Mutual Information based Rigid Medical Image registration using Normalized Tsallis entropy and Type II fuzzy index.

Mohanalin, Prem Kumar Kalra and Nirmal Kumar

***Abstract*— We investigated the registration of medical images based on the Normalized Tsallis entropy using mutual information measure. A prerequisite for successful registration is unambiguous maximum of mutual information. We discuss the framework of our algorithm with Normalized Tsallis entropy as the core. Further we propose a type II fuzzy based technique to select the optimal Tsallis parameter q which provides the best alignment. Consequently, specific instances of image registration involving rigid affine transformations were studied. Registration was applied to clinically acquired mammogram. The accuracy was compared with several other techniques. Our algorithm shows promising results. Further, the Need for Pre-registration in mammogram is discussed in detail. Our algorithm can be effective enough to replace Shannon and Tsallis entropy based affine registration.**

***Index Terms*— Tsallis entropy, Shannon entropy,Normalized Tsallis entropy, Joint intensity distribution,image registration, Powell optimization, Mammogram.**

I. INTRODUCTION

Image registration is the determination of geometrical transformation that aligns points of one view of an object with the corresponding points in another view of that object. i.e. output is a geometrical transformation which is simply a mathematical mapping from one points to points in second. There are many image registration methods and they can be classified into many ways. Mutual information (MI)based technique is the most popular technique, because MI does not rely on the intensity values directly to measure correspondence between different images, but on their relative occurrence in each of the images separately and co-occurrence in both images combined. MI is insensitive to one-to-one intensity transformations and is capable of managing positive and negative intensity correlations simultaneously. MI is not based on intensity differences or intensity correlation, like other pixel-based registration criteria. The purpose of the registration (rigid) using MI is to reduce global spatial differences between corresponding images caused by the positioning difference during their acquisition. By finding a set of parameters, tx, ty, and θ, capable of maximizing the MI between the two images, the best registration location is found. It has been found that MI based similarity measure can still fail for certain clinical images. Improved performance is detected by various normalization schemes. Current Validation also supports this theory. This paper shows the application of normalized Tsallis entropy (NTE) as a new method of rigid registration instead of traditional Shannon entropy (SE). The Paper is organized as follows. Section 2 Deals with the literature review of recent work in rigid image registration. Section 3 explains basics of our algorithm, and details of Normalized entropy. A new method to find optimal Tsallis parameter using Type II fuzzy set is proposed. The proposed algorithm is checked for mammograms with known simulated deformation and results with several validation techniques were shown in section 5. Section 6 contains the conclusion.

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II. BRIEF SURVEY OF EXISTING WORKS IN IMAGE REGISTRATION:

The Shannon-MI has received tremendous attention **[1]** and is robust and accurate in registering images. Viola et al. **[2,3]** was the first one to propose Image registration based on MI. Studholme et al. **[4]** introduced normalized mutual information (NMI) to rigidly register multi-modal images with different fields of view; Pluim et al. **[5]** used a gradient-based term along with the MI to avoid the problems of local maxima. Rueckert et al. Maintz et al. **[6]** used correlation for multi-modal image registration. In recent years NMI has proven to be a robust and accurate similarity measure for image registration **[7].** In **[8],** Rangarajan et al. proposed feature point registration with MI. Rényi entropy **[9, 10]** and Tsallis mutual information (Tsallis-MI) **[11,12,13],** are the other two entropy techniques showing promising results, consequently their properties make them conductive to medical image registration. MI, was commonly used to solve global spatial differences between mammograms **[14].**

III. REGISTRATION ALGORITHM

Let one of the images selected be the reference image, R(x,y) and other image be target image T(x,y). The MI based methods state that, for two images that are to be registered, the value of their MI will be maximal if the images are geometrically aligned. The NMI of two images is expressed in terms of the entropy of the images. Entropy is a measure of uncertainty of the prediction of the intensity of a point in an image: for example, the entropy of a homogeneous image is zero since there is no uncertainty about the intensity of any of its pixel. On the contrary, an image containing a large number of equally distributed intensities has high entropy. The entropy terms needed for the computation of the NMI can be derived from the joint histogram, which is an estimation of the joint probability distribution of the intensities of two images. Joint histogram denotes the number of time that intensity couples occur at corresponding positions in the images. To test the algorithm, images were transformed by a rigid-body transformation scheme given a vector T equal to [tx, ty, θ], which corresponds to translations over x- and y-axes and a rotation through the centre of the image with an angle θ . Spline interpolation was used to fit the Target image T(x,y) to the grid of the reference image R(x,y) after each transformation.

A. Important steps of our algorithm:

1) Vary Tsallis parameter ‘q’ from 0.1 to 0.9.
2) Find MI using NTE for every possible θ and translational distances t_x, t_y. Calculate the best 9 possible transformation for various q.
3) Use Type II fuzzy set to calculate the fuzzy indices of floating images and reference image.
4) Find the maximum of ratio between fuzzy index of original image and floating images.
5) The maximum index denotes the correct Tsallis parameter q and the final θ and translational distances t_x, t_y 6) Interpolate to the new location using Spline based approach.

B. Normalized Tsallis entropy as similarity measure:

The generalized form of TE is written as:

$$S_q = \frac{1}{1-q} * \left(\sum_{i=1}^{K} (p_i)^q - 1 \right) \qquad \textbf{(1)}$$

Where K is the total number of possibilities of the system and the real number q is an entropic index that characterizes the degree of non-extensivity. The entropic index q characterizes the degree of non-extensivity of the system through the pseudo-additivity. NTE can be written from **(1)** as follows

$$S_A{}^{NT}{}_q[p] = \frac{S^A{}_q[p]}{\sum_{i=1}^{k} (p_i)^q} \qquad \textbf{(2)}$$

$$= \left(\frac{1}{1-q}\right) * \left(1 - \frac{1}{\sum_{i=1}^{k} (p_i)^q} \right)$$

NTE Based MI=

$$S^{NT}{}_q(u) + S^{NT}{}_q(v) - (1-q) * S^{NT}{}_q(u) S^{NT}{}_q(v) - S^{NT}{}_q(u,v) \qquad \textbf{(3)}$$

The NTE measure tends to become minimum when the two distributions become equal. The registration procedure is an iterative process, and is terminated when NTE becomes sufficiently small. [15,16,17] Proves, TE, and the NTE all converge to the Boltzmann–Gibbs–Shannon entropy in the limit q → 1. Secondly, NTE are concave only for q € (0, 1), whereas the TE is always concave for any positive values of q, which makes more difficult to select the optimal value of q. In literature it is often found that value of q is calculated based on trial and error method.

C. Type II fuzzy set for perfect image registration.

As the selection of Tsallis parameter is not crisp, the selection of optimal q is done using Type II fuzzy set. Otherwise selection of automatic values of q seems to be impossible. To remove the uncertainty in q selection fuzzy theory seems to be beneficial. **[18]** Explained that type I fuzzy is still fuzzy, and termed it as ultra fuzziness. The basic idea behind this phase is to fuzzify the fuzzified image and find the fuzzy number which indicates how much the fuzzified image is fuzzy. Let θ_A be the best fit declared by using MI. But vagueness prevails about selection of q parameter as it can vary from 0.1 to 0.9. This implies 9 best fits will be declared.

i.e
$$\left\{ \begin{array}{l} q_{0.1} \text{ for } q = .1 \\ q_{0.2} \text{ for } q = .2 \\ q_{0.3} \text{ for } q = .3 \\ . \\ . \\ . \\ q_{0.9} \text{ for } q = .9 \end{array} \right\}$$

The fuzzy index for original image is found using type II fuzzy set as follows.

$$\Upsilon\left(A\right) = \frac{1}{(M*N)} \sum_{g=0}^{L-1} h(g) * [\mu_U(g) - \mu_L(g)] \qquad \textbf{(4)}$$

The fuzzy index for floating images is found using type II fuzzy set as follows.

$$\Upsilon\left(B_i\right) = \frac{1}{(M*N)} \sum_{g=0}^{L-1} h(g) * [\mu_U(g) - \mu_L(g)] \qquad \textbf{(5)}$$

$$\begin{array}{l} where \\ \mu_U(g) = [\mu_B(g)]^{1/\alpha}, \\ \mu_L(g) = [\mu_B(g)]^{\alpha}, \\ \alpha \in (0,2] \\ i \in [0.1 ... .9] \end{array}$$

$$\Psi = \max \left\{ \frac{\Upsilon(A)}{\Upsilon(B_1)}, \frac{\Upsilon(A)}{\Upsilon(B_2)}, \ldots\ldots, \frac{\Upsilon(A)}{\Upsilon(B_9)} \right\} \qquad \textbf{(6)}$$

μu(g), μL(g) Upper, Lower Membership function found from Gaussian Membership function respectively. Γ(A) is the fuzzy index of original image, γ(Bi) is the fuzzy indices of i transformed images, h(g)-histogram of image, for a M*N-Dimensions of image. Ratio between the indexes gives the similarity measure between reference image and floating image. The ratio will become unity in ideal case when images are aligned perfectly. Optimal value of q corresponding to the maximum index denotes optimal θ, tx, and ty. **(6)** Calculates the max index of the vagueness of image under study for various q values ranging from 0.1 to 0.9 for best θ and translational distances tx, ty. **Figure 1** shows the fuzzy and ultra fuzzy membership functions **[18]**. Choosing a proper membership function is an application dependent problem. Some most commonly used membership functions are cone, exponential, and Cauchy function. Two factors are considered when we select the membership function for our algorithm: registration accuracy and computational intensity for evaluation a membership function.

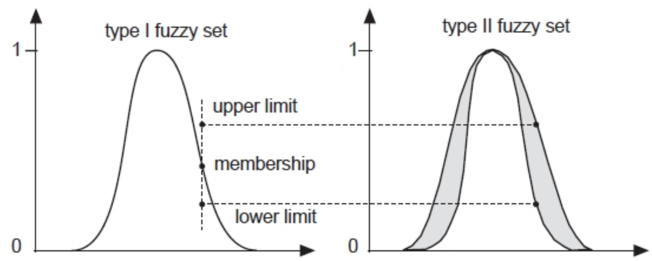


Figure1 shows Type I fuzzy set and Type II fuzzy set. [18]

We chose the Gaussian function due to its good expressiveness and high-computational efficiency. A cone function will not ensure that all inputs are fuzzified in some class. The intensities of an image are fuzzified to an interval [0, 1]. The Gaussian function can be written as

$$\mu_{m,n} = \exp\left(-\frac{\left|\left(x_{avg} - x_{m,n}\right)\right|^2}{2 * f_h^{\,2}}\right) \qquad \textbf{(7)}$$

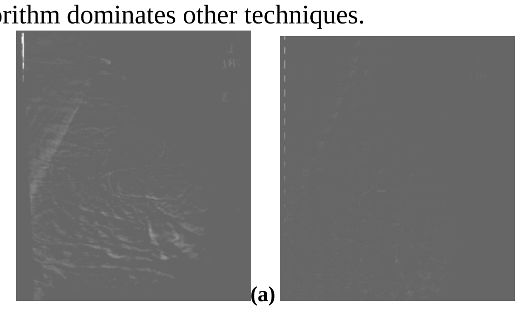
$X_{m,n}$ -Pixel value at (m,n) position, X_{avg} - Average of image and f^2_{h} - width of the Gaussian membership function. The value of f^2_h could not be too large or too small. We chose $f^2_h=2$. The algorithm can be substantially sped up if the θ ,t_x, and t_y can be guessed approximately from the knowledge of images. Powell's method is used to minimize distance between two functions **[19].** This is reasonable between robustness and speed. Because Powell is the most frequently used algorithms in this context, we used only this.

IV. APPLICATION TO MAMMOGRAMS. A NEED FOR PRE-REGISTRATION:

Mammogram registration is an important step in automatic detection of breast cancer. One critical step in mammogram registration is pre-registration step. Pre-registration is done to reduce the global differences, such that to make the Non-rigid registration procedure effective and accurate. The goal of the pre-registration is to find the optimal transformation to minimize the distance between the template image and floating image. To perform accurately, non-rigid registration schemes must need a good starting point and consequently, in general, some sort of pre-registration has to be performed **[20]**. Typically, an affine linear registration scheme is performed as pre-registration step. If the initial alignment is wayward, the non-linear matching procedure may perform poorly, i.e., does not converge to the needed result. Therefore, a good pre-registration step can be an important issue before performing the non-rigid approaches. In this research, a new MI based image registration technique is developed. The distance is measured using the measure based on NTE. Experimental results are shown in **Figure 2** by comparing with several other techniques, the proposed method is computationally more efficient without sacrificing registration accuracy. We preprocess mammograms using **[21,22]** to remove the unwanted noise from mammogram thus becoming noise free.

V. VALIDATION OF RESULTS:

Correlation Coefficient, Sum of squared differences, L-norm, Kernel Density Estimate plot, Difference image are the various important measures used in image registration to validate. For a perfectly aligned image the L-Norm of difference between original image and simulated deformed image should be zero. But due to interpolation effects it may not be zero but closer to zero. Table 1 indicates the different values of L-norm of images before and after rigid registration is performed. Kernel Density Estimate KDE is a perfect tool to measure the perfect ness of rigid registration. For a perfectly registered image the KDE plot will be concentrated as a single line as shown in figure **(4)**. **Figure (4)** shows KDE plot of unregistered image and results of other registering techniques. NTE shows almost a perfect result when compared to other techniques. Variation of NTE's KDE plot from ideal plot is due to interpolation effects. . The difference of registered image and original image gives the visual measure of registration. **Figure (3)** shows the difference image of mdb 026 of MIAS database. The **table (1)** displays various validation measures. Results show that our algorithm dominates other techniques.

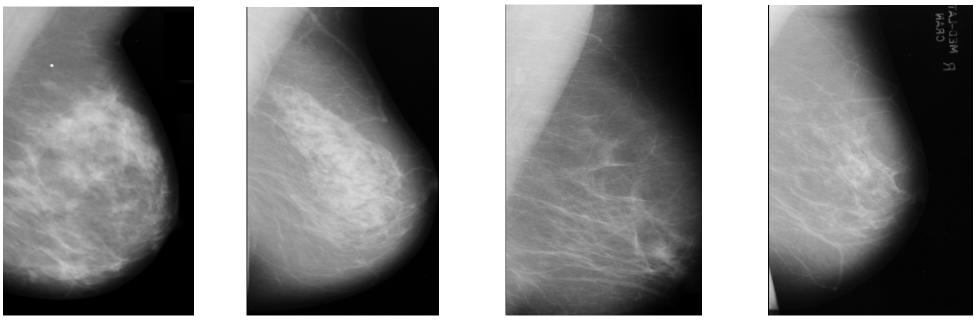


(a) **(b)**

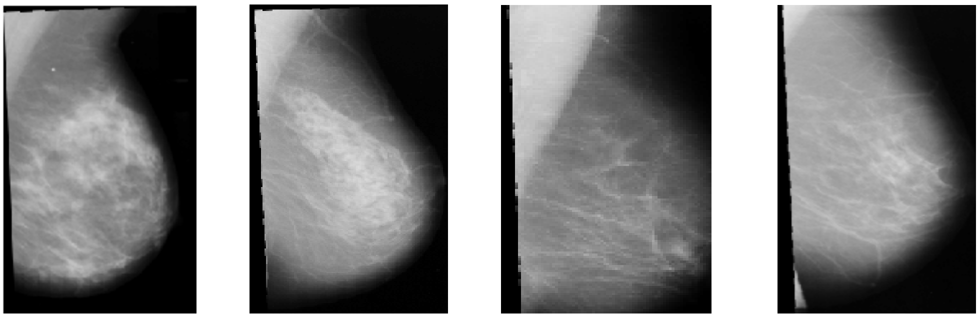
Figure 3 shows an example (mdb 026) of difference image before (a) and after Rigid registration (b) is performed.

VI. OBSERVATION AND CONCLUSION FROM EXPERIMENT RESULTS:

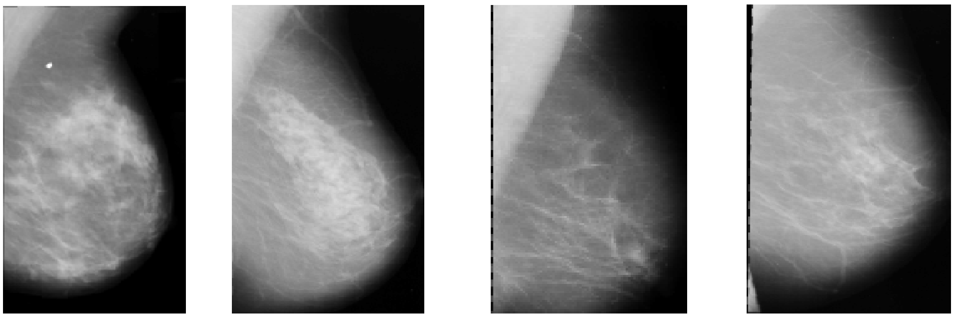
In this paper, we showed a new way to register image by using NTE and fuzzy type II technique. The proposed approach is very efficient for registering mammograms. The advantages of the proposed approach are: 1) Registration is lot better than existing SE NSE and TE technique; 2) Selection of Tsallis parameter is often an ill defined problem in existing techniques, forcing to employ trial and error method. We used Type II fuzzy technique to select q parameter 3) Predicting approximate θ, tx, and ty by visual inspection of misalignment of images leads to faster registration. 4) The proposed approach is validated for its perfect ness using several techniques like L-Norm, CC, SSD, θ error. It is found that our algorithm can register images more accurately and efficiently than some of existing rigid algorithms and it will be useful as Pre-registration step in Non- rigid mammogram registration. The original mammograms and the simulated deformed images were shown. Also the image registration results of Shannon entropy based MI, NSE based MI, TE based MI, NTE based MI registration results were shown.



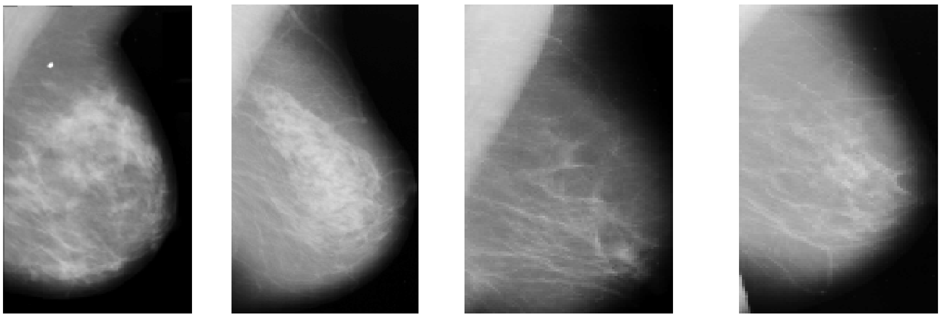
(a)



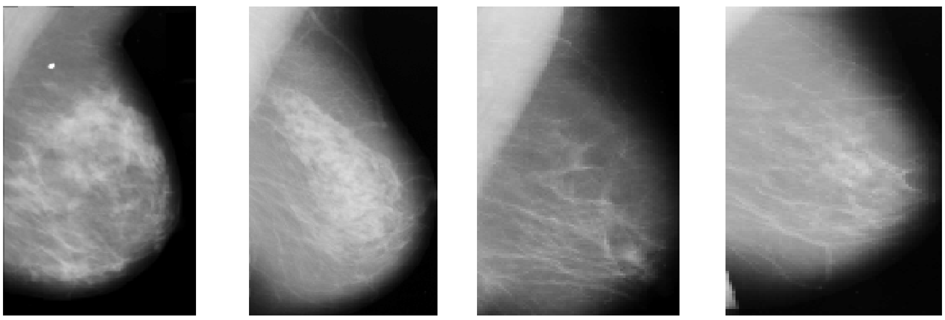
(b)



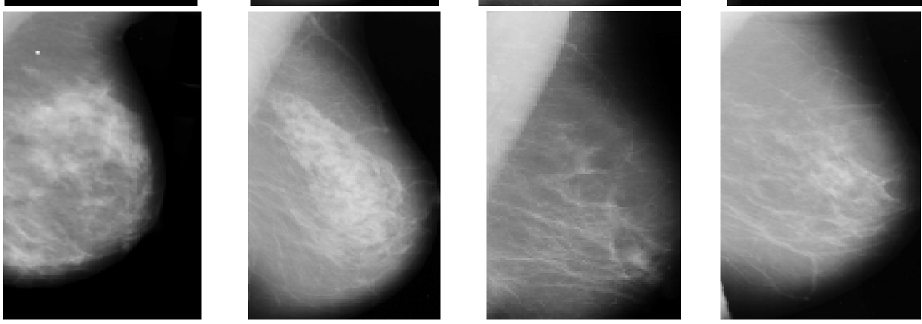
(c)



(d)

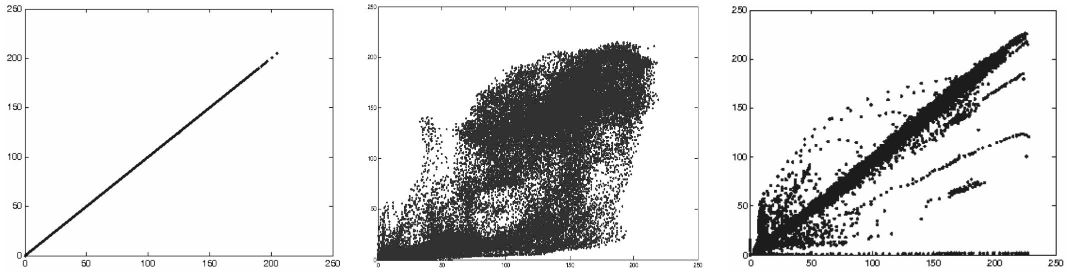


(e)

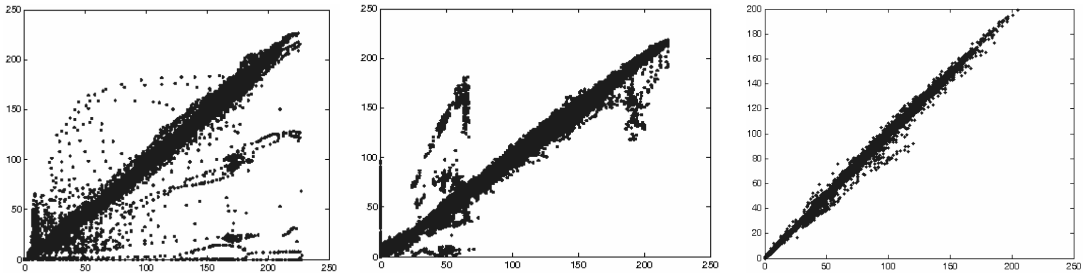


(f)

Figure 2 shows original mammograms(a), temporal mammograms with simulated deformations(b),performance of Shannon MI[2,3] (c), performance of Normalized Shannon MI[4] (d), performance of Tsallis MI [11](e), performance of Normalized Tsallis MI (f),



(a) (b) (c)



(d) (e) (f)

Figure 4 shows the KDE plot of ideal registered mammogram(a), unregistered image(b), Shannon MI(c), Normalized Shannon MI(d), Tsallis MI(e), and Plot of Normalized MI mammograms(f).

TABLE 1 COMPARES OUR APPROACH WITH OTHER APPROACHES [2,3,4,11]

MIAS Mammogram with 512*512 dimension	**Type of technique used**	**SSD**	**CC**	**Timing In secs**	**L-Norm Before Registration**	**L-Norm After Registration**	**θ simulated**	**θ corrected**
Mdb 252	SE**[2,3]**	0.08	0.929	274.283		1163		-2
	NSE**[4]**	0.067	0.948	263.68	$2.779*10^3$	1022	2	-2
	TE***[11]**	0.063	0.952	268.35		1110		-2
	NTE(q=0.9)	0.057	0.973	235.298		783		-2
Mdb022	SE	0.083	0.918	312.23		1533		-3
	NSE	0.065	0.925	303.68	$1.59*10^3$	1125	3	-3.1
	TE*	0.0621	0.956	311.5		1013		-3
	NTE(q=0.9)	0.050	0.963	285.98		851		-3
Mdb026	SE	0.082	0.913	224.3		3163	3	-2.8
	NSE	0.06	0.918	210.02	$2.309*10^3$	989		-3
	TE*	0.067	0.942	218.51		936		-3
	NTE(q=0.8)	0.056	0.953	205.07		533		-3
Mdb024	SE	0.063	0.923	344.65		2635		-2.8
	NSE	0.078	0.908	327.87	$3.339*10^3$	1352	3	-3
	TE*	0.063	0.928	337.33		1105		-3
	NTE(q=0.9)	0.055	0.933	315.8		713		-3

***Indicates q= 4 for TE**

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where we procured MIAS database.

REFERENCES

[1] L. G. Brown, "A survey of image registration techniques," ACM Computing Surveys, vol. 24, no. 4, pp. 325-376, Dec. 1992.

[2] Viola, P., & Wells, W.M., Alignment by maximization of mutual information, 1995, Medical Image Analysis, 1, 1, 17-34.

[3] W. M. Wells, P. Viola, H. Atsumi, S. Nakajima, and R. Kikinis, "Multi-modal volume registration by maximization of mutual information," Med. Image Anal., vol. 1, pp. 35–51, 1996.

[4] C. Studholme, D.L.G. Hill, D.J. Hawkes, An overlap invariant entropy measure of 3D medical image alignment, Pattern Recognition 32 (1999) 71–86..

[5] J. P.W. Pluim, J. B. A. Maintz, and M. A. Viergever, "Image registration by maximization of combined mutual information and gradient information," IEEE Trans. Med. Imag., vol. 19, pp. 809–814, Aug. 2000

[6] D. Rueckert, M. J. Clarkson, D. L. G. Hill, and D. J. Hawkes, "Non-rigid registration using higher order mutual information," Proc. SPIE, Medical Imaging: Image Processing, vol. 3979, pp. 438–447, 2000.

[7] Knops, Z.F., Maintz, J., Viergever, M., Pluim, J.: Registration using segment intensity remapping and mutual information. In: Barillot, C., Haynor, D.R., Hellier, P. (eds.) MICCAI 2004. LNCS, vol. 3216, pp. 805–812. Springer, Heidelberg (2004)

[8] A. Rangarajan, H. Chui, and J. S. Duncan, "Rigid point feature registration using mutual information," Medical Image Analysis, vol. 3, no. 4, pp. 425–440, 1999.

[9] Y. He, A. B. Hamza, and H. Krim, "A generalized divergence measure for robust image registration," IEEE Trans. Sig. Processing, vol. 51, no. 5, pp. 1211–1220, 2003.

[10] M. P. Wachowiak, R. Smolikova, and T. M. Peters, "Multiresolution biomedical image registration using generalized information measures," in Proceedings of Medical Image Computing and Computer Assisted Intervention (MICCAI), Montréal, 2003, pp. 846–853.

[11] S. Martin, G. Morison, W. Nailon, and T. Durrani, " 'Fast and accurate image registration using Tsallis

[12] entropy and simultaneous perturbation stochastic approximation," Electron. Lett., vol. 40, no. 10, pp.595-597, 2004.

[13] C. Tsallis, "Possible generalization of Boltzmann-Gibbs statistics," J. Stat. Phys., vol. 52, pp. 479-487, 1988.

[14] N. Cvejic, C N. Cangarajah, and Bull D. R, "Image fusion metric based on mutual information and Tsallis entropy," Electron. Lett., vol. 42, no. 11, pp.626-627, 2006.

[15] Collignon, A.: Multi-modality medical image registration by maximization of mutual information. Ph.D. thesis, Catholic University of Leuven, Leuven Belgium (1998)M. Portes de Albuquerque etal"Image thresholding using Tsallisentropy"PatternRecognitionLettersVolume 25, Issue 9, 2 July 2004, p 1059-1065

[16] El-Feghi, M. Galhoud, M.A. Sid-Aadhmed, M. Ahmadi, "Three-Level Gray-Scale Images Segmentation using Non-extensive Entropy," Computer Graphics, Imaging and Visualisation (CGIV 2007), p.304-307, 2007.

[17] Abe, S.: Tsallis entropy: How unique? to appear in Continuum Mechanics and Thermodynamics (2003) [cond-mat/0305087].

[18] M. Mendel and R.I. John, "Type-2 fuzzy sets made simple," IEEE Trans. on Fuzzy Systems, vol. 10, pp. 117–127, April 2002

[19] J. L. Bernon, V. Boudousq, J. F. Rohmer, M. Fourcade, M. Zanca, M. Rossi, and D. Mariano-Goulart, "A comparative study of Powell's and downhill simplex algorithms for a fast multimodal surface matching in brain imaging," Comput. Med. Imaging Graph., vol. 25, pp. 287–297, 2001.

[20] J.Mohanalin, P.K.Kalra, Nirmal Kumar"A novel Hyper elastic based Image registration" ; pp.178-183, nscmit,2007.

[21] J.Mohanalin, P.K.Kalra, Nirmal Kumar " A simple Region growing technique for extraction ROI from Mammograms" ICQMOIT, p.231-238; 2008.

[22] R. J. Ferrari, Member, IEEE, R.M. Rangayyan, Fellow, IEEE, J.E.L. Desautels, R.A. Borges, and A.F. Frere, "Automatic Identification of the Pectoral Muscle in Mammograms", IEEE Transaction on Medical Imaging, p: 232 – 245, v.23, n.2, 2004.

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