL JOURNAL OF COMPUTER THEORY AND ENGINEERING reserves the right to do the final formatting of your paper.

## II. Resource Planning

Resource planning has three phases–*(1) predicting future growth in customers, (2) predicting future volume of traffic in regional branch, and (3) planning resource upgrades for the future.* In the first phase, the marketing team estimates how many new customers will join the service to particular branch and how many old customers will leave the service. The marketing team can use historical growth patterns, advertising budget, channel strength and customer satisfaction reviews, etc., to determine future growth and churn. This allows prediction of total number of customer in the particular branch. In the second phase, manager translates the number of customers into possible customer passage. This helps identify hot-spots in the branch. Once the hot-spots are identified, in the third phase, the service provider must decide where investments are necessary in order to provide a good service to customers [4].

For example, one simple approach to make investment decisions could be the following. The service provider sets a policy that all unit of branch should have overall utilization less than a threshold, $\tau_{suggested}$. Consider branch l with **resource** $\beta_l$ that has a projected volume of traffic $b_{l,future}$ bits over the future time period $T_{future}$. Therefore, overall utilization of **l** denoted by $\tau_{l,future}$ be computed as $b_{l,future} / (T_{future} * \beta_l)$. If this exceeds $\tau_{suggested}$ then the service provider marks the unit to be upgraded. The quantum of upgrade, $\chi_l$ should be greater than $(1/\tau_{suggested}) * (b_{l,future} / T_{future}) - \beta_l$. This is obtained by solving the following inequality representing the service provider's policy decision:

$$\tau_{suggested} \geq \frac{b_{l,future}}{T_{future}(\beta_l + \chi_l)} \qquad (1)$$

Using some such mechanism, the service provider can construct a set of resources ***S*** that need to be upgraded/purchased, and also determine how many se resources are required for particular branch. In this paper, we assume that the service provider has done some analysis to determine this initial set ***S*** of resources that need to be upgraded /purchased [4], [11], and [15].

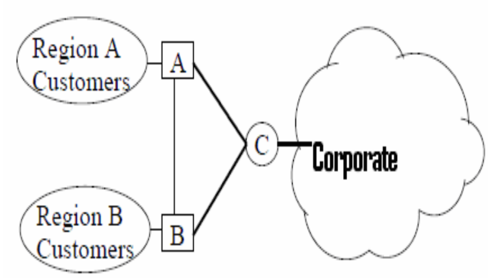

Fig. 1: Resource Planning Example

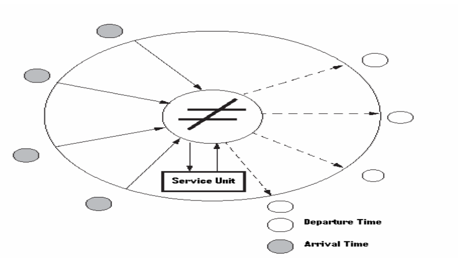

Fig. 2: Customer Service for Branch

## III. Statement of the Problem

We consider branch operations within time period ***T*** during one specific day. The system considered in this paper is comprised of $n_{af}$ *customer* arrival time and $n_{df}$ *customer* departure time and a branch resource system. A simple layout of the branch area along with the arrival and departure time is shown in Figure [1], [2]. The arrival time serve only the arrival flow of customers and the departure time support only the departure flow of customers. The arrival customers have to pass through pre-assigned arrival time before coming to branch, and the departing customer also have to pass through pre-assigned departure time after they leave the resources. The branch resource system, however, handles both arrival and departure flows. The arrival queues are formed at the arrival time. After passing through the arrival time, the arriving customers are accepted at the resource without any further delays.

The departure queues are formed before the branch resource system and the departure customer can therefore be delayed either at their gates or on the parking stand. The arrival and departure capacity indicates the maximum number of customers that can go through a branch in a time unit. Times usually have different utilization; i.e., traffic is usually not allocated evenly among times. Appropriate coordination among resources and times seems to be necessary in order to properly use available resources' and their capacities.

The problem considered in this paper can be defined as follows: For a given customer departure and arrival demand, and approximately known branch resource and customer service, calculate the real values of branch arrival and departure capacities to be used in operations and flows through the arrival and departure time in order to minimize total customer delay over a considered time period.

The proposed Fuzzy Mathematical Model for Utilization of the resources to calculates the optimal values of the branch arrival and departure capacity of customers, and optimal customer flows through departure and arrival time in 60-minute intervals.

## IV. Proposed Solution to the Problem

Generally make a trade-off between arrival and departure capacities of the Branch.

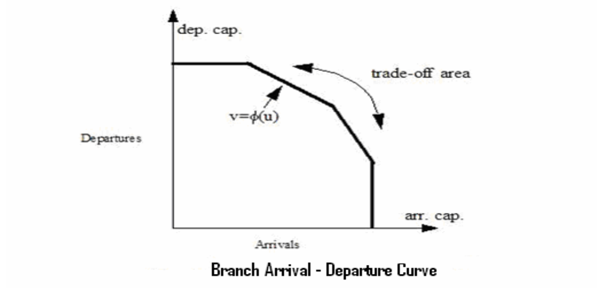


Fig. 3: Branch Capacity Curve

In other words, the number of arrivals and departures actually taking place is somewhere between the maximum number of arrivals and the maximum number of departures. Let us divide time period $\boldsymbol{T}$ into $\boldsymbol{N}$ discrete time intervals whose length is $\boldsymbol{\delta t}$ . We assume that the branch arrival capacity at the $i^{th}$ time interval, $\boldsymbol{U_i}$ is characterized by uncertainty. This capacity can be represented as a triangular fuzzy number $\boldsymbol{U_i = (u_{1i}, u_{2i}, u_{3i})}$, where $\boldsymbol{u_{1i}}$ is lower (left) boundary of the triangular fuzzy number, $\boldsymbol{u_2 i}$ is number corresponding to the highest level of presumption, and $u_{3i}$ is upper (right) boundary of the fuzzy number in figure 4

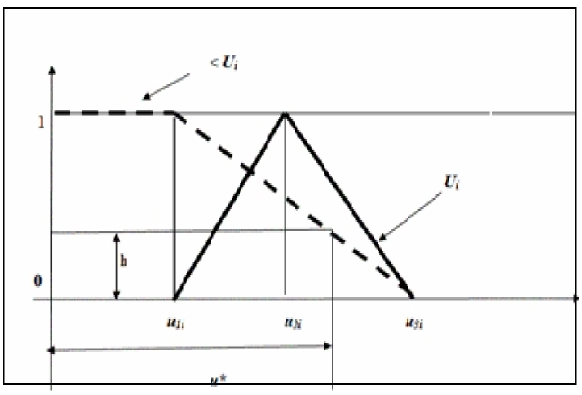


Fig. 4: The Triangular Fuzzy Number Ui that represents Arrival Capacity, and the Fuzzy Number "Less than Ui"(<Ui)

Based on experience or intuition, an expert is able to state that, for example, branch capacity is "around 30 customers per hour". The branch arrival capacity at the $\boldsymbol{i}$-th time interval $\boldsymbol{U_i}$ and the branch departure capacity at the $\boldsymbol{i}$-th time interval $\boldsymbol{V_i}$ are interdependent. The branch departure capacity at the $\boldsymbol{i}$-th time interval $\boldsymbol{V_i}$ is also represented by a triangular fuzzy number, $\boldsymbol{V_i = (v_{1i}, v_{2i}, v_{3i})}$ The values $\mathbf{v_{1i}, v_{2i}, v_{3i}}$, , can be calculated using fuzzy arithmetic rules [10], [18] once we know the values of $u_{1i}$, $u_2 i$, *and* $u_{3i}$. Similarly, we represent the capacity of the $j$-th arrival $\boldsymbol{F^j_{Ai} = (f^j_{A1}, f^j_{A2}, f^j_{A3})}$, the capacity of the $k$-th departure $F^j_{Di} = (f^j_{D1}, f^j_{D2}, f^Z_{D3})$, at the $\boldsymbol{i}$-th time interval as triangular fuzzy numbers.

Let us explain, the difference between "crisp" and fuzzy" constraints are using the following example. The following crisp constraint is used in the traditional resource capacity utilization models:

$$\sum_{i=1}^{n_{af}} w_i^j \le u_i \tag{1}$$

This constraint states that the sum of the arrival flows $\sum_{i=1}^{n_{af}} w_i^j$ during the $i^{th}$ time interval must be less than or equal to the branch arrival capacity $\boldsymbol{(u_i)}$. In this case the branch capacity is treated as a deterministic quantity. In an attempt to adequately represent uncertainty, we treat the branch arrival capacity ($u_i$) as the fuzzy number $\boldsymbol{U_i}$. Figure 4 also shows on the ordinate axis the level of satisfaction $\boldsymbol{h\ (0 \le h \le 1)}$ that we wish to achieve. This level of satisfying the constraint, $h$, can be achieved when the $\sum_{i=1}^{n_{af}} w_i^j$ is less than or equal to the highest value $\boldsymbol{(u^*)}$ of the fuzzy number $\boldsymbol{< U_i}$ for this level of satisfaction. The highest value $\mathbf{u^*}$ of the fuzzy number $< \boldsymbol{U_i}$ for this level of satisfaction can be obtained from the similarity of triangles (Figure 4). Therefore:

$$\sum_{j=1}^{n_{af}} w_i^j \le u_{3i} - h(u_{3i} - u_{1i}) \tag{2}$$

In the same way, we assume that the capacity of the $\boldsymbol{j}$-th arrival fix and the capacity of the $k$-th departure fix at the $\boldsymbol{i}$-th time interval are characterized by uncertainty. We also represent these capacities as triangular fuzzy numbers. When optimizing utilization of resources, we tried to minimize the total customer delay during the considered time period while taking care of branch and their capacities.

**Objective Function:**

$$\sum_{i=1}^{N} \gamma_i \left[ \sum_{p=1}^{i} \left( \alpha_i \sum_{j=1}^{n_{af}} w_p^j + (1-\alpha_i) \sum_{k=1}^{n_{af}} z_p^k \right) \right]$$

It is represents the total flow through all departure and arrival customer into branch for all the intervals. The weight given to the arrivals is denoted by $\boldsymbol{\alpha_{i.}}$, the weight for the departures equals $\mathbf{(1- \alpha_{i.})}$, while $\gamma_i$ represents the weight given to the *i*-th time interval **($0 \le \gamma_i \le 1$)**. Total arrival flow through any time can never exceed the sum of total demand and the initial arrival queue. Similarly, the total arrival flow cannot exceed the arrival capacity of the branch. Similar constraints must also exist for the departure flows. Arrival and departure flows must always be less than or equal to the arrival and departure capacities of branch respectively. Let us introduce "acceptable total flow" into the discussion. In other words, instead of maximizing total flow, we will try to generate acceptable total flow with a level of satisfaction at least equal to ***h***. We define "acceptable total flow" as a triangular fuzzy number **ATF = $(t_1, t_2, t_3)$.**

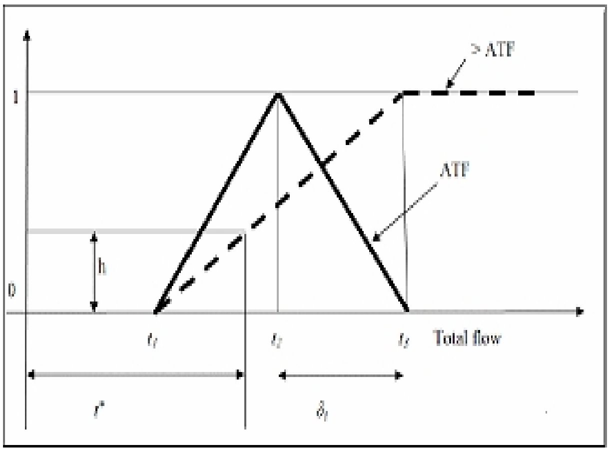


Fig. 5: - "Acceptable Total Flow" (ATF) and Total Flow Greater than "Acceptable" (>ATF)

From Figure:5 we see that a "flow greater than satisfactory" will be achieved with a level of satisfaction at least equal to ***h***, if:

$$\sum_{i=1}\gamma_i\left[\sum_{p=1}^{i}\left(\alpha_i\sum_{j=1}^{n_{af}}w_p^j+(1-\alpha_i)\sum_{k=1}^{n_{df}}z_p^k\right)\right]\ge t_1+h(t_3-t_1) \qquad (3)$$

In other words, our objective function has become a constraint, which agrees completely with Bellman and Zadeh (1970) whereby both objective functions and constraints are in a fuzzy environment are treated in the same way. Since we have transformed the objective function into a constraint, the question arises of defining a new objective function. We will naturally try to find a solution that maximizes the level of satisfying both the objective function and the constraint, ***h***. In other words, in order to determine the optimal solution that satisfies both the objective function and constraint by the maximum possible degree ***h***, a fuzzy optimization principle is applied by which ***h*** is maximized:

**Subject to:**

$$\sum_{i=1}^{N}\gamma_i\left[\sum_{p=1}^{i}\left(\alpha_i\sum_{j=1}^{n_{af}}w_p^j+(1-\alpha_i)\sum_{k=1}^{n_{df}}z_p^k\right)\right]\ge t_1+h(t_3-t_1) \qquad (4)$$

$$\sum_{p=1}^{i}w_p^j\le X_1^j+\sum_{p=1}^{i}a_p^j\ ,\ i\in I, j\in J \qquad (5)$$

$$\sum_{j=1}^{n_{af}}w_i^j\le u_{3i}-h(u_{3i}-u_{1i}),\quad i\in I \qquad (6)$$

$$\sum_{p=1}^{i}z_p^k\le Y_1^k+\sum_{p=1}^{i}d_p^k\ ,\ i\in I, k\in K \qquad (7)$$

$$\sum_{k=1}^{n_{df}}z_i^k\le v_{3i}-h(v_{3i}-v_{1i}),\ i\in I \qquad (8)$$

$$w_i^j\le f_{A_{3i}}^j-h(f_{A_{3i}}^j-f_{A_{1i}}^j),\ i\in I, j\in J \qquad (9)$$

$$z_i^k\le f_{D_{3i}}^k-h(f_{D_{3i}}^k-f_{D_{1i}}^k),\ i\in I, k\in K \qquad (10)$$

The obtained flows in the fuzzy optimization model correspond to a certain level of satisfaction of *h*. By shifting the "acceptable total flow" to the left, the level of satisfaction could be increased. In other words, the achieved level of satisfaction *h* highly depends on the "acceptable total flow" set up by the decision maker. If we are prepared to accept the fact that all constraints are not completely satisfied, we can considerably increase the total flow. Every pair (*h*, *t2*) corresponds to a certain traffic flow pattern. In this manner, a large number of different traffic flow patterns are generated for the decision maker. The telecom and banking sector are used in this paper to demonstrate model performance. This model can be use in shopping mall or big store where customer strength is very high.

## V. Conclusion

Customer satisfaction does not come about overnight or by accident, it requires careful planning and execution. Owing to its supreme importance, it would be necessary to have some ways of assessing or measuring its degree or extent of achievement. In recent years, customers, suppliers, and survey practitioners have different views on assessing and measuring customer satisfaction because the approach and emphasis might vary with different market sectors, dependent upon the individual product types and commercial considerations.

Now days, customer expectation are very high. They are looking prompt service from service provider. These days, they are not satisfied with services which are expecting from any service provider. Resource planning is a critical task for management. It identifies how much resources are needed to match future customer traffic demand. It directly affects customer satisfaction and revenues. In this paper, we have described a new approach to utilization of resources. We illustrated the power of combining marketing, revenue, and customer usage information. Commercial company always interested more profit and they always target to put less investment on resources. In this paper, fuzzy mathematical model for utilization of the resources has been developed and it has been tested on telecom and banking sector. The result obtain by this model is very promising.

## REFERENCES

[1] Bellman, R., Zadeh, L. “Decision making in a fuzzy environment”, Management Science, 17, B144-B164, (1970).
[2] D. A. Menasce and V. A. F. Almeida, “Capacity Planning forWeb Services: Metrics, Models, and Methods”, Prentice Hall PTR, 2002.
[3] Demeulemeester E, Herroelen W. Project scheduling: a research handbook. Boston: Kluwer Academic Publishers; 2002. ISBN: 1402070519
[4] Drexl A, Kimms A. Optimization guided lower & upper bounds for the resource investment problem. Journal of the operational Research Society. 2001;52(3): 340–351. OI: 10.1057 /palgrave. jors. 2601099
[5] IPAT, http://www.wandl.com/html/ipat/IPAT new.cfm
[6] J. Altmann and L. Rhodes, “Dynamic Netvalue Analyzer - A Pricing Plan Modeling Tool for ISPs Using Actual Network Usage Data”, IEEE WECWIS2002, International Workshop on Advance Issues of E-Commerce and Web-Based Information Systems, 2002.
[7] J. Case, K. McCloghrie, M. Rose and S. Waldbusser, “Protocol Operations for Version 2 of the Simple Network Management Protocol (SNMPv2)”, RFC 1905, 1996.
[8] J. Hershberger and S. Suri, “Vickrey Prices and Shortest Paths: What is an Edge Worth?”,IEEE Symposium on Foundations of Computer Science, 2001.
[9] J.Hershberger, S. Suri and A. Bhosle., “On the Difficulty of some Shortest Path Problems,” STACS, Berlin, 2003.
[10] Kaufmann, A., Gupta, M. “Introduction to Fuzzy Arithmetic”, Van Nostrand Reinhold Company, New York, (1985).
[11] Najafi AA, Niaki STA. A genetic algorithm for resource investment problem with discounted cash flows. Applied Mathematics and Computations. 2006;183(2): 1057-1070. DOI:10. 1016/ j.amc.2006.05.118
[12] Nadjafi BA, Shadrokh S. An algorithm for the weighted earliness-tardiness unconstrained project scheduling problem. Journal of Applied Sciences. 2008; 8(9): 1651-1659. DOI: 10.3923 /jas .2008.1651.1659
[13] NetRule, http://www.analyticalengines.com/
[14] Opnet, http://www.opnet.com .
[15] S. Keshav, “An Engineering Approach to Computer Networking: ATM Networks, the Internet, and the Telephone Network”, Addison-Wesley, 1997.
[16] Shadrokh S, Kianfar F. A genetic algorithm for resource investment project scheduling problem, tardiness permitted with penalty. European Journal of Operational Research. 2007;181(1): 86-101. DOI:10.1016/j.ejor.2006.03.056
[17] T. Robertazzi, “Planning Telecommunication Networks”, Wiley-IEEE Press, February1999.
[18] Teodorovic, D., Vukadinovic, K. “Traffic Control and Transport Planning: A Fuzzy Sets and Neural Networks Approach”, Kluwer Academic Publishers, Boston/Dordrecht/London, (1998).
[19] Waligora G. Discrete-continuous project scheduling with discounted cash flows-a tabu search approach. Computers and Operations Research. 2008;35(7): 2141-2153. DOI:10.1016/ j.cor.2006.09.022
[20] Y. Diao, J. L. Hellerstein, and S. Parekh, “A Business-Oriented Approach to the Design of Feedback Loops for Performance Management”, Distributed Operations and Management,2001.