Vague Metagraph

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Abstract—Aim In this paper the we introduce a new concept of vague metagraph in field of vague theory. Authors presented how to represent this new construct inside the computers memory, and various properties of vague. Method Metagraph. Metagraph is a graph theoretic construct in which set-to-set mapping in place of node to node as in conventional graph structure. Result Vague metagraph is a advanced vaguefication of fuzzy metagraph.

Index Terms— Vague set (VS), vague relation (VR), Metagraph, Fuzzy Metagraph(FM), Vague Metagraph.

I. INTRODUCTION

Graphs are ubiquitous in simplest form a graph consists of set of elements (or nodes) and a set of ordered and unordered pairs of nodes (or edges). A graph is defined by a pair G = {X, E} where X = {x1, x2, x3...xn} is a finite set of vertices and E a collection of edges that happen to connect these vertices. The edge set graphically represented as X û X. The concept of fuzzy graph is the “fuzzyfication of the crisp graphs using fuzzy sets.

A. Fuzzy Graphs

A fuzzy graph G can be defined as triplet (X, Õ, Ý) where Õ is a fuzzy set on X and Ý a fuzzy relation on X û X. A Fuzzy set X on X is completely characterized by its membership function M : X Ñ [0, 1]. For each x õ X, M (x) illustrates the true value of the statement “x belongs to Õ “. A fuzzy edge set Ý is defined as a function P :[0, 1]. Therefore, a fuzzy graph can be described by two functions M and P. For the sake of notational convenience, X is omitted and, thus, the notation G = (Õ, Ý) or G = (M, P) is used where both vertices and edges have membership values. Often, the membership value of an edge is called certainty factor (CF) of the edge. For Simplicity, assign Õ\(x_i\) denoting (x, M ( x)) and Ý\(e_k\) denoting (\(e_k\), CFk), i.e. (\(e_k\), P (\(e_k\))). Over the past years, a number of fuzzy graphs have been proposed to represent uncertain relationship between fuzzy elements and sets of fuzzy elements however, as mentioned before; existing fuzzy graphs are not capable of effectively modeling and directed relationships between sets of fuzzy elements. It is critical that all the graphs lack powerful algebraic analysis methods for manipulating the directed relationships between sets of elements. This motivates the development of FM i.e. Fuzzy Metagraphs. The major distinction between FMs and traditional graph-theoretic constructs is that an FM describes the directed relationships between sets of elements inside of single elements. Each edge in the FM is an ordered pair of sets of elements. So, the edge is neither an ordered pair of elements in a fuzzy directed graph nor a disordered set of elements in a fuzzy hypergraph[11].

II. MATERIAL AND METHOD

A. Metagraph

In 1992 in the classical paper [7], Basu and Blanning introduced the concept of metagraph. In this section we reproduce the preliminaries about metagraph. We begin with few definitions:-

B. Generating set

The generating set of a metagraph is the set of elements X = {x1, x2, . . . xn}, which represent variables of interest.

C. Edge

An edge e in a metagraph is a pair e = ⟨ve, we⟩ õ E (where E is the set of edges) consisting of an invertex Ve õ X and an outvertex We õ X, each of which is a set and may contain any number of elements. The different elements in the invertex (outvertex) are co-inputs (co-outputs) of each other.

D. Metagraph

A metagraph S = ⟨X, E⟩ is a graphical representation consisting of two tuples X and E. Here X is its generating set and E is the set of edges defined on generating sets. The generating set X of the metagraph S i.e. the set of elements X = {x1, x2, x3, ………xn} represents variables and occurs in the edges of the metagraphs [9].

Example 2.1 A metagraph can be understood by the following example:-

S = ⟨X, E⟩ is a metagraph when

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X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\} is the generating set, and E = \{(e_1, e_2, e_3, e_4)\} is the set of edges. The edge set can be specified as shown in Figure-1

E = \{\{\{x_1, x_2\}, \{x_4\}\}, \{\{x_2, x_3\}, \{x_5\}\}, \{\{x_4, x_7\}, \{x_6, x_7\}\}, \{\{x_5, x_7\}\}\}

In-vertex is an operation (function) having one argument which can find out the internal vertices from a given set. For example, In-vertex (\{\{x_4, x_5\}, \{x_6, x_1\}\}) = \{x_4, x_5\}. Out-vertex is another operation (function) having one argument which can find out what are the out vertices from the given set. For example, Out-vertex (\{\{x_4, x_5\}, \{x_6, x_1\}\}) = \{x_6, x_1\}.

The edges of the metagraph are labeled as

\[ e_1 = \{\{x_1, x_2\}, \{x_4\}\}, \]
\[ e_2 = \{\{x_2, x_3\}, \{x_5\}\}, \]
\[ e_3 = \{\{x_4, x_5\}, \{x_6, x_7\}\}, \]
\[ e_4 = \{\{x_5, x_7\}\}. \]

\[ \text{Figure-1 Metagraph} \]

**III. FUZZY METAGRAPH**

The fuzzy metagraph is a generalization of metagraph concept proposed, can be defined as follows [11].

**A Definition Fuzzy Metagraphs**

Consider a finite set X = \{x_i, i = 1, 2, …., I\}. A fuzzy metagraphs is a triplet \( \tilde{S} = \{X, \tilde{X}, \tilde{E}\} \) where \( \tilde{X} \) is a fuzzy set on X and \( \tilde{E} \) is a fuzzy edge set (\( \tilde{e}_k, k = 1,2,3,…….K\)). Each Component of \( \tilde{e}_k \) in \( \tilde{E} \) is characterized by an ordered pair \( \{\tilde{V}_k, \tilde{W}_k\} \), in pair \( \tilde{V}_k \subseteq \tilde{X} \) is the invertex of \( \tilde{e}_k \) and the \( \tilde{W}_k \subseteq \tilde{X} \) is the outvertex[7][8]. The co-input of any \( \tilde{x} \in \tilde{V}_k \) is \( \tilde{V}_k \{\tilde{x}\} \) and the co-output of any \( \tilde{x} \in \tilde{W}_k \) is \( \tilde{W}_k \{\tilde{x}\} \).

\[ \text{Figure-2 Fuzzy Metagraph without Cycle} \]

**IV. VAGUE SETS**

Vague sets, defined by Gau and Buehrer have an extra potential edge over fuzzy sets of Zadesh. Vague sets are higher order fuzzy sets. Application of higher order fuzzy set make the solution procedure more complex, but if the complexity on computation-time, computation -volume or memory -space are not the matter of concern then a better results could be accomplished. Let U be a universe, say the collection of all students of Delhi High School. Let A be a vague set of all “good-in-maths students” of the universe U, and B be a fuzzy set of all “good-in-maths students” of U.

Suppose that an intellectual Manager M1 proposes the membership value \( \mu_B(x) \) for the element x in the fuzzy set B by his best intellectual capability. On the contrary, another intellectual Manager M2 proposes independently two membership values \( t_A(x) \) and \( f_A(x) \) for the same element in the vague set A by his best intellectual capability. The amount \( t_A(x) \) is the true-membership value of x and \( f_A(x) \) is the false-membership value of x in the vague set A. Both M1 and M2 being human agents have their limitation of perception, judgment, processing-capability with real life complex situations. In the case of fuzzy set B, the manager M1 proposes the membership value \( \mu_B(x) \) and proceed to his next computation. There is no higher order check for this membership value in general. In the later case, the manager M2 proposes independently the membership values \( t_A(x) \) and \( f_A(x) \), and makes a check at this base-point itself by exploiting the constraint \( t_A(x) + f_A(x) \leq 1 \). If it is not honored,
the manager has a scope to rethink, to reshuffle his judgment procedure either on ‘evidence against’ or on ‘evidence for’ or on both. They are mathematically not independent. They are mathematically constrained. This is the breaking philosophy in Gau and Buehrer’s theory of vague sets [4]. There are a number of applications of vague theory reported in the literature, viz. Chen[2] suggested a method of calculating system reliability using vague set theory. Wang, Liu and Zhang[3] measured roughness of a vague set. In the present paper we introduce the notion of vague relations, study some operations on them, define compositions of vague relations, and show an application of vague relations.

V. RESULT

A. Vague Metagraph

In this section we make some characterizations of the nonlinear data structure vague metagraphs. Since the notion of this new data structure is still at baby stage, we need to introduce a number of new terminologies which will be useful in our work here and in future, specially to study the various properties of vague metagraphs.

B. Vague Metagraphs

Consider a finite set $X = \{x_1, x_2, \ldots, x_n\}$. A vague metagraph is a triplet, $\tilde{S}_v = \{X, \tilde{X}_v, \tilde{E}_v\}$ where $\tilde{X}_v$ is a vague set on X and $\tilde{E}_v$ is a vague edge set $\{\tilde{e}_{kv}, k = 1,2,3,\ldots, K\}$. Each component of $\tilde{e}_{kv}$ is characterized by an ordered pair $\langle \tilde{V}_k, \tilde{W}_k \rangle$, $\tilde{V}_k \subseteq \tilde{X}_v$ is the invertex of $\tilde{e}_{kv}$ and $\tilde{W}_k \subseteq \tilde{X}_v$ is the outvertex. The co-input of any $\tilde{x}_v \in \tilde{V}_k$ is $\tilde{V}_k \{ \tilde{x}_v \}$ and the co-output of any $\tilde{x}_v \in \tilde{W}_k$ is $\tilde{W}_k \{ \tilde{x}_v \}$. The VM adjacency matrix of Figure -3 will be as follows.

$$
\begin{array}{ccc}
\tilde{x}_v & \tilde{x}_w & \tilde{x}_z \\
\{\{v_1, \{x_1, x_2\}\}\} & \phi & \phi \\
\{\{v_2, \{x_3, x_4\}\}\} & \phi & \phi \\
\{\{v_3, \{x_5, x_6\}\}\} & \{\{v_4, \{x_7, x_8\}\}\} & \{\{v_5, \{x_9, x_{10}\}\}\}
\end{array}
$$

C. Incidence Matrix of a Vague Metagraph

The incidence matrix $I$ of a vague metagraph has one row for each of the element in the vague generating set $\tilde{X}_v$ and one column for each edge. The $ij$th component of $I$, $i\neq j$, is -1 if $\tilde{x}_v$ is an invertex of $\tilde{e}_{jv}$, it is +1 if $\tilde{x}_v$ is in the outvertex of $\tilde{e}_{jv}$, and it is $\phi$ otherwise. The incidence matrix of Figure-3 will be as follows.

$$
\begin{array}{cc}
\tilde{x}_v & \tilde{x}_w \\
-1 & \phi \\
-1 & \phi \\
-1 & -1 \\
\phi & -1 \\
\phi & +1 \\
\phi & +1
\end{array}
$$

3. Representation of a Vague Metagraph inside Computer Memory

Vague Metagraph is a new concept of data structure in computer science. Therefore, for any kind of hard and soft computation with vague metagraph, we need to know how to store a vague metagraph in computer memory, how to access it or its components from computer memory.

Vague Metagraph can be represented in computer memory in the form of any of the two matrices:-

(i) The Incidence matrix
(ii) The Adjacency matrix

Each of these matrices is a complete representation of vague metagraph and can be derived from them. Our basic assumption here is that X and E are finite sets.

Adjacency matrix of VM

The Adjacency matrix A of an VM is a square matrix with rows and columns labeled by the elements in the vague sets. Each entry $a_{ij}$ in the matrix is a null set or a few ordered triplet according to whether $\tilde{x}_v$ and $\tilde{x}_w$ are adjacent or not and how many edges connect $\tilde{x}_v$ to $\tilde{x}_w$. In each triplet
the first component is the co-input of $\tilde{x}_{iv}$ and the second is the co-output of $\tilde{x}_{jv}$, third is a simple path from $\tilde{x}_{iv}$ to $\tilde{x}_{jv}$ with a length of one.

C Closure of Adjacency Matrix of Vague Metagraph

Given a finite Set Xv = \{ $\tilde{x}_{iv}$, i = 1, ..., L \} and an VM \( \tilde{S}_v = \{ X_v, \tilde{x}_v, \tilde{E}_v \} \) with $\tilde{E}_v$ being a fuzzy edge set \( \{ \tilde{e}_{kv}, k = 1, ..., K \} \), the VM adjacency matrix $A$ of $\tilde{S}_v$ is an $I \times 1$ matrix. For $i,j \in \{ 1, ..., I \}$, each entry $a_{ij}$ is defined as:

$$a_{ij} = \bigcup_k (\alpha_k)_{kv}$$

Where

\( (\alpha_k)_{kv} = \{ (v_{kv} \setminus x_{iv}), (w_{kv} \setminus x_{iv}), \{ x_{iv} \}, \{ E_{kv} \} \} \)

$\phi$ Otherwise

The closure of adjacency matrix

The adjacency matrix only shows the adjacent linkages in the graph. There may be many other paths existing, but not visible from the adjacency matrix of VM. If $\tilde{x}_{iv}$ is adjacent to $\tilde{x}_{jv}$ and $\tilde{x}_{jv}$ is adjacent to $\tilde{x}_{kv}$, then it can be inferred that there is a path with the length two from $\tilde{x}_{iv}$ to $\tilde{x}_{kv}$. The closure of adjacency matrix can be developed by all paths of any length connecting two arbitrary vertices $\tilde{x}_{iv}$ to $\tilde{x}_{jv}$ if exists. The Vague Metagraph closure matrix $A^*$ is formed by adding the successive power to Adjacency matrix, namely, the multiplication by itself. Square of Adjacency matrix $A^2$ of vague Metagraph shown in Table-3

<table>
<thead>
<tr>
<th>$S_v$</th>
<th>$S_v^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{S}_v$</td>
<td>$\tilde{S}_v^2$</td>
</tr>
</tbody>
</table>

Closure matrix $A^*$ of vague Metagraph of Figure-1 is represented in Table-4.

<table>
<thead>
<tr>
<th>$S_v$</th>
<th>$S_v^2$</th>
<th>$S_v^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{S}_v$</td>
<td>$\tilde{S}_v^2$</td>
<td>$\tilde{S}_v^3$</td>
</tr>
</tbody>
</table>

Every graph traversal algorithm is based on the closure and adjacency matrix of a graph, so the these two matrices have lots of advancements and applications in the various fireld.

D. Singular Vague Metagraph

A single edge Vague metagraph is called a singular Vague metagraph.

E. Null Vague Metagraph

A Vague metagraph $\tilde{S}_v = \{ X_v, \tilde{x}_v, \tilde{E}_v \}$ is called a null Vague metagraph if $\tilde{E}_v = \emptyset$ and $\tilde{E}_v = \emptyset$.

A null Vague metagraph is denoted by the symbol $\Phi$. Clearly, $\tilde{S}_v = \{ \tilde{x}_v, \emptyset \}$ is a Vague metagraph but not a null Vague metagraph. But $\tilde{S}_v = \{ \emptyset, \tilde{E}_v \}$ is not a defined Vague metagraph, and hence is not a null Vague metagraph.

F. Vague Submetagraph

A Vague metagraph $\tilde{S}_v = \{ \tilde{x}_v, \tilde{E}_v \}$ is called to be a Vague submetagraph of the Vague metagraph $\tilde{S}_{2v}$ = $\{ \tilde{x}_{2v}, \tilde{E}_{2v} \}$ if the following conditions are true:

i) $\tilde{x}_{iv} \subseteq \tilde{x}_{2v}$, and

ii) $\tilde{E}_{iv} \subseteq \tilde{E}_{2v}$

Using conventional notation (as in Set Theory), we write $\tilde{S}_v \subseteq \tilde{S}_{2v}$ to mean that $\tilde{S}_v$ is a fuzzy submetagraph of $\tilde{S}_{2v}$. In that case, we can also say that $\tilde{S}_{2v}$ is a Vague super metagraph of the Vague metagraph $\tilde{S}_v$. It is important to mention here that if $\tilde{S}_v = \{ \tilde{x}_v, \tilde{E}_v \}$ is a Vague metagraph then any subset $\tilde{A}_v \subseteq \tilde{x}_v$ together with any subset $\tilde{F}_v \subseteq \tilde{E}_v$ need not necessarily produce a Vague submetagraph $\{ \tilde{A}_v, \tilde{F}_v \}$ of $\tilde{S}_v$.

VI Conclusion

Vague Metagraph is a graphical model that not only visualized the process of any system but also their formal analysis where the analysis will be accomplished by means of an algebraic representation of the graphical structure.

The graphical structure can be represented by the adjacency and incidence matrix of a Vague metagraph. Vague Metagraphs have lot of applications in the field of information processing systems; decision support systems, models Management of database and rule base Management of work flow systems in which the work consists of information processing tasks to

REFERENCES


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